# NUMERICAL SIMULATION OF 3D TURBULENT STRATIFIED FLOWS

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Abstract. This work presents numerical simulation methods for two types of three-dimensional fluid flows: the free and the stratified turbulent flows, both applied to study and forecast the environmental impacts caused by hydro-electrical power plant reservoir floodings. In this type of problems the correct parametrization of the turbulent fluxes is essential to obtain realistic simulations. The numerical simulation methods employ the Finite Element Method (FEM) approximation of Reynolds-averaged Navier-Stokes (RANS) equations. The convective terms are discretized employing the Semi-Lagrangian method and the spatial discretization is based on the Galerkin method. The time discretization is semi-implicit, resulting in an unconditionally stable scheme. Eddy-Viscosity models for free and for stratified turbulent flows are incorporate into the RANS equations. The computational results of reservoir simulations employing the implemented methods are presented and the effect of the parametrization is discussed.

*Keywords:* Numerical simulation, Reynolds averaged Navier Stokes (RANS), Finite Element Method (FEM), eddy viscosity, turbulence stratified flow.

# 1. INTRODUCTION

Several fluid motion phenomena, such as circulation in the atmosphere and the ocean (Fernando and Hunt, 1996), applications in lakes (Imberger and Ivey, 1991), in estuaries (Huang et al., 2003), and engineering application such as thermal nuclear reactors (Andreania et al., 2008), are stratified turbulent flows. The velocity gradient can lead to generation of turbulence in the usual way through the action of inertia forces, and the density gradient of the fluid due to the difference of temperature or salinity, depending on its sign, can provide an additional source of energy for the turbulence (Tritton, 1988).

The modeling of stratified flows normally involves the modeling of internal turbulence processes. This constitutes a difficulty, since there is a lack of reliable and efficient models to account for the effects of turbulence (Sotiropoulos, 2005).

There is a large literature about model turbulent flows. Although, classical techniques such as the Direct Numerical Simulation can be used, they requires large computational resources in practical engineering situations. A more realistic and useful (for engineering purposes) tool is based on statistical turbulence models. In this article, we perform the numerical simulation of the turbulent stratified flows using the Reynolds-averaged Navier-Stokes (RANS) equations with an appropriate turbulence model. This provides a better understanding and realistic prediction of the flows, which is necessary to study and forecast the environmental impacts caused by hydro-electrical power plant reservoir flooding.

In this context, the main objective of this article consists in describing the methodology employed for the turbulent exchanges at momentum and scalar quantities in cases of stable and unstable density gradients.

Initially, the RANS equations and the turbulence model used for the free and stratified flow are presented in Section 2. Afterward, in Section 3, we show the discretization method to solve the differential equations system. Specifically the MINI tetrahedral element for the momentum equations and continuity, and the linear tetrahedral element for the scalar quantities are used. In the Section 4, some results of the numerical simulations, on a simple 3D geometry are presented.

# 2. TURBULENCE EQUATION

In this section, we present the derivation of the RANS equations and then discuss the eddy viscosity model, specifically the algebraic model applied to numerical simulation. At last the turbulent modeling for stratified flow is shown.

#### 2.1 Derivation of RANS equations

The instantaneous motion of an incompressible and Newtonian fluid denoting mass and momentum conservation are governed by the 3D, incompressible Navier-Stokes (NS) equations:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial (u_i u_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_j} + g_i \tag{2}$$

where  $u_i(i = 1, 2, 3)$  are the instantaneous velocity components,  $x_i(i = 1, 2, 3)$  are the coordinate axes with direction 3 vertically upward, p is the instantaneous pressure,  $\rho$  is the density,  $\tau_{ij}(i, j = 1, 2, 3)$  are the components of the viscous stress tensor and  $g_i$  is the gravitational acceleration.

The stratification-inducing scalar  $\Theta$  (temperature or salinity) is considered governed by:

$$\frac{\partial\Theta}{\partial t} + \frac{\partial\Theta u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha \frac{\partial\Theta}{\partial x_j} \right) + S_\Theta \tag{3}$$

where  $S_{\Theta}$  is a source or sink of  $\Theta$  and  $\alpha$  is the molecular (heat or mass) diffusivity coefficient. For a Newtonian fluid the stress tensor is related to the rate of strain tensor as follows

$$\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{4}$$

where  $\mu$  is the molecular viscosity.

