3-D NUMERICAL HEAT TRANSFER ANALYSIS IN A INDUSTRIAL VAT CHEESE

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Abstract. *During cheese production, one of the most important process variables to be controlled and monitored is the cooking temperature. Therefore, a suitable temperature distribution inside the cheese vat and milk is an important point to guarantee the final cheese characteristic, because there is a close relationship between homogeneity, temperature, concentration and cheese quality. Attempting for temperature distribution control, thermal sensors must be carefully mounted and those response must be representative for entire system. This paper presents a heat transfer analysis for a closed double "O" model cheese vat, with 5000 liters capacity operating in batch process. The initial process of cheese production occurs in transient heat transfer process, and this stage was formulated using an analytical solution based on first thermodynamics law, where mean time and temperature in heating process were calculated for entire milk mass, and for permanent regime heat transfer process the finite element method were used for detail the hot water distribution inside the equipment channel. Several boundary conditions for inlet hot water mass flow rate and temperature were considered, isotropic and constant materials properties were also considered. Temperature distribution charts for transient stage of milk heating, temperature distribution at equipment walls for permanent regime stage, water velocity distribution inside equipment distribution channel and Nusselt number along symmetric base line of the equipment are presented as results of this study.*

Key words: *CFD, cheese vat, food industry, cheese production.*

1. INTRODUCTION

In general, greater the system, greater are the difficulties to keep it stable. In industrial equipment control, this is not exception. Large scale productions are always working with large mass quantity, volume, flow rate, power consumption, and of course large chemical, thermal and mechanical inertia. Those are some of the variables that brings difficulty to stabilize any system and with it, industrial process error control are more often. Control process instruments just like manometers, thermometers, fluid flow measuring devices are used together with valves, pumps, motors and computer systems to assure that the right operation condition is really happening. Information about how is going the process is vital to any efficient control system works and a better result will be consequence of how many and what kind of information is available. In a vat cheese, the large amount of product that is processed at time, passing through heating and agitation, gives to the equipment an inertial characteristic that may difficult the temperature and homogenization control of milk, two of the main properties that is important to observe during the milk cooking and that will have consequences for cheese quality. This is the importance of knowing how the heat transfer process and temperature distribution happens in the equipment.

The use of computational tools for food industry equipment design and optimization is something new compared to other engineering areas, but is clear that the detail level of information is grater than relative analytical solutions, and it's really helpful to find better solutions. Scott and Richardson (1997) presents a brief review about some cases in food industry where the use of computational tools applied to industrial oven design, it could be optimized after a detail study of heat and mass transfer simulation inside the equipment. Xia and Sun (2002) also presents a wider discussion about the use of computational tools to industrial equipment design, like ovens, refrigerated spaces as freezers and cold chambers, spray dryers, heat exchange equipment, pumping systems and fluid flow inside tubes, drying and sterilization equipment and clean room air distribution. Some of the main advantages of using computational tools for engineering problems are listed by Wanot (1996) as follow:

- The computational analysis allows a detailed understanding of the fluid flow, heat, mass and particulate distribution, mass balance, etc. With this kind of information, engineers can understand more deeply what is really happening in a particular process.
- Makes possible an evaluation of behavior change if geometrical changes are implemented in less time and cost compared to traditional experimental methods.
- Makes possible to answer "If" questions in a short time and with high quality of results.
- Makes possible the reduction of scale problems because mathematical models are based on fundamental equations, which are scale independent.

- The use of computational tools are particularly interest to simulate conditions where direct evaluation of some property is not possible by traditional techniques of measuring, as high temperature and corrosive mean.

2. CHEESE PRODUCTION PROCESS DESCRIPTION

In cheese production process, one of the stages is the cooking of milk. Some acidifying agents like enzymes and bacterial culture are addicted to produce the cheese curd with particular characteristics like smell, flavor, texture, that later will be separated into solids and liquid whey. After this, the mass is cut, drained, salted, flavored, pressed and molded. The flow chart presented in Fig. 1, describes the initial stages of cooking and draining.

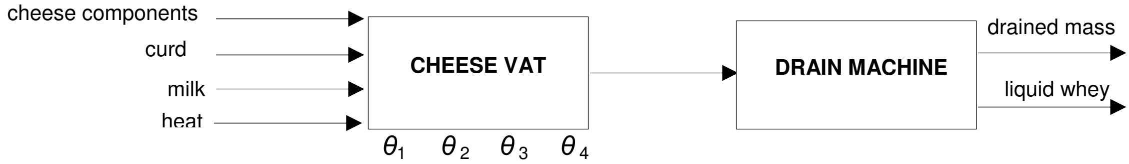


Figure 1. Cooking process and draining fluxogram

At time θ_1 all the ingredients are added to the equipment, wich have an agitation system that keeps the milk homogeneous at the same time thath is cooking. At time θ_2 the agitaion is stoped, the milk mass is put to rest for a little time to growing grains. At time θ_3, begins the cutting process of the grains with rotating knives, and at time θ_4 is finished the cooking time, the cutted grains is transported to drain machine, where part of the liquid whey is separated, then the drained mass is ready to go to the press stage, cut in the final size , salted and molding.

