A PARAMETRIC RELATION FOR SEPARATING BOUNDARY LAYERS OVER ROUGH HILLS

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Abstract. *A parametrization method used to account for the effects of flow separation and wall roughness on the lower boundary condition for turbulent boundary layers is investigated against some DNS and LDA data. The numerical simulation represents flow over a smooth, flat surface with a prescribed external adverse pressure gradient. The experiments cover flow over smooth and rough hills, on two specified Reynolds numbers. Global optimization algorithms based on four different direct search methods are used to assess the parametrization function – C – in terms of local mean velocity profiles and the parametrization parameters u_* (friction velocity), $\partial_x p$ (local pressure gradient), z_0 (effective roughness) and d (displacement in origin). The work investigates regions of attached and reversed flows. Forty two velocity profiles are compared with the proposed expression for the function C, including two profiles that satisfy the solution of Stratford (1959).*

Keywords: *Hill, separation, roughness, lower boundary condition*

1. Introduction

Any description of turbulent flow over natural land should ideally contemplate two very difficult modelling issues: wall roughness and flow separation. In particular, since these two subjects often occur simultaneously, their effects should be considered together and not separately.

Unfortunately, this has not been the practice in literature. The excessive level of complexity involved in the account of both subjects has usually forced authors to approach the problem individually. The present work, on the other hand, proposes a new treatment for the lower boundary condition that, in principle, can be used in regions of attached and separated flows over rough surfaces. The specification of wall functions to represent near-wall turbulence is particularly helpful for flows over rough surfaces since the fine-scale complexities generated by the flow around roughness elements cannot be resolved even in the present day environment of optimized algorithms and high computing power.

The treatment of presented in this work is not susceptible to the deficiencies encountered by the classical logarithmic law of the wall. Close to a separation point, the proposed solution reduces to the square-rooted law of Stratford (1959). In that particular, the relevant scales of the flow are arranged so that far away from a separation point the skin-friction velocity ($u_* = \sqrt{\tau_w/\rho}$, τ_w = local wall shear stress) is the prescribed scale of the flow, whereas near to a separation point a new scale based on the local pressure gradient is introduced ($u_p = ((z_0/\rho)\partial_x p)^{1/2}$, z_0 = effective roughness, $\partial_x p$ = local pressure gradient).

One major difficulty with all derivations of the law of the wall is the appearance of an additive parameter that must be determined experimentally. The physical modelling of this parameter is necessarily very complex for it should incorporate, for example, the diversity of details that define a rough wall. In the present work, the additive parameter – C – is parametrized by considering the limiting form of the proposed solution as $\tau_w \gg (1/\rho)\partial_x p$. The implication is that C is written in terms of u_*, $(1/\rho)\partial_x p$, z_0 and the von Kármán constant, $\varkappa$ (= 0.4). The clear attractiveness of this approach is the possibility of defining C from, as mentioned before, the vast collection of z_0-values that has been quoted in literature. The apparent weakness, on the other hand, is the lack of support of the theory in regions where the above inequality is not satisfied, that is, close to a separation point and in regions of reverse flow.

The purpose of the present work is to investigate in detail the proposed parametrization of C for the region close to a separation point and for the zone of reverse flow. This will be made through a direct estimate of C from some DNS and experimental data. Four sets of data will be considered here: the smooth wall DNS data set of Na and Moin (1998), the smooth wall LDA data set of Loureiro et al. (2007a, 2007b) and the two rough wall LDA data sets of Loureiro et al. (2008a, 2008b). In particular, one of the data sets of Loureiro et al. (2008a, 2008b) permits C to be evaluated at a point of separation over a rough wall.

2. Theory

This section briefly reviews some basic arguments regarding the specification of the lower boundary condition for separating flow as well as flow over a rough surface.

Asymptotic expansion methods have been used abundantly in the past to split the turbulent boundary layer into regions where dominant physical effects can be isolated and local solutions found. These methods naturally uncover the relevant scaling parameters of the flow and the functional nature of solutions. The works of Yajnik (1970) and Mellor (1972) have centered their arguments on the two-layered flow structure of Prandtl (1925) and v. Kármán (1930). The result is the establishment of a flow model where the sum of the viscous and turbulent stresses is considered to remain invariant across the entire near wall region, the inner region. In the adjacent layer, the outer layer, the solution is a small perturbation to the external potential flow.