The statistical description for a turbulent flow based on Reynolds decomposition (in order to describe the velocity  $u_i$ , pressure p and scalar  $\Theta$ ) decomposes each state variable into a mean value plus a random fluctuating part. Therefore, the decomposition of the velocity  $u_i(\mathbf{x}, t)$  takes the form

$$u_i(\mathbf{x},t) = \overline{u}_i(\mathbf{x}) + u'_i(\mathbf{x},t) \tag{5}$$

where  $\overline{u}_i$  is the mean velocity and  $u'_i(\mathbf{x}, t)$  is the random fluctuation part. The mean velocity is given by

$$\overline{u}_i(\mathbf{x}) = \lim_{T \to \infty} \frac{1}{T} \int_t^{t+T} u_i(\mathbf{x}, t) dt$$
(6)

where the averaging interval T is taken to be much longer than the longest turbulent fluctuations in the flow.

Using the Eq. (5) and averaging the Eq. (1)-(3), we obtain the following Reynolds-averaged Navier-Stokes (RANS) equations:

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{7}$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial (\overline{u}_i \overline{u}_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_i} + \frac{1}{\rho} \frac{\partial}{\partial x_j} \left( \overline{\tau_{ij}} - \rho \overline{u'_i u'_j} \right) + g_i \tag{8}$$

$$\frac{\partial\overline{\Theta}}{\partial t} + \frac{\partial\overline{\Theta}\overline{u}_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha \frac{\partial\overline{\Theta}}{\partial x_j} - \overline{\Theta' u'_j} \right) + S_{\Theta}$$
(9)

The quantity  $\left(-\rho \overline{u'_i u'_j}\right)$  in the Eq. (8) is known as the Reynolds stress tensor. This tensor is symmetric, and thus has six independent components. Consequently the RANS equations system is unclosed. In order to make the system (7)-(9) solvable, we use the Boussinesq eddy-viscosity approach.

#### 2.2 Eddy-viscosity model

The Boussinesq eddy-viscosity approximation is used to compute the Reynolds stress tensor as the product of an eddy viscosity and the mean strain-rate tensor.

The eddy-viscosity hypothesis, is mathematically analogous to the strain-rate tensor relation for a Newtonian fluid. According to the hypothesis, the deviatoric Reynolds stress  $\left(-\rho \overline{u'_i u'_j} + \frac{2}{3}\rho k \delta_{ij}\right)$  is proportional to the mean strain-rate,

$$-\rho \overline{u'_i u'_j} + \frac{2}{3}\rho k \delta_{ij} = 2\rho \nu_t \overline{S}_{ij}, \quad \overline{S}_{ij} = \frac{1}{2} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right)$$
(10)

where the mean turbulent kinetic energy per unit mass in the fluctuation velocity field is defined as  $k = \frac{1}{2}\overline{u}_i^2$ ,  $\delta_{ij}$  is the unit tensor (Kronecker's delta), and the positive scalar coefficient  $\nu_t$  is the turbulent viscosity (also called the eddy viscosity). The mean-momentum equation incorporating the eddy-viscosity hypothesis (i.e., Eq. (10)) substituted into Eq. (8) is

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial (\overline{u}_i \overline{u}_j)}{\partial x_j} = -\frac{1}{\rho} \frac{\partial}{\partial x_i} \left( \overline{p} + \frac{2}{3} \rho k \right) + \frac{\partial}{\partial x_j} \left[ \nu_{eff} \left( \frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i} \right) \right] + g_i \tag{11}$$

where

$$\nu_{eff}(\mathbf{x},t) = \nu + \nu_t(\mathbf{x},t) \tag{12}$$

is the effective viscosity, with kinematic viscosity  $\nu = \frac{\mu}{\rho}$ . This is the same as the Navier-Stokes equations with  $\overline{u}_i$  and  $\nu_{eff}$  in place of  $u_i$  and  $\nu$ , with  $\overline{p} + \frac{2}{3}\rho k$  the modified mean pressure (Pope, 2000).