3. GOVERNING EQUATIONS, GEOMETRY AND BOUDARY CONDITIONS

The cooling law of Newton, is a general formulation that can be used to describe a general process of convective heat transfer between solid and fluid means. This equation applied to hot water stream and internal side of the equipment wall during the heating process, can be written as follow, Eq. (1):

$$q_{hw} = h_{hw} A_{hw} \Delta T_{hw} \tag{1}$$

where the subscript indicates hot water interaction with internal metallic plate.

For transient formulation, the 1st Law of Thermodynamics can be applied considering a closed control volume where the milk is inside, and the energy balance can be written as:

$$d\dot{q} = \frac{dq}{d\theta} = m_{milk} c_{milk} \frac{dt}{d\theta} = UA_{hw} \Delta t \tag{2}$$

Integrating the Eq. (2) at time θ, leads to a global transient formulation that describes the heating process in a jacket recipient, found in Kern (1999).

$$\ln\left(\frac{T_1 - t_1}{T_1 - t_2}\right) = \frac{UA_{hw}\theta}{m_{milk} c_{milk}} \tag{3}$$

where: T_1, T_2, t_1, t_2, are respectively inlet, outlet hot water temperatures, initial, final milk temperature
U is the global coefficient of heat transfer between internal metallic plate and milk
A_{hw} is the convective heat transfer area between internal metallic plate and milk
m_{milk} is the milk mass quantity
c_{milk} is the milk specific heat
θ is the heating time

Again, the 1st Law applied to hot water stream, leads to:

$$\dot{q}_{hw} = \dot{m}_{hw} c_{hw} (T_2 - T_1) \tag{4}$$

The energy balance for hot water stream and milk, results that the energy liberated by hot water is the same quantity of energy that is received by the milk, then:

$$\dot{m}_{hw} = \frac{UA_{hw}(t_2 - t_1)}{c_{hw}(T_2 - T_1)} \tag{5}$$

The conditions considered to Eq. (2) are:

- U, constant and homogeneous for entire process of heating.
- Constant properties.
- Constant inlet hot water temperature.
- The equipment provides a homogeneous distribution for the milk, temperature, concentration, etc.
- No phase change.
- Small external heat loss.

After the heating process, the milk is maintained under a controlled temperature condition for a period of time that is characteristic for each kind of cheese, and the energy lost in the equipment is associated to convective heat lost to environment and water milk evaporation. Since the equipment is covered by a layer of isolate thermal material and is closed by a superior metallic shell, is reasonable to considering that the heat transfer to environment is very small compared to heat transfer associated to water vapor lost by milk evaporation and convective heat transfer to environment. Under these conditions, this stage was considered as a permanent regime, and the analysis was performed using finite element method.

The water and milk thermal-physical properties considered are listed on Tab. (1).

Table 1. Water and milk thermal-physical properties

	$\rho\left[\frac{kg}{m^3}\right]$	$k\left[\frac{W}{mK}\right]$	$c_p\left[\frac{J}{kgC}\right]$	$\mu\left[\frac{Ns}{m^2}\right]$
Water (80 °C)	971,80	0,57	4210,00	0,32x10^{-3}
Milk-integral (60 °C) *grass (4,5%), solid (22%)*	1030,00	0,56	3900,00	2,12x10^{-3}

The hot water flow inside the distribution channel inside the equipment, and the temperature distribution were solved considering the following equations:

- Mass balance equation: $$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0 \tag{6}$$
- *Momentum* equation, for **x** coordinate:

$$\frac{\partial(\rho uu)}{\partial x}+\frac{\partial(\rho vu)}{\partial y}+\frac{\partial(\rho wu)}{\partial z}=\rho g_x-\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) \tag{7}$$

similar equations can be written for **y** and **z** coordinates.

- Energy equation

$$\frac{\partial}{\partial x}(\rho uc_pT)+\frac{\partial}{\partial y}(\rho vc_pT)+\frac{\partial}{\partial z}(\rho wc_pT)=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) \tag{8}$$

A schematic figure of a double "O" cheese vat, showing the agitation system, the hot water inlet and outlet, structured wall composition and inside water distribution system is presented in Fig. (2) and Fig. (3).

