A SEMI-EMPIRICAL CORRELATION FOR THE PICKUP VELOCITY

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Abstract. *The project of a pneumatic transport system must consider parameters as the pressure drop, the gas velocity, particles velocity, the wall roughness, the particle-particle interaction, among others. One aspect of great importance in the operational stabilization of a pneumatic transport system is the transport of particulates materials at minimum possible velocity. In this work, a semi-empirical correlation is developed for the pickup velocity of particles in rest at the bottom of pipeline. This new technique proposed for prediction of pickup velocity was developed based on a model for incipient motion of a single particle in a vertical direction and the Archimedes number. This correlation presents good results for particles with sizes ranging from 30 to 4000 µm.*

Keywords: pickup velocity; minimal velocity; Archimedes number; pneumatic transport.

1. INTRODUCTION

Pneumatic conveying systems are very well established on industries such as ore processing, chemical industries and grains processing industries. On the other hand, the phenomena related to pneumatic conveying are complex, due to the great number of variables involved (Salman *et al.*, 2002).

In accordance to Cabrejos and Klinzing (1992), the minimum velocity of transport is one of the most important parameters in pneumatic transport of solids. The velocity higher than the necessary to the steady transport of solid particles leads to a greater energy consumption due system pressure drop, added to the solids degradation and pipe erosion problems. On the other hand, the velocity below of this boundary-value certainly will result the deposition of solid particles on the pipe bottom and consequently blockage (Cabrejos and Klinzing, 1994)

In literature, several terms are employed to refer the minimum transport velocity: pickup velocity, saltation velocity, suspension velocity, critical velocity, among other (Herbreteau and Bouard, 2000).All of these definitions presents similar numerical values and are based on visual observations or through measurements, for example, the pressure drop. These velocities characterize an alteration of the flow regime, changing from a steady to an unstable phase. Also indicates some transition on the way that the particles are moving or beginning their movements.

The pickup velocity is relevant in wide range of applications. For example, some pharmaceutical industries are focused on dry powder inhalers for drug delivery, the movement of sand dunes and soil deposition in river and ocean flows (Kalman *et al.*, 2005).

In this paper, a semi-empirical correlation is proposed. A model for incipient motion of a single particle on vertical direction is developed and correlated with a particles layer using some experimental data. The results are compared with experimental data showing good agreement.

2. LITERATURE REVIEW

Cabrejos and Klinzing (1992) developed a model for incipient motion of a single sphere initially in rest at the bottom of a horizontal pipe and subjected to a steady fully developed turbulent flow of air. They assumed that the single particle begins to move when the forces in the horizontal direction are zero (sliding). They found that for large single particle:

$$U_{puo} = \left[1 - \left(\frac{d_p}{D}\right)^{1.5}\right]\sqrt{\frac{4}{3}\frac{f_s g d_p}{C_D}\left(\frac{\rho_p - \rho}{\rho}\right)} \qquad (1)$$

and for a small single sphere:

$$1.54.10^{-4}\left[1-\frac{d_p}{D}^{1.5}\right]^{-2} C_D \rho d_p^{\,4}\left(\frac{U_{puo}^{7}}{\nu^3 D}\right)^{1/2}$$
$$= f_s\left[\frac{\pi}{6} g d_p^{\,3}\left(\rho_p-\rho\right)+1.302.10^{-6} d_p - 6.35.10^{-3}\rho d_p^{\,3}.\left(\frac{U_{puo}^{21}}{\nu^5 D^3}\right)^{1/8}\right] \tag{2}$$

Thus, a semi-empirical correlation was developed for pickup velocity of a layer of particles for sizes ranging from 10 to 1000 μm:

$$U_{pu}=\left[1.27Ar^{-1/3}+0.036Ar^{1/3}+0.45\right]\left[0.70Ar^{-1/5}+1\right]U_{puo} \tag{03}$$

where:

$$Ar=\rho g\frac{\rho_s-\rho}{\mu^2}d_p^{\,3}\text{, Archimedes number} \tag{04}$$

Cabrejos and Klinzing (1994), applying dimensional analysis, found that the following π groups can describe the pickup mechanism of the coarse particles in horizontal tubes

$$f\left(\frac{U}{\sqrt{g\,d_p}},\frac{\rho_p}{\rho},\phi,\frac{D}{d_p},\frac{d_p U\ \rho_p}{\mu}\right)=0 \tag{05}$$