Following the same argument, the turbulent flux  $(-\overline{\Theta' u'})$  is proportional to the mean scalar gradient, i.e.,

$$-\overline{\Theta' u'} = \alpha_t \frac{\partial \overline{\Theta}}{\partial x_j} \tag{13}$$

where  $\alpha_t$  is the turbulent diffusivity. Substituting into Eq. (9), we have

$$\frac{\partial\overline{\Theta}}{\partial t} + \frac{\partial\overline{\Theta}\overline{u}_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha_{eff} \frac{\partial\overline{\Theta}}{\partial x_j} \right) + S_{\Theta}$$
(14)

where

$$\alpha_{eff}(\mathbf{x},t) = \alpha + \alpha_t(\mathbf{x},t) \tag{15}$$

is the effective diffusivity.

Eddy-viscosity models express  $\nu_t$  as the product of a turbulence length scale,  $l_t$ , and a turbulence velocity scale,  $u_t$ :

$$\nu_t = l_t u_t \tag{16}$$

and the task of specifying  $\nu_t$  is generally approached through specifications of  $l_t$  and  $u_t$ . In algebraic models,  $l_t$  is specified on the basis of the geometry of the flow.

#### 2.3 Algebraic models

Algebraic models rely on Prandtl's mixing-length hypothesis. By drawing an analogy with the molecular momentum transport process and replacing the molecular thermal velocity and mean free path with characteristic turbulent velocity and length scale. For 3D flows, the mixing length model, used in conjunction with Eq. (10), can be generalized as follows (Sotiropoulos, 2005):

$$\nu_t = 2l_t^2 \sqrt{\overline{S_{ij}}\overline{S_{ij}}} \tag{17}$$

The mixing-length  $l_t$ , is an empirical quantity that is typically assumed to be proportional to some characteristic length scale of the flow and needs to be specified using input from experiments.

#### 2.4 Turbulence modeling for stratified flows

For problems in which the density gradients are not large, the governing equations can be greatly simplified by invoking the so-called Boussinesq approximation and accounting for density variations only in the gravity term. The Boussinesq form of the Reynolds-averaged transport equations for mass, momentum and scalar (temperature or salinity) transport read as follows (Sotiropoulos, 2005):

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \tag{18}$$

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial (\overline{u}_i \overline{u}_j)}{\partial x_j} = -\frac{1}{\rho_r} \frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j} \left( 2\nu_{eff} \overline{S}_{ij} \right) + g_i \frac{\rho - \rho_r}{\rho_r}$$
(19)

$$\frac{\partial\overline{\Theta}}{\partial t} + \frac{\partial\overline{\Theta}\overline{u}_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \alpha_{eff} \frac{\partial\overline{\Theta}}{\partial x_j} \right) + S_{\Theta}$$
(20)

where  $\rho$  is the local fluid density and  $\rho_r$  is a reference density. The local density is related to the temperature or salinity via an equation of state of the following form:

$$\frac{\rho - \rho_r}{\rho_r} = -\beta(\Theta - \Theta_r) \tag{21}$$

where, in case the local density is related to the temperature,  $\beta$  is the thermal expansion coefficient.

Stratification effects are typically parametrized in terms of the gradient Richardson number, the ratio of the local production of turbulence due to buoyancy effects to that due to mean shear:

$$Ri = -\frac{g}{\rho} \frac{\partial \rho / \partial z}{\left(\partial \overline{u} / \partial z\right)^2}$$
(22)

where z is the direction of stratification. This number depends on the sign of the density gradient but not on that of the velocity gradient. Negative Richardson number corresponds to a destabilizing density gradient; both shear and buoyancy give rise to turbulence generation. Positive Richardson number corresponds to a stabilizing density gradient; turbulent motion cannot be sustained when Ri becomes large (Tritton, 1988). By setting a critical level of Ri,  $Ri_c$ , beyond which turbulence cannot be sustained, a mixing-length model modified for stratification effects can be formulated as follows (Sotiropoulos, 2005):

Stable stratification (Ri > 0)

$$\nu_t = \begin{cases} 2l_t^2 \sqrt{\overline{S}_{ij}\overline{S}_{ij}} \left(1 - Ri/Ri_c\right)^2 & 0 \le Ri \le Ri_c \\ 0 & Ri \ge Ri_c \end{cases}$$
(23)

Unstable stratification

$$\nu_t = 2l_t^2 \sqrt{\overline{S}_{ij}\overline{S}_{ij}} \left(1 - Ri\right)^{1/2} \tag{24}$$

Choosing  $Ri_c$  to be a small value (say one or less) suppresses the eddy viscosity to a very small level in regions of the flow away from shear zones (Sotiropoulos, 2005). The turbulence is almost completely suppressed when Ri reaches 0.45 (Tritton, 1988).