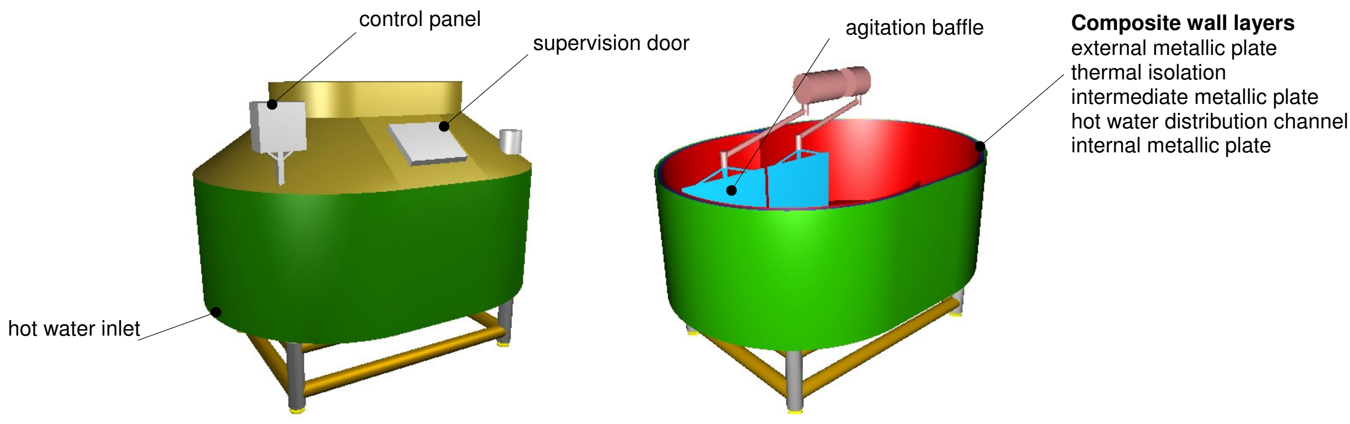


Figure 2. Cheese vat sketch.

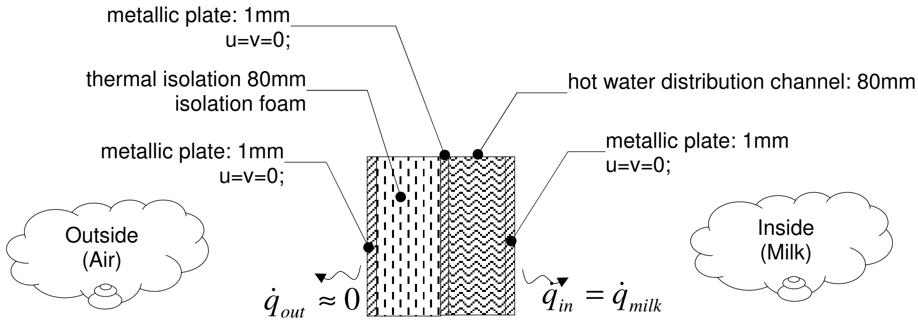


Figure 3. Composite wall layers and finite element boundary conditions

Finite element method was used to solve the hot water distribution inside the multi layer wall and the thermal interaction with inside metallic plate through that the heat is transferred to milk. The general boundary conditions used to this solution are:

- No slip wall: $\vec{V} = 0$
- Adiabatic external wall.
- Hot water inlet conditions are known, described at Tab. (2).

Table 2: Hot water flow rate boundary conditions for finite element solution

	$\dot{m}_{in}$ [kg/h]	T_{in} [°C]	h_{hw} [W/m^2]	$\dot{m}_{out}$ [kg/h]	$\left.\frac{dT}{dn}\right\|_{out}$
Case 1	435,0	80	50	obtained by mass balance equation for entire model	0
Case 2	435,0	70	50		0
Case 3	435,0	60	50		0
Case 4	290,0	80	50		0
Case 5	290,0	70	50		0
Case 6	290,0	60	50		0

h_{hw}: convective heat transfer coefficient associated to hot water/internal metallic plate interaction

4. NUMERICAL SOLUTION METHOD

For numerical solution based on finite element method, commercial computational software was used and a global convergence criteria considered was a global iteration error less than 10^{-8}. The computational mesh generated for this problem has about 640109 elements and 614955 nodes. Some mesh details are shown on Fig. (4).