Sychev and Sychev (1987) offer a different interpretation to the flow asymptotic structure. In their analysis, an additional intermediate layer is considered where a balance of inertia forces, the pressure gradient and turbulent friction forces occurs. Thus, they argue that the inclusion of this new layer is essential to explain flow separation under an adverse pressure gradient. In fact, the two-layered structure does not permit turbulent friction forces to act on the velocity defect region to a first-order approximation. Also, in the inner region, it does not permit the pressure gradient to act on the dominating friction forces. These two difficulties must then be remedied by the inclusion of a third layer.

In view of the above arguments, near wall solutions for flows subject to pressure gradients can be obtained if a balance between friction forces and the pressure gradient is considered to exist in some region of the flow. This region corresponds to an intermediate limit process so defined as to match the outer limit of the inner solution with the inner limit of the intermediate solution of Sychev and Sychev's flow structure. Thus, we may write for this region

$$\mu \partial_z^2 u - \partial_z (\rho \overline{u'w'}) = \partial_x p \,. \tag{1}$$

In the sense of Kaplun (1967) (see also Lagerstrom and Casten (1972), Cruz and Silva Freire (1998)) – and under some appropriate intermediate limit process – Eq. (1) contains the approximate equations for two distinct limiting conditions: flow very near to a wall and flow approaching the intermediate region (Sychev and Sychev (1987)). In the former limit, just the viscous and pressure terms are retained. Then, for flows under a zero-pressure gradient, a double integration of Eq. (1) yields the linear solution $u^+ = z^+$ with $u^+ = u/u_*$, $z^+ = z/(\nu/u_*)$ and $u_* = \sqrt{\tau_w/\rho}$.

The assumption implicit in the linear solution is the very existence of a near wall viscous region. Surfaces with a well defined viscous sublayer are termed smooth surfaces.

Moving away from the wall, the turbulent term becomes dominating in Eq. (1). Then, by summoning the mixing-length hypothesis, the solution for zero-pressure gradient flows can be obtained from a double integration as

$$u^+ = \varkappa^{-1} \ln z^+ + A, \qquad u^+ = u/u_*, \qquad z^+ = z/(\nu/u_*), \tag{2}$$

where $\varkappa$ (= 0.4) and A (= 5.0) are parameters that have to be found experimentally. In fact, the value of A is fixed through the empirically verified hypothesis that the logarithmic and linear profiles intercept at $z^+ = 11$.

The immediate implication of the linear and logarithmic solutions is that, for flow over a smooth surface, the near wall relevant scales for velocity and length are u_* and ν/u_*.

At a separation point, where $u_* = 0$, the approximate solution given by Eq. (2) breaks down. Hence, a more appropriate solution needs to be considered to incorporate the pressure effects and the condition $\tau_w = 0$.

In the viscous sublayer, Goldstein (1948) showed the local solution to be

$$u^p = (1/2)\,(z^p)^2, \tag{3}$$

with $u^p = u/u_{p\nu}$, $z^p = z/(\nu/u_{p\nu})$, $u_{p\nu} = ((\nu/\rho)\partial_x p)^{1/3}$.

Stratford (1959) considered the limiting solution for distances away from the wall, approaching the intermediate region of Sychev and Sychev (1987). Again, use of the mixing length hypothesis, the no-slip boundary condition and the fact that at a separation point $\tau_w = 0$ gives

$$u^p = (2\,\varkappa^{-1})\,(z^p)^{1/2}, \tag{4}$$

with u^p and z^p defined according to Eq. (3).

The implication is that the relevant velocity and length scales for flows at a separation point are $u_{p\nu}$ and $\nu/u_{p\nu}$.

Strictly speaking, the no-slip condition should not have been used by Stratford, for Goldstein's solution is the solution that is supposed to be valid at the wall. Equation (4) should then include an integration constant. Stratford also incorporated an empirical factor – β (= 0.66) – to Eq. (4) to correct pressure rise effects on $\varkappa$.

The worth note is that all approximate solutions derived so far have been obtained from local equations reduced from Eq. (1). Therefore, the global solution of Eq. (1) should also reduce, under the relevant limiting processes, to the local approximate solutions.

A double integration of Eq. (1) in the fully turbulent region furnishes (see, e.g., Cruz and Silva Freire (1998))

$$u = 2\varkappa^{-1}\sqrt{\Delta_w} + \varkappa^{-1}u_*\ln\left((\sqrt{\Delta_w} - u_*)/(\sqrt{\Delta_w} + u_*)\right) + C, \tag{5}$$

with $\Delta_w = \rho^{-1}\tau_w + (\rho^{-1}\partial_x p)z$.