## 3. FINITE ELEMENT METHOD

In this section, we show the variational approach of the presented equations. Then, the Galerkin method for spatial discretization and semi-Lagrangian method for the convective terms discretization are briefly presented.

#### 3.1 Variational approaches

Rewriting the Reynolds-averaged transport equations for mass (Eq. (18)), momentum (Eq. (19)) and the stratificationinducing scalar (Eq. (20)) using the vectorial notation, read:

$$\nabla \cdot \mathbf{u} = 0 \tag{25}$$

$$\frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho_r} \nabla p + \nabla \cdot (2\nu_{eff} \mathbf{S}) + \mathbf{g} \frac{\rho - \rho_r}{\rho_r}$$
(26)

$$\frac{D\Theta}{Dt} = \nabla \cdot (\alpha_{eff} \nabla \Theta) + S_{\Theta}$$
<sup>(27)</sup>

where, the substantive derivative operator D/Dt is defined as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$$
(28)

Equations (25)-(27) are defined in a domain  $\Omega \subset \mathcal{R}^m$  with the following boundary conditions

$$\mathbf{u} = \mathbf{u}_{\Gamma} \text{ on } \Gamma_1 \tag{29}$$

$$p = p_{\Gamma} \text{ on } \Gamma_2 \tag{30}$$

$$\Theta = \Theta_{\Gamma} \text{ on } \Gamma_3 \tag{31}$$

The variational approach of the problem read: Seek  $\mathbf{u}(\mathbf{x},t) \in {\mathbf{u} \in \mathcal{H}^1(\Omega)^m : \mathbf{u} = \mathbf{u}_{\Gamma} \text{ on } \Gamma_1}, p(\mathbf{x},t) \in {p \in \mathcal{H}^1(\Omega) : p = p_{\Gamma} \text{ on } \Gamma_2}$  and  $\Theta(\mathbf{x},t) \in {\Theta \in \mathcal{H}^1(\Omega) : \Theta = \Theta_{\Gamma} \text{ on } \Gamma_3}$ , such that

$$\int_{\Omega} \left[ \nabla \cdot \mathbf{u} \right] q \, d\Omega = 0 \tag{32}$$

$$\int_{\Omega} \frac{D\mathbf{u}}{Dt} \cdot \mathbf{w} \, d\Omega - \frac{1}{\rho} \int_{\Omega} p \nabla \cdot \mathbf{w} \, d\Omega + \int_{\Omega} 2\nu_{eff} \mathbf{S} : \nabla \mathbf{w} \, d\Omega - \int_{\Omega} \frac{\rho - \rho_r}{\rho_r} \mathbf{g} \cdot \mathbf{w} \, d\Omega = 0$$
(33)

$$\int_{\Omega} \frac{D\Theta}{Dt} r \, d\Omega + \int_{\Omega} \left( \alpha_{eff} \nabla \Theta \right) \nabla r^T \, d\Omega - \int_{\Omega} S_{\Theta} r \, d\Omega = 0 \tag{34}$$

where,  $\mathbf{w} \in {\mathbf{w} \in \mathcal{H}^1(\Omega)^m : \mathbf{w} = \mathbf{0} \text{ on } \Gamma_1}$ ,  $q \in {q \in \mathcal{H}^1(\Omega) : q = 0 \text{ on } \Gamma_2}$ ,  $r \in {r \in \mathcal{H}^1(\Omega) : r = 0 \text{ on } \Gamma_3}$ , and  $\mathcal{H}^1(\Omega)$  is the Sobolev space of degree 1, which is subset of functions that possess square-integrable generalized derivatives through order 1.