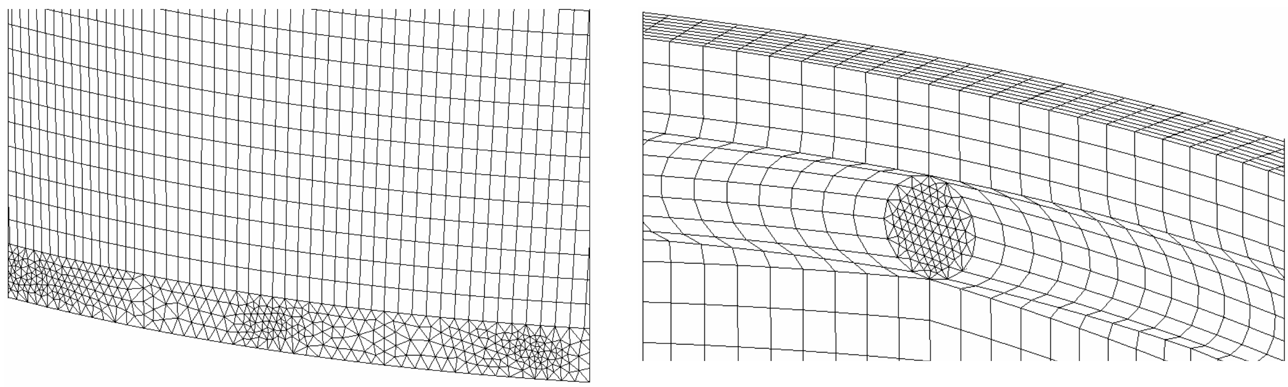


Figure 4. Detail of computational mesh for hot water distribution channel

The temperature difference between the two faces of a metallic plate, that is made with inox steel ANSI 304 with 1,0 mm thickness, was considered very small to thermal analysis because the high thermal conductivity of the steel, $k=14,9$ $[W/mK]$, but it was considered for fluid dynamics effects.

5. RESULTS

At the beginning of the productive process, the heat transferred to milk is characterized by a transient phenomena and the analysis at this stage was made using Eq. (2) and (5). The heating process described by Eq. (2), after the respective boundary conditions and material properties applied, can be written as:

$$\theta = \frac{m_{milk} c_{milk}}{UA_{hw}} \ln\left(\frac{T_1 - t_1}{T_1 - t_2}\right) = \frac{1539080{,}45}{U} \ln\left(\frac{T_1 - 30}{T_1 - t_2}\right) \quad (9)$$

$$\dot{m}_{hw} = \frac{UA_{hw}(t_2 - t_1)}{c_{hw}(T_1 - T_2)} = 3{,}09 \times 10^{-3} U \frac{(t_2 - 30)}{(T_1 - T_2)} \quad (10)$$

for: m_{milk}=5150,0 kg, A_{hw}=13,05m^2, T_1 according to Tab.(2) data; t_1=30 ^{o}C is the milk initial temperature

Figure (5) and (6) shows the Eq. (9) and (10) solutions, that describes the transient heating process. The sequence of use for these charts is presented below:

1. The final milk temperature is chosen as well your respective process time.
2. With item (1) values, the nearest curve UxT_1 is found in transient heating chart.
3. With UxT_1 curve, at the hot water flow charts, reads the water flow rate necessary as function of the final milk temperature and outlet water temperature.