Equation (5) must be viewed with much discretion for depending on the relative values of τ_w and $(\partial_x p)z$ the discriminant Δ_w might become negative, thus rendering the solution undetermined. Furthermore, the argument of the logarithmic term cannot become negative. In Cruz and Silva Freire (1998), three different cases have been identified and explicitly quoted.

In general, however, Eq. (5) can be seen as a generalization of the classic law of the wall for separating flows. In the limiting case $(\partial_x p)z \ll \tau_w/\rho$, Eq. (5) reduces to the logarithmic expression

$$u^+ = \varkappa^{-1}\ln z^+ + b_m, \qquad b_m = 2\varkappa^{-1} + \varkappa^{-1}\ln((u_{p\nu}^3/4u_*^3)e^{\varkappa C}). \tag{6}$$

Near a point of separation Stratford's solution is recovered.

In principle, Eq. (5) can be used indistinctly in all flow regions – including regions of reversed flow – provided the domain of validity of its discriminant is respected and appropriate integration constants are determined. Equation (5) cannot be written in terms of the similarity variables u_* and $u_{p\nu}$ for in situations where any of these two parameters approach zero, a singularity occurs. Thus, it will be kept in its present form.

The effects of roughness on a boundary layer can be dramatic. Provided the characteristic size of the roughness elements are large enough, a regime can be established where the flow is turbulent right down to the wall. One important consequence is that the viscous sublayer is completely removed so that the linear and Goldstein's solutions do not apply anymore. The roughness also distorts the logarithmic profile acting as if the entire flow is displaced downwards.

The manner in which the logarithmic law is expressed to describe flow over a rough surface depends on the field of application. In meteorology, the common practice is to write

$$u^+ = \varkappa^{-1}\ln((z-d)/z_0), \tag{7}$$

where z is the distance above the actual ground surface.

The specification of the lower boundary condition on rough walls depends thus on two unknown parameters: the aerodynamic surface roughness, z_0, and the displacement height, d. Many works have attempted to relate the magnitude of d and z_0 to geometric properties of the surface. Garratt (1992) mentions that the simple relation d/h_c = 2/3 (h_c = height of canopy) seems to offer good results for many of the natural vegetation of interest. However, since d is known to depend strongly on the way roughness elements are packed together, much discretion must be considered in using this relation. Garratt (1992) also mentions that many texts suggest considering $z_0/h_c = 0.1$. Typical natural surfaces satisfy $0.02 < z_0/h_c < 0.2$.

The derivation that leads to Stratford's law needs not be changed for flow over a rough surface except that, if the equation is to be written in a non-dimensional form, then $z^p = (z-d)/z_0$, $u_{p\nu} = ((z_0/\rho)\partial_x p)^{1/2} + B$ and the integration constant, B, must be used to account for the roughness effects.

The derivation of Eq. (5) has disregarded any detail of the wall roughness. This equation is, in fact, supposed to be valid not in the region adjacent to the wall where the complicated flow around the individual roughness elements is apparent, but, instead, in a region where the flow statistics are spatially homogeneous. Hence, inasmuch as for the classic law of the wall, the characteristics of the rough surface must enter the problem through the integration constant C. In addition, the coordinate system must be displaced by d. The immediate conclusion is that Eq. (5) can be used to model separating flow over a rough surface provided d and C are adequately modeled.

Parameter C is a general function of τ_w, $(\rho^{-1}\partial_x p)$ and z_0 that must be determined by a consistent analysis of experimental data. However, an estimate of its functional behaviour might be obtained by considering the limiting form of Eq. (5) as $\tau_w >> (\partial_x p)z$. The resulting expression is

$$C = \varkappa^{-1}u_*\left[\ln\left(4u_*^2/((z_0/\rho)\partial_x p)\right) - 2\right]. \tag{8}$$

Equation (8) is strictly valid for attached flows in neutral conditions. At a separation point, Eq. (8) furnishes $C = 0$. This result is, as we shall see, not consistent with the findings of Loureiro et al. (2008) for flow over a rough wall.