They also analyzed the effect of several parameters that affect the pickup velocity of coarse particles in horizontal pipes. By changing one parameter and keeping the others parameters constant, the following relationships were found:

$$U_{pu}\propto d^{1/2};U_{pu}\propto \rho_p^{\,3/4};U_{pu}\propto\rho^{-1/2};U_{pu}\propto D^{1/4} \tag{06}$$

The authors also verified that the influence of the conveying gas viscosity on the pickup velocity is minimal. Finally, they postulated the following relationship to determinate the pickup velocity:

$$\frac{U_{pu}}{\sqrt{g\,d_p}}=0,0428\,\mathrm{Re}_p^{\,0,175}\left(\frac{D}{d_p}\right)^{0,25}\left(\frac{\rho_p}{\rho}\right)^{0,75} \tag{07}$$

This relationship is valid for $25<R_{ep}<5000$, $8<(D/d_p)<1340$ e $700<(\rho_p/\rho)<4240$.

Hayden *et al.* (2003) applied a force balance on a single small particle initially in rest at a flat plate in a shear flow. They assumed that the pickup velocity occurs in vertical lift-off and found that:

$$U_{puo}=\frac{2,62\nu^{\frac{13}{21}}D^{\frac{3}{21}}}{\mu^{\frac{8}{21}}}\left(\frac{\pi}{6}g\left(\rho_p-\rho\right)+\frac{1,302.10^{-6}}{d_p^2}\right)^{\frac{8}{21}} \tag{08}$$

Kalman *et al.* (2005) developed a relationship for the pickup velocity in terms of modified Reynolds number as a function of modified Archimedes number. They modified the Reynolds number empirically to take account the pipe diameter:

$$\mathrm{Re}_p^{\,*}=\frac{\rho U_{pu} d_p}{\mu\left(1,4-0,8.e^{-\frac{D/D_{50}}{1,5}}\right)} \tag{09}$$

Then, they presented the following correlation:

$$\frac{U_{pu}}{U_{pu50}} = 1,4 - 0,8.e^{-\frac{D/D_{50}}{1,5}} \tag{10}$$

By this correlation, all measured pickup velocity found in the literature can be converted to the pickup velocity in a 2 inch pipe for further investigation. In accordance with the authors, this equation agrees well with the measurements conducted with pipe diameters ranging from about 1 up to 6 in.

Using their own experiments and data of others researchers they developed three relationships, each one applied to a specific flow situation. The three zones model was in reasonable according with the Geldart classification. The first zone applies to large particles and the cohesive forces are negligible.

$$\mathrm{Re}_p^{\ *} = 5\,Ar^{\frac{3}{7}}, \text{ for Ar} > 16{,}5 \tag{11}$$

In second zone the cohesive forces are considerable, but still the particles are picked up individually.

$$\mathrm{Re}_p^{\ *} = 16,7, \text{ for } 0{,}45 < \text{Ar} < 16{,}5 \tag{12}$$

In third zone the cohesive forces are very strong, the particles are very small and the capture occurs in agglomerates instead of individual particles.

$$\mathrm{Re}_p^{\ *} = 21,8.Ar^{\frac{1}{3}}, \text{ for Ar} < 0{,}45 \tag{13}$$

The relationships are valid for $0{,}5 < \mathrm{Re}_p^* < 5400$, $2.10^{-5} < \mathrm{Ar} < 8{,}7.\ 10^7$, $0{,}53 < d_p < 3675$ µm, $1119 < \rho_p < 8785$ kg/m^3 and $1{,}18 < \rho < 2{,}04$ kg/m^3.

3. MODEL FOR INCIPIENT MOTION OF A SINGLE SPHERE

The significant forces acting on a single sphere in rest at the bottom of a horizontal tube are: drag force, friction force, cohesive force, buoyancy force, lift force and gravitational force. Cabrejos and Klinzing (1992) showed that the horizontal movement occurs before the vertical lift-off. By Hayden e*t al*. (2003), the pickup velocity is considered to be the velocity at which the particle becomes entrained in the moving fluid, which implies vertical movement.