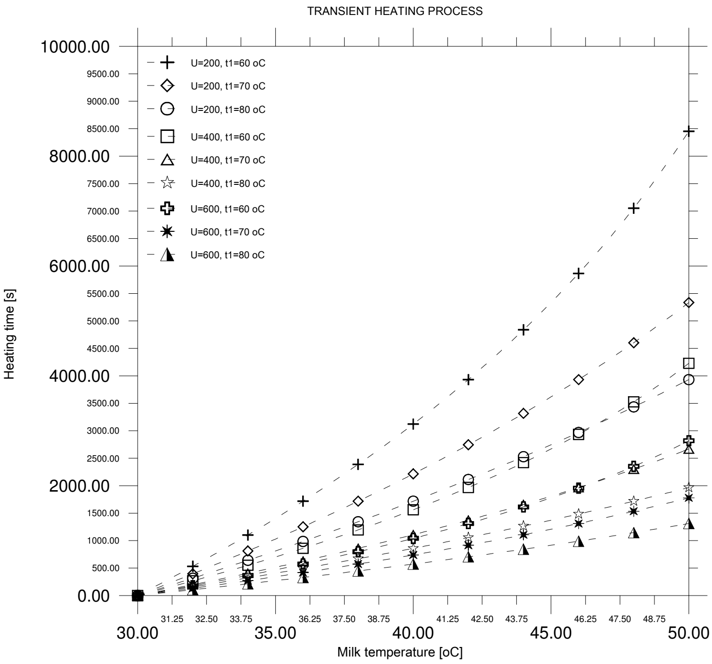
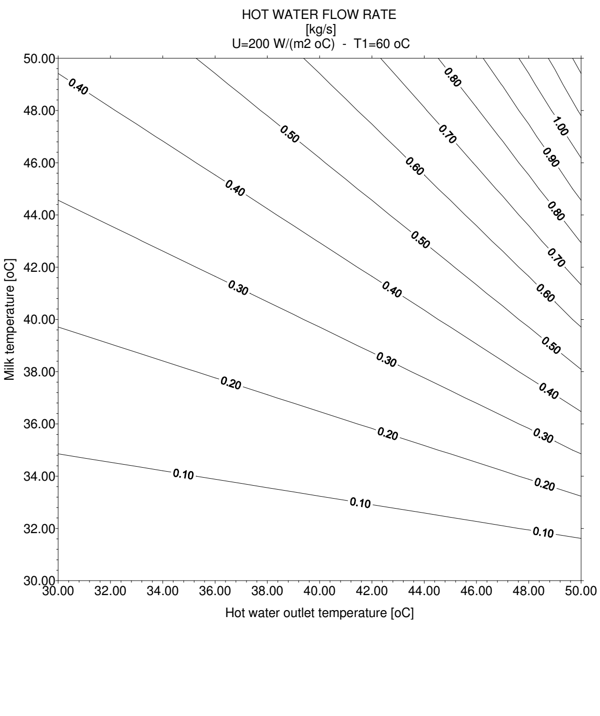
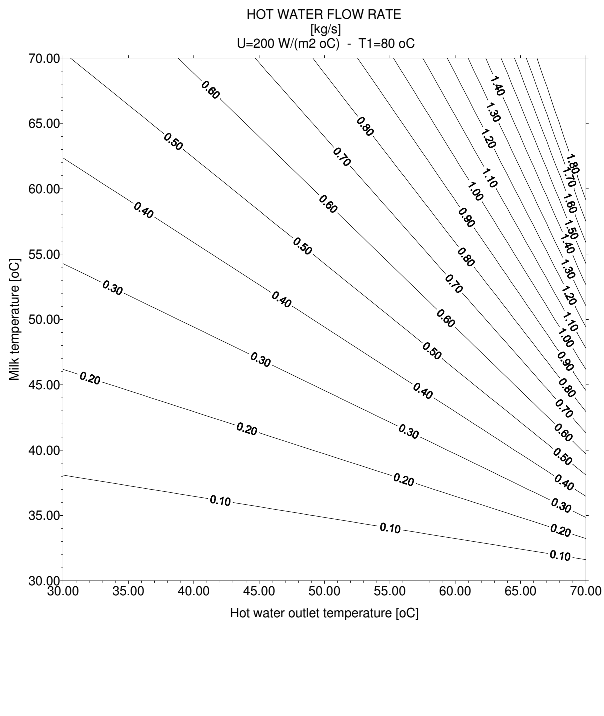