The obvious implication is that Eq. (8) needs to be specialized in regions where $\tau_w \approx 0$. This is by no means a simple task. While many works can be found in literature on separated flows (see, e.g., the review paper of Simpson (1996) for a large collection of references), very few deal with flow over rough surfaces and pay attention to the flow features

Table 1. Properties of undisturbed profile.

Property	NM	SS	RSA	RSB
Boundary layer thickness (δ, mm)	42.8	100	90	100
Displacement thickness (δ_1, mm)	1.9	09	16	15
External velocity (U_δ, ms^{-1})	4.19	0.0482	0.0497	0.3133
Friction velocity (u_*, ms^{-1})	0.2208	0.0028	0.0047	0.0204
Displacement height (d, mm)	0.0	0.0	2.1	2.0
Roughness length (z_0, mm)	0.01	0.08	0.83	0.33
Reynolds number (R_δ)	11,446	4,772	4,425	31,023
Reynolds number (R_{z_0})	2.82	0.22	3.88	6.65

at or near a separation point. The work of Simpson (1996) is a typical example: no mention to rough surfaces or to the solution of Stratford is made. Of course, roughness and separation effects are particularly relevant in micrometeorological applications. Most contributions to this subject, however, have focused on other issues including velocity speed-up and scalar dispersion. Representative works are Britter et al. (1981), Arya et al. (1987), Athanassiadou and Castro (2001) and Ohba et al. (2002).

The lack of experimental data on flows over rough walls and near to a separation point clearly hampers any serious attempt at establishing a definitive expression for C. An assessment on the applicability of C on conditions to which it is not supposed to hold is thus a matter of utmost importance.

3. Experiments

Equations (5) and (8) will be tested against the smooth wall DNS data of Na and Moin (1998) – NM – the smooth wall LDA data of Loureiro et al. (2007a, 2007b) – SS – and the two rough wall LDA data sets of Loureiro et al. (2008) – RSA, RSB.

For a detailed description of the numerical simulations and of the experiments the original texts are referred. Succinctly, the simulations of Na and Moin (1998) were conducted for flow over a flat, smooth surface with a prescribed adverse pressure gradient. The LDA measurements of Loureiro et al. (2007a, 2007b, 2008a, 2008b) were conducted in a water channel for flows over steep, smooth and rough hills. In all, 36 flow positions were characterized by Loureiro et al. (2007a, 2007b, 2008a, 2008b): 13 for the SS-conditions, 10 for the RSA-conditions and 13 for the RSB-conditions.

The main characteristics of the laser-Doppler system varied between experiments. Most measurements were carried out with a two-component Dantec system fitted with an Ar-ion tube laser that was set to operate in the forward scatter mode. A Bragg cell unit was used to resolve flow direction. Typical uncertainties associated with the mean velocity data are below 0.2% of U_δ (the free stream velocity). In regions of reverse flow, the uncertainties increase to about $0.3\% U_\delta$. The hill was 60 mm high and followed a Witch of Agnesi profile. The roughness elements consisted of rigid rubber strips 3 mm wide by 3 mm high that were spaced by 9 mm. The rough surface extended from 1.5 m upstream of the hill top to 1.5 m downstream. The main properties of the undisturbed profiles are shown in Table 1.

4. Results

The applicability of Eq. (8) is assessed through the following procedure. First, parameters u_*, $\partial_x p$ and C are estimated through Eq. (5) from given profiles of u versus z. Next, parameters u_* and $\partial_x p$ are again estimated but with C given by Eq. (8). The results are then compared with the directly calculated smooth surface values of Na and Moin (1998) and of Loureiro et al. (2007a, 2007b). For flow over a rough surface, where directly measured values of u_* are not available, results are compared with each other.

To estimate the parameters in Eqs. (5) and (8) the same procedure described in Loureiro et al. (2008) was adopted. Global optimization algorithms based on direct search methods were used. Despite their tendency to converge more slowly, direct search methods can be more tolerant to the presence of noise in the function and constraints. Four different methods were used for solution search: Nelder Mead, Differential Evolution, Simulated Annealing and Random Search. Only when all four methods furnished consistent results, with accuracy down to the sixth decimal fraction, the search was stopped.

The fitting results are illustrated by the selection of curves presented in Figs. 1 to 4. All shown curves were drawn with the estimated values of C. Curves were chosen so as to represent conditions upstream of the separation point and in the region of reversed flow for every flow condition.