In this paper, the force balance is made for the motion particle in vertical direction. To achieve the vertical pickup, the resultant force in this direction is equal the zero.

$$F_g + F_a = F_l + F_b + F_d \tag{14}$$

The lift, buoyancy and adhesive forces are not appreciated in this demonstration. Thus

$$F_g = F_d = C_D A \rho \frac{v^2}{2} \tag{15}$$

Since $v \sim \mu_\tau$ (friction velocity) and $\mu_\tau^{\ 2} = \tilde{c}\bar{v}^2 f^2$

$$F_g = C_D A \rho \frac{\mu_\tau^{\ 2}}{2} = C_D A \rho \tilde{c}\bar{v}^2 \frac{f^2}{2} \tag{16}$$

where $\tilde{c}$ is a constant related the turbulent fluctuations.

Rewriting the Eq. (16), in terms of friction factor,

$$f^2 = \frac{2P}{C_D A \rho \tilde{c}\bar{v}^2} \Rightarrow f = \sqrt{\frac{2P}{C_D A \rho \tilde{c}\bar{v}^2}} \tag{17}$$

Making use of Blasius relation,

$$f = \frac{0.316}{\mathrm{Re}^{1/4}} \tag{18}$$

Since

$$P = \frac{g\rho_p \pi d_p^3}{6} \tag{19}$$

and

$$A = \frac{\pi}{4} d_p^2 \tag{20}$$

Substituting the Eqs. (18), (19) and (20) into (17), we have

$$\frac{0.316}{\left(\dfrac{\rho_p U_{pu} D}{\mu}\right)^{1/4}} = \sqrt{\frac{4}{3}\frac{g\rho_p \pi d_p}{C_D \rho \tilde{c} \bar{v}^2}} \tag{21}$$

Making $\bar{v}$ = U_{pu} and solving the Eq. (21) for the gas velocity U_{puo}, the velocity of the uniform flow required to the initiate motion of a single particle in a horizontal pipe can be written as

$$U_{puo} = 5.628289 \frac{g^{2/3} \rho_p^{\ 2/3} d_p^{2/3} D^{1/3}}{C_D^{2/3} \rho^{1/3} \tilde{c}^{2/3} \mu^{1/3}} \tag{22}$$

This relationship shows us the dependency of the pickup velocity of the properties of the particle and conveying gas and also of the duct diameter.

4. THE GENERAL SEMI-EMPIRICAL CORRELATION

Combining the single particle model with experimental results for pickup velocity of a pile of particles in a pipe on the horizontal direction, it was possible to determinate a general semi-empirical correlation which it was simple and practical for to determinate the pickup velocity of a layer of particles. The semi-empirical correlation developed is valid over the range of particles sizes from 30 to 4000 μm and can be written in the following form:

$$U_{pu} = \left(2.7Ar^{-1/4} + 0.03Ar^{-1/14} + 0.45\right)\left(1.8Ar^{-1/4} + 1\right)U_{puo} \tag{23}$$

The Archimedes number was used to correlate the particle-particle interactions and particle shape. It characterizes the particle-gas properties and is related with the Froude number, the Reynolds number and the ratio of gravity to buoyant force.

5. RESULTS

Figures 1-2 was obtained with the use of Eq. (22). In first figure, the effect of particle diameter is illustrated when keeping the others parameters constant, i.e. particle density, conveying gas density and viscosity, and pipe diameter. It can be seen also the influence of density in the pickup velocity, i.e. higher densities demands a greater fluid velocity for entrainment of particles.

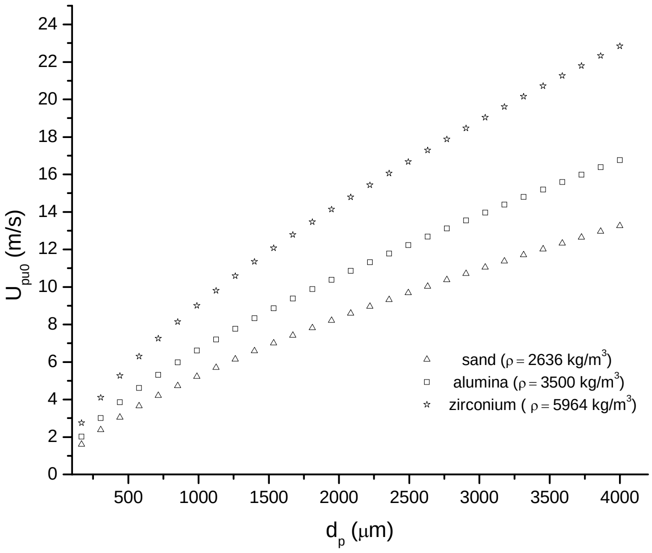

Figure 1. The pickup velocity as function of the particle diameter of for single spheres.