Figure 5. Transient heating process





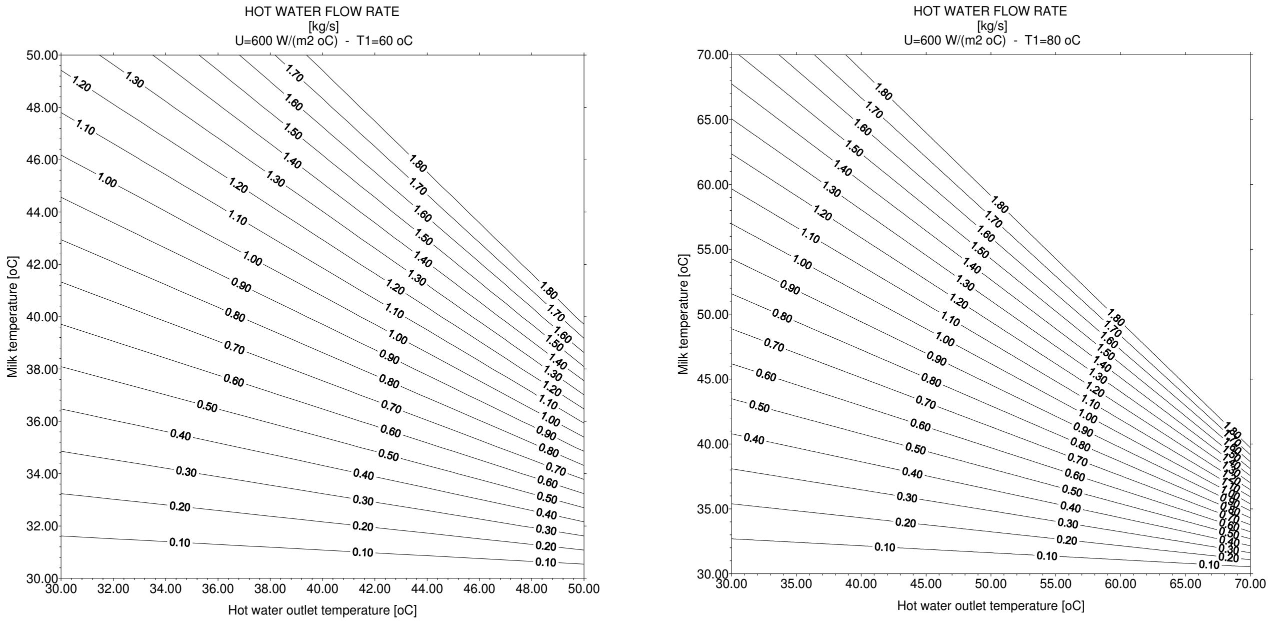
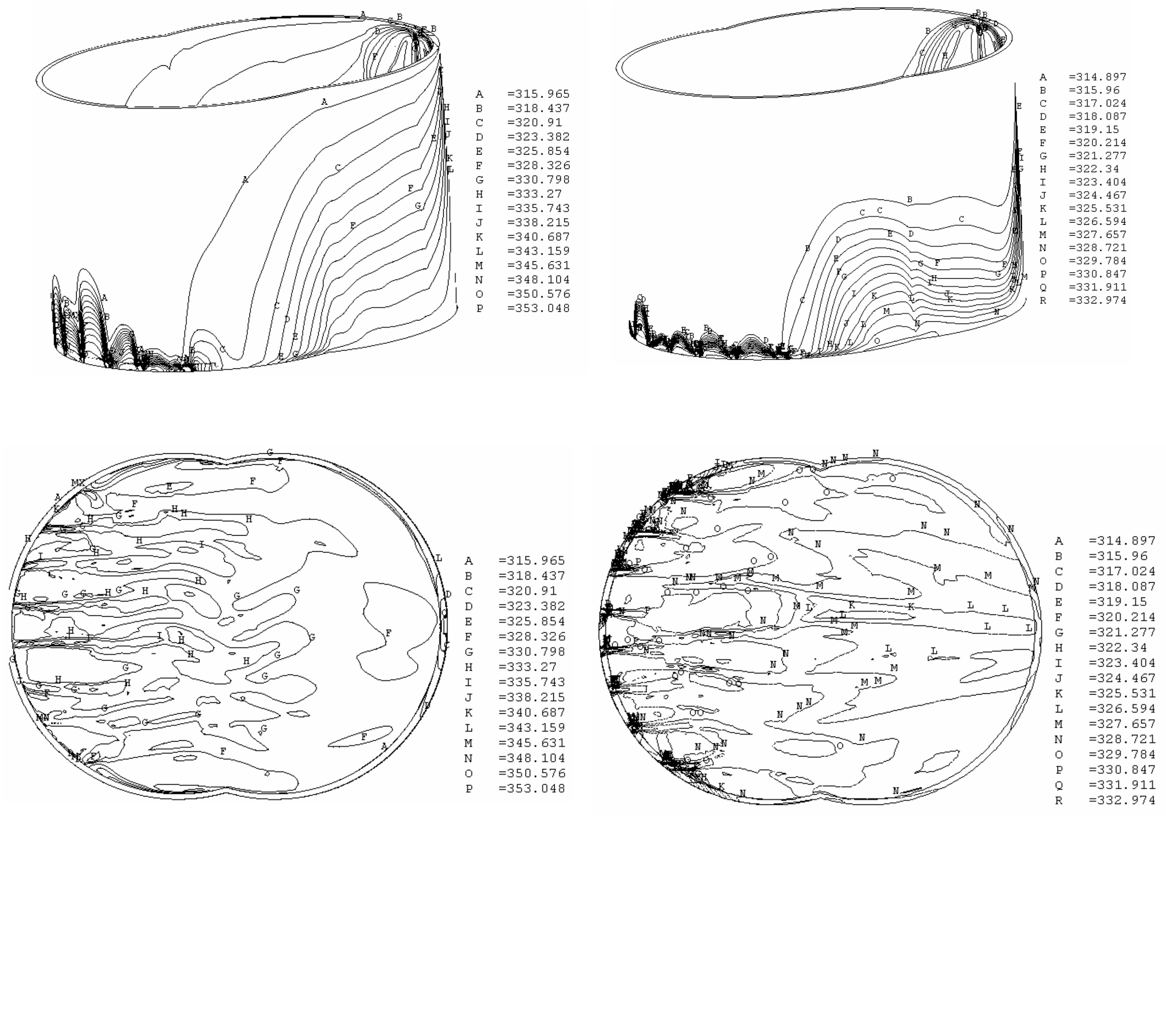