For flow over a smooth wall – Figs. 1a and 2a – the viscous, fully turbulent and external inertial regions are well

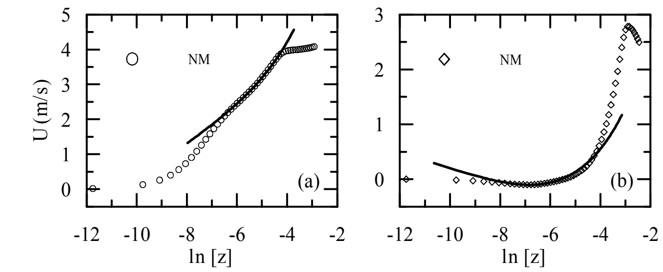

Figure 1. Typical curve fits to the data of Na and Moin (1998). a: upstream profile at station x/δ^*_{in} = 100, b: reverse flow region profile at station x/δ^*_{in} = 250. z is plotted in m.

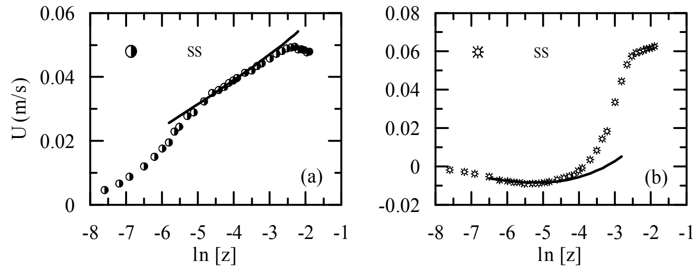

Figure 2. Typical curve fits to the data of Loureiro et al. (2007a, 2007b). a: upstream profile at station x/H = -12.5, b: reverse flow region profile at station x/H = 3.75. z is plotted in m.

discerned in the upstream profiles. Equation (5) is observed to fit very well the fully turbulent region, thus capturing the log-dominated shape of the local velocity profiles. In the regions of reverse flow, the picture is different. The detailed very near wall DNS data of Na and Moin (1998) (Fig. 1b) captures the viscous solution of Goldstein (1948) for ln[z] < -9. Away from the wall, a typical region of reverse flow follows, which is observed to have the mean velocity profile well represented by Eq. (5). The data of Loureiro et al. (2007a, 2007b) are also close enough to the wall to have the viscous region well discriminated. The consequence is that the curve fit resulting from Eq. (5) represents very well the data but the first five nearest to the wall points.

The curve fits for flows over a rough surface are shown in Figs. 3 and 4. The destruction of the viscous layer by the roughness elements is evident. The consequence is that the set of points to be fitted extends right down to the wall. Agreement between Eq. (5) and the data is very good, indistinctly of the flow station, whether in regions of attached or reverse flow.

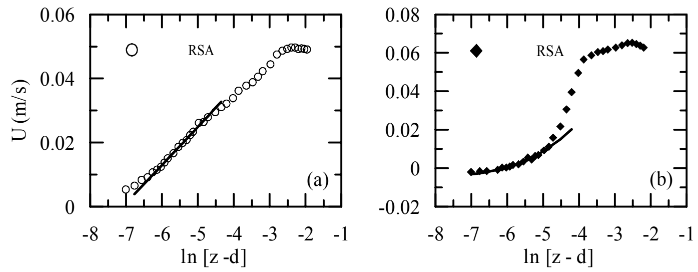

Figure 3. Typical curve fits to the data of Loureiro et al. (2008), RSA conditions. a: upstream profile at station x/H = -5.8, b: reverse flow region profile at station x/H = 1.31. z is plotted in m.

Predictions of u_* based on Eqs. (5) and (8) are shown in Fig. 5. For the smooth wall data, three sets of points are shown: predictions with estimated C's (= C_{est}) from Eq. (5), predictions with parametrized C's (= C_{par}) from both Eqs. (5) and (8), and the reference data of Na and Moin (1998) and of Loureiro et al. (2007a, 2007b).

For the work of Na and Moin (1998) (Fig. 5a), both predictions agree well with the DNS data, except at two positions close to the reattachment point. Actually, predictions with C_{est}'s are almost exact for all flow domain, but very near the reattachment point where a very high value is found. The parametrization of C yields slightly lower values of u_* upstream of the separation point, an almost exact prediction at the separation and reattachment points, and an uncharacteristic very low value just before the reattachment.