## 3.2 Galerkin method

For the implementation, the Eq. (32)-(34) is discretized using the standard Galerkin method, where the spaces of functions are replaced by finite dimensional subspaces. The following linear spatial approximation of the variables in space is used:

$$u_i(\mathbf{x},t) \approx \sum_{n=1}^{NV} N_n(\mathbf{x}) u_{in}(t) \ (i=1,2,3)$$
 (35)

$$p(\mathbf{x},t) \approx \sum_{n=1}^{NP} P_n(\mathbf{x}) p_n(t)$$
(36)

$$\Theta(\mathbf{x},t) \approx \sum_{n=1}^{N\Theta} R_n(\mathbf{x})\Theta_n(t)$$
(37)

where, NV is the number nodes of velocity, NP is the number nodes of pressure,  $N\Theta$  is the number nodes of scalar and  $N_n(\mathbf{x})$ ,  $P_n(\mathbf{x})$  and  $R_n(\mathbf{x})$  are the shape functions. On employing the Galerkin weighting to Eq. (32)-(34), is obtained the following linear system of ordinary differential equations:

$$D\tilde{u} = 0 \tag{38}$$

$$M\dot{\tilde{u}} + K_M\tilde{u} - G\tilde{p} + b = 0 \tag{39}$$

$$M_{\Theta}\dot{\tilde{\Theta}} + K_{\Theta}\tilde{\Theta} + c = 0 \tag{40}$$

where,  $\tilde{u} = [u_{11}, \dots, u_{1NV}, u_{21}, \dots, u_{2NV}, u_{31}, \dots, u_{3NV}]^T \in \mathcal{R}^{3NV}$ ,  $\tilde{p} = [p_1, \dots, p_{1NP}]^T \in \mathcal{R}^{NP}$  and  $\tilde{\Theta} = [\Theta_1, \dots, \Theta_{1N\Theta}]^T \in \mathcal{R}^{N\Theta}$  are the unknowns at nodes of velocity, pressure and scalar respectively,  $D \in \mathcal{R}^{NP} \times \mathcal{R}^{3NV}$  is the divergence matrix,  $G \in \mathcal{R}^{3NV} \times \mathcal{R}^{NP}$  is the gradient matrix,  $M \in \mathcal{R}^{3NV} \times \mathcal{R}^{3NV}$  is the mass matrix,  $K_M \in \mathcal{R}^{3NV} \times \mathcal{R}^{3NV}$  is the momentum diffusion matrix,  $M_{\Theta} \in \mathcal{R}^{N\Theta} \times \mathcal{R}^{N\Theta}$  is the scalar mass matrix,  $K_{\Theta} \in \mathcal{R}^{N\Theta} \times \mathcal{R}^{N\Theta}$  is the scalar diffusion matrix,  $b \in \mathcal{R}^{3NV}$  is the forcing vector due to buoyancy force and  $c \in \mathcal{R}^{N\Theta}$  is the forcing vector due to the scalar source. Notice that b is related with scalar  $\Theta$  by the Eq. (21) and  $\dot{\tilde{u}}$  and  $\tilde{\Theta}$  represent the substantive derivatives of  $\tilde{u}$  and  $\tilde{\Theta}$ .

The tetrahedral MINI element is selected to discretize the velocity and the linear tetrahedral for the pressure and the scalar  $\Theta$ .

#### 3.3 Semi-Lagrangian method

The ordinary differential equations system (Eq. (38)-(40)) is solved employing the semi-Lagrangian method for time discretization. Because of the larger allowable time step, the semi-Lagrangian technique contributes to a significant enhancement of the efficiency of the semi-implicit integration scheme (Robert et al., 1984). Using a function  $\phi$ , the substantive derivative of this function at the point  $x_i$  can be discretized using a first order scheme as

$$\frac{D\phi}{Dt} = \frac{\phi_i^{n+1} - \phi_d^n}{\Delta t} \tag{41}$$

where,  $\phi_i^{n+1} = \phi(\mathbf{x}_i, t^{n+1})$  is the image of  $\phi$  at the point  $\mathbf{x}_i$  and the time step n+1 and  $\phi_d^n = \phi(\mathbf{x}_d, t^n)$  is the image of  $\phi$  at the point  $\mathbf{x}_d$  and the time step n, obtained by interpolating the solution on the mesh nodes at time step n. The position  $\mathbf{x}_d$  is obtained using the expression

$$\mathbf{x}_d = \mathbf{x}_i - \mathbf{v}\Delta t \tag{42}$$

where  $\mathbf{v} = \mathbf{v}(\mathbf{x}_i, t^n)$  is the velocity vector at the point  $\mathbf{x}_i$  and time step n.