Figure 6: Hot water flow rate

The results obtained for permanent stage of productive process, describing the temperature distribution of the hot water stream and composite wall, internal and external plate, isothermal curves at the equipment base cutting plane, and hot water velocity distribution at the stream are shown at Fig. (6). The Nusselt distribution presented at Fig. (7), was calculated along the symmetry base line of the equipment and used numerical results obtained by finite element solution.



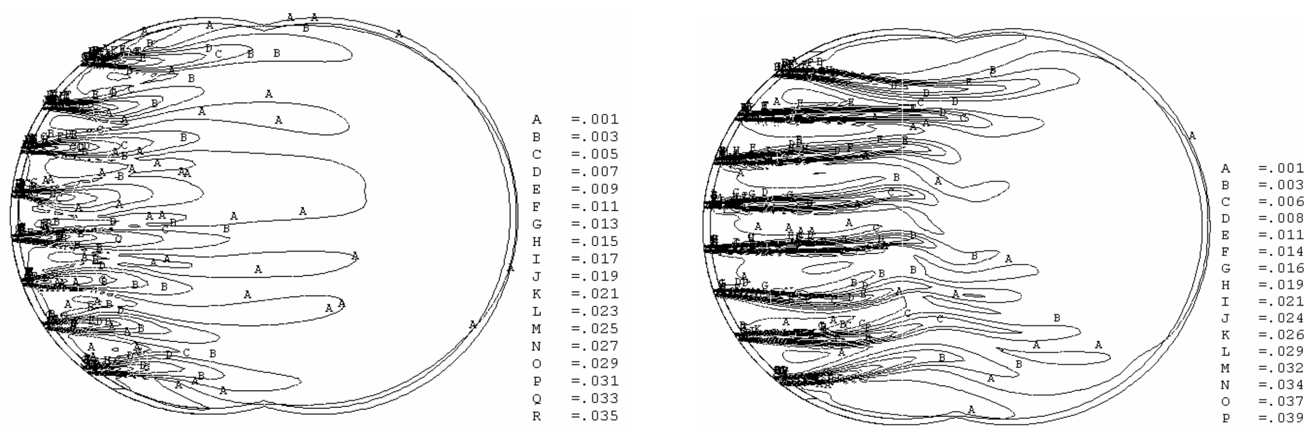


Figure 7. Surface isothermal curves, base temperature and velocity distribution. Left (case 1), right (case 6). Velocity [m/s], Temperature [K]

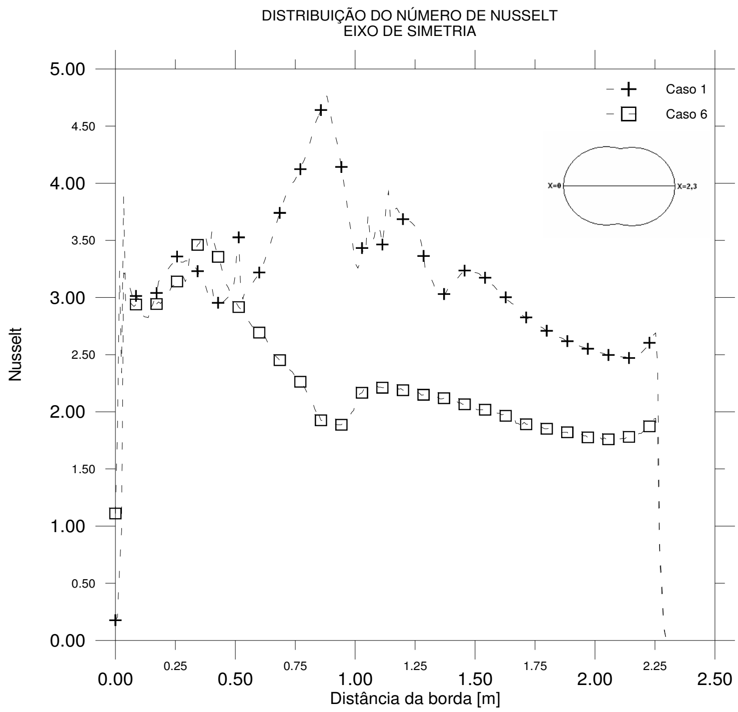


Figure 8. Nusselt distribution along the symmetry line.