Figure 2 shows the pickup velocity of sand particles. It can be seen the effect of pipe diameter that is an important fact to be considered in project of pneumatic conveying systems.

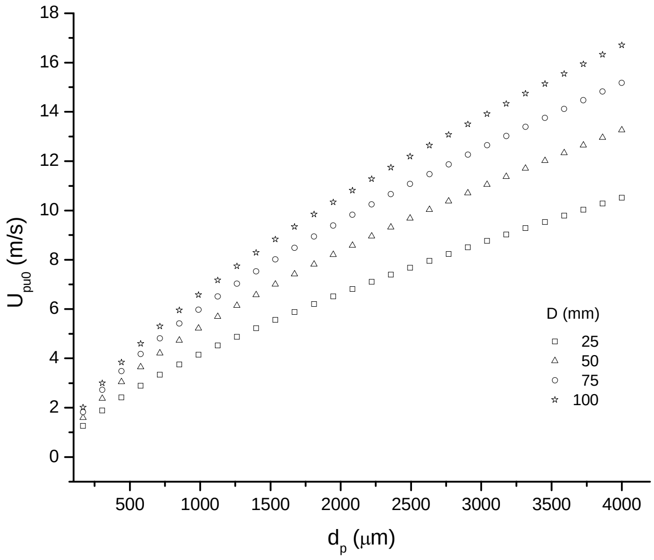

Figure 2. The pickup velocity of sand particles as function of the particle diameter.

In order to evaluate the accordance of the correlation developed by us, the pickup velocity was calculated and plotted against the experimental result of Cabrejos and Klinzing (1994), Hayden *et al.* (2003), Kalman *et al.* (2005) and Gomes and Mesquita (2006). The predicted values with the use of present correlation showed a very good agreement with the experimental data (Figs. 3-5).

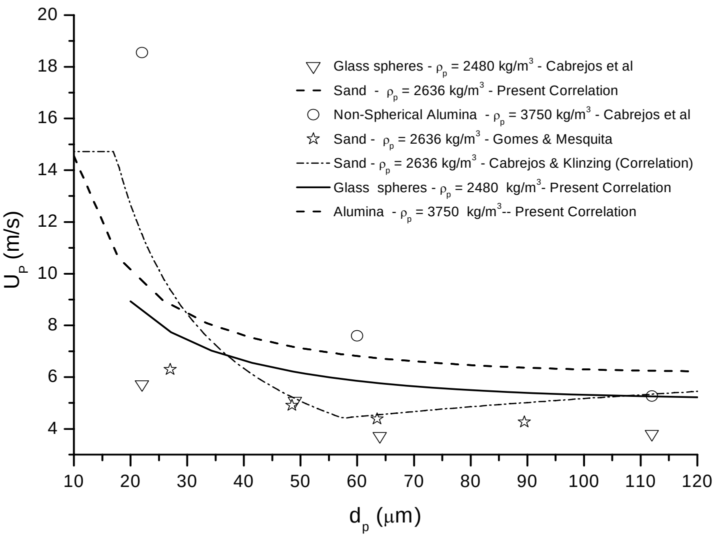

Figure 3. The pickup velocity D = 52 mm.

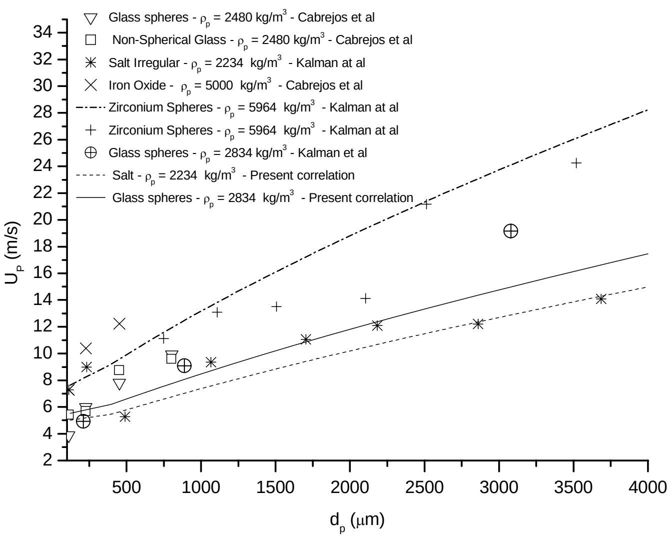

Figure 4. The pickup velocity - D = 52 mm.