Equations (38)-(40), with an implicit time discretization read

$$D\tilde{u}^{n+1} = 0 \tag{43}$$

$$M\left(\frac{\tilde{u}^{n+1}-\tilde{u}_d^n}{\Delta t}\right) + K_M \tilde{u}^{n+1} - G\tilde{p}^{n+1} + b = 0$$

$$\tag{44}$$

$$M_{\Theta}\left(\frac{\tilde{\Theta}^{n+1} - \tilde{\Theta}_d^n}{\Delta t}\right) + K_{\Theta}\tilde{\Theta}^{n+1} + c = 0$$
(45)

At each time step n, Eq. (43) and (44) are solved employing a discrete projection method based on a block LU approximate factorization, and Eq. (45) is solved separately.

#### 4. NUMERICAL SIMULATION

In this section we present the results obtained by the numerical simulation, on a simply 3D geometry.

This simulation considers a flow through a channel with a real fluid. The size of channel is  $160 m \ge 3 m \ge 30 m$  and number of nodes is  $41 \ge 2 \ge 11$  nodes with 2400 elements. We consider no-slip condition for the three velocity components on the bottom of the channel and the y and z components of the velocity is 0 on the side walls and the top of the channel respectively (slip walls). The inflow velocity is 1 m/s in x-direction, while for pressure at the outflow is prescribed as hydrostatic distribution. This distribution of the pressure at the outflow is necessary for more realistic simulation because the distribution of the scalar concentration, related with the fluid density by the Eq. (21), produces vertical acceleration of the fluid. The inflow scalar concentration is  $\Theta = 10$  for z < 15 m and 0 otherwise. The boundary conditions and the geometry of the problem are shown in the Fig. 1 below.



Figure 1. Geometry and boundary conditions of the simulation of flow through a channel

We have considered two type of flows, one corresponding to a stable stratification (Fig. 2) and the other an unstable one (Fig. 3). In both, we specify the critical level of Richardson number  $Ri_c = 0.25$  and the turbulent length scale  $l_t = 15 m$ .

Figure 2 shows the velocity field, the scalar concentration and the pressure for stable stratification, while, Fig. 3 shows the velocity field and the scalar concentration for unstable stratification. For stable stratification we have used  $\beta = -0.06$  and for unstable stratification  $\beta = 0.06$ .



Figure 2. Flow evolution for stable stratification

The left sides of Fig. 2 and 3 show the flow after a short time of the beginning of the simulation, and the right sides of Fig. 2 and 3 show the flow patterns that develop at long times. After the initial transient phase, the stably stratified flow case settles for a steady state solution with low turbulent diffusion. On the other hand, the unstably stratified case shows, after the initial unsteady phase, the presence of internal gravity waves propagating downstream from the inflow, and a

very strong mixing across the water column. Notice that the diffusion is due to a combination of the turbulence diffusivity and the numerical diffusion.



Figure 3. Flow evolution for unstable stratification

# 5. CLOSING REMARKS

This article describes the initial work to study and forecast the environmental impacts caused by hydro-electrical power plant reservoir flooding, including simple stratified turbulence models. This inclusion provides a more realistic and useful tool for engineering purposes. Realistic simulation can be obtained with low computational cost adjusting adequately a small number of parameters. This type of model is suitable for automatic parameter estimation by adaptive learning, employing streams of field data.

In this work we have obtained qualitative results using a algebraic equation for the eddy viscosity models for free and stratified flows. These results are important to future investigations and implementations of more complex models, such as one equation models (Spalart-Allmaras model) and two equation models. Additionally, the analyzed cases provide a benchmark for parameter validation based on experimental and field data.

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