The Nusselt definition says that this number means a non dimensional temperature gradient at wall, and according to Chang (1987), it can be numerically calculated using an interpolation scheme presented in Fig. (9).

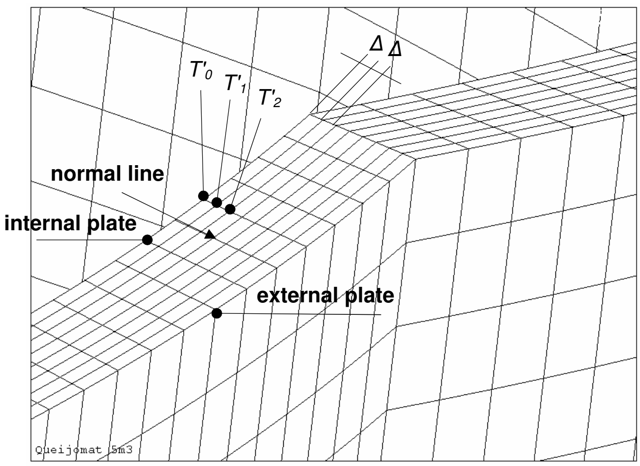


Figure 9. Numerical scheme for numerical Nusselt calculation on surface wall.

$$Nu_{wall} = \frac{h_{wall} L}{k_{fluid}} = \frac{1}{2\Delta}\left[-3T'_0 + 4T'_1 - T'_2\right] \quad (11)$$

where T' is non dimensional temperature and Δ is non dimensional step along the surface normal line.

The mean outlet water temperature and total heat flux in the equipment for each case at permanent regime, are presented at Tab. (3).

Table 3. Mean hot water outlet temperature and heat flux

	T_{in} [C]	T_{out} [C]	Q [kW]
Case 1	80,00	63,61	8,28
Case 2	70,00	57,54	6,29
Case 3	60,00	52,85	3,61
Case 4	80,00	59,91	10,15
Case 5	70,00	55,79	7,18
Case 6	60,00	50,78	4,66

6. CONCLUSION

The numerical analysis presented in this paper, provides an easy way to determine the heat balance to supply the energy needs in a cheese production process, specifically the cooking milk stage. The heating process which is the measure of the temperature raising can be determined by the charts presented at Fig. (4) and (5), and this is an important point to control the cheese final quality. The hypothesis of homogeneous temperature of the milk, can be achieved if is used an agitation baffles, which one is used to cut the curd as well. In fact, this solution can be applied to any heating process that follows the same conditions presented here. The wall temperature distribution charts, reveals that upper and closest regions from the hot water inlet point are the coldest regions of the equipment, and this happens because there is a bad water distribution inside the equipment, which could be get better if some baffles were used to lead the water stream. The temperature distribution is a important point to be observed in question to positioning the temperature sensor that controls the water valves that controls the bath temperature.

The Nusselt number along the symmetry line at the base of the equipment presents a irregular distribution, but keeps between 2,0 and 4,0, and this happens because the temperature distribution at the base is irregular as well, the water flow is not well behavior. The use of computational tools has been very important to obtain a detail solution of temperature distribution of the equipment, and with this kind of information is possible to suggest project changes to get better results.

7. REREFENCES

Chang, M. W., and B. A. Findlayson; "Heat Transfer in Flow Past Cylinders at Re<150: Part 1, Calculations for Constant Fluid Properties", Numerical Heat Transfer, vol. 12, pp. 179-195, 1987.

Incropera, F. P., DeWitt, D. P,; "Fundamentos da Transferência de Calor e Massa", LTC Editora, 4a Edição, 1998.

Kern, D. Q.; "Processos de Transferência de Calor", Compañia Editorial Continental, S.A. de C.V., México, 31ª Reimpressión, 1999.

Perry, R. H.; "Perry´s Chemical Engineer´s Handbook", 7th Edition, Mc Graw Hill, 2000.

Scott, G., Ricahrdson, P.; "The Application of Computational Fluid Dynamics in the Food Industry", Trends in Food Science and Technology, vol. 8, pp. 119-124, April 1997.

Wanot, J.; "Computational Fluid Dynamics Methods in Ship Design", R&D Projects, Germany, 1996.

Xia, B., Sun, Da-Wen.; "Applications of Computational Fluid Dynamics (CFD) in the Food Industry: a Review", Computers and Electronics in Agriculture, vol. 34, pp. 5-24, 2002.