Our correlation presents good results for particle densities ranging from 1500 to 6000 kg/m^{3}. Other interesting aspect is the fact it present also very good results for particles sizes ranging from 30 to 4000 μm, i.e. a large interval in densities and sizes. Almost all particles handled by industry, in a general way, are in this size range, it making evident the great applicability of this correlation.

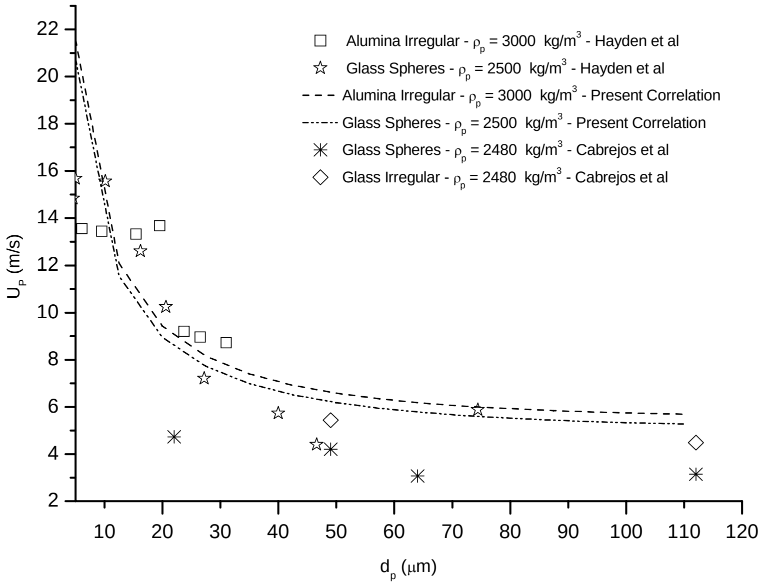

Figure 5. The pickup velocity - D = 26 mm.

6. CONCLUSION

A new correlation was proposed for prediction of pickup velocity and successfully tested. This new semi-empirical correlation was developed, based on a model for incipient motion of a single particle in a vertical direction and the Archimedes number.

The pickup velocity for different solids cannot be easily predicted because this parameter is influenced by many diverse variables. Among these, the characteristics of the material itself, such as particle size, density and shape, the coefficient of sliding friction, and the particle interaction with other particles are the most important variables that affect pickup velocity.

This correlation did not present results very good for particles diameter and densities above of 4000 μm and 6000 kg/m^3, respectively. More studies must be made to verify this inconsistency. However, the results obtained for particles with sizes ranging from 30 to 4000 μm are in good concordance with experimental measures carry out by Cabrejos and Klinzing (1992), Hayden *et al.* (2003), Kalman *et al* (2005) and Gomes and Mesquita (2006).

Considering the large range size and density and precision of numerical values evaluated with this correlation we expect it could an efficient tool to predict the pickup velocity.

7. ACKNOWLEDGEMENTS

The authors thank the financial support from CAPES without which this work would be impossible.

8. NOMENCLATURE

A	projected area normal to flow [m^2]
Ar	Archimedes number
C_D	drag coefficient
$\tilde{c}$	constant related the turbulent fluctuations
D	pipe diameter [m]
d_p	particle diameter [m]
f_s	friction coefficient between particles and surface
F_a	adhesive force [N]
F_b	buoyancy force [N]
F_d	drag force [N]
F_g	gravitational force [N]
F_l	lift force [N]

g	acceleration due to gravity [m/s^2]
Re_p	particle Reynolds number
Re_p^*	Reynolds number modified by the pipe diameter
U	mean gas velocity [m/s]
U_{pu}	pickup velocity (gas velocity when pickup from a layer of particles begins) [m/s]
U_{puo}	pickup velocity (gas velocity when particles start to move on a surface) [m/s]
μ	gas dynamic viscosity [kg/ms]
μ_τ	friction velocity
ρ	gas density [kg/m^3]
ρ_p	particle density [kg/m^3]
ν	fluid kinematic viscosity, m^2/s
ϕ	particle sphericity

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