Predictions for the smooth wall data of Loureiro et al. (2007a, 2007b) are shown in Fig. 5b. Upstream of the hill, at

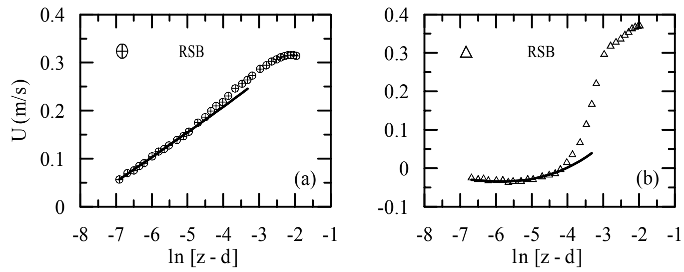

Figure 4. Typical curve fits to the data of Loureiro et al. (2008), RSB conditions. a: upstream profile at station x/H = -17.87, b: reverse flow region profile at station x/H = 3.06. z is plotted in m.

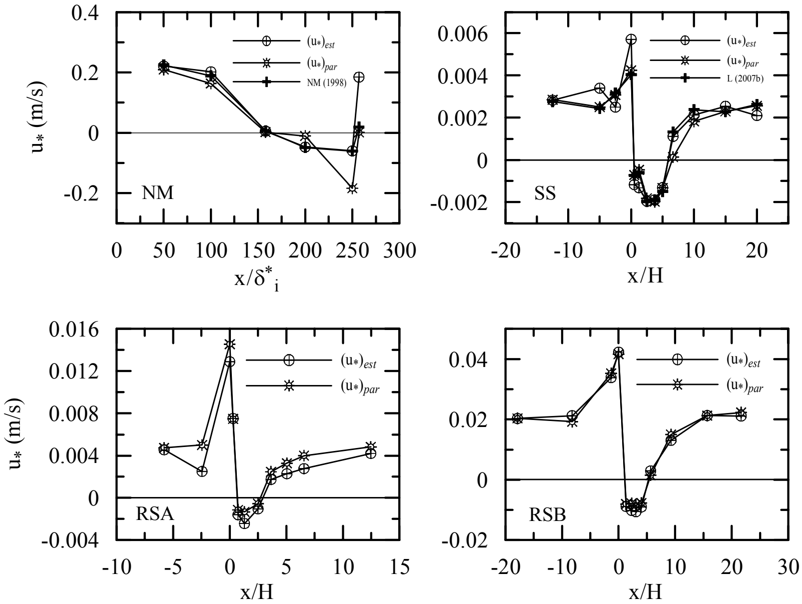

Figure 5. Predictions of u_*(ms^{-1}) estimated through Eq. (5), $(u_*)_{est}$, in comparison to values calculated from Eqs. (5) and (8), $(u_*)_{par}$.

x/H = -12.5, both predictions coincide with the reference experimental value of u_*. On the upwind side of the hill and on the hill top, predictions with C_{par} give an almost exact result. The use of C_{est} overshoots u_* at the hill foot (x/H = -5) and top (x/H = 0), and gives a lower value at x/H = -2.5. In the region of reverse flow, predictions through both procedures agree very well with the experiments. Downstream of the hill, predictions with C_{par} exhibit a slow recovery to the undisturbed values of u_*. Position x/H = 6.67, in particular, shows the worst agreement. Use of C_{est} results in very good predictions for $x/H \geq$ 6.67.

The rough wall data of Loureiro et al. (2008) are shown in Figs. 5c and 5d. For the lower Reynolds number (RSA conditions), Eq. (8) gives consistently higher values of u_* when compared with values obtained through a direct estimation of C. The worst agreement is noted to occur at position x/H = -2.45, the mid-length of the upstream slope of the hill. Results for the higher Reynolds number (RSB conditions) offer a very distinct perspective (Fig. 5d) on the usefulness of Eq. (8). The differences between values of u_* given by both procedures are almost unnoticed. The parametrization works very well even in regions where it is supposed to fail, that is, close to separation and attachment points.

Velocity profiles in the coordinates of Stratford are shown in Fig. 6 to conditions NM and RSB. The resulting best fits are given respectively by U = 13.7455$z^{1/2}$ - 0.2420 and U = 1.1120$z^{1/2}$ - 0.0471. Hence, in the limit $\tau_w \rightarrow 0$, the additive parameter C is different from zero, in opposition to Eq. (8).

To better understand the role of C on u_*-predictions, Fig. 7 is in order. In general, higher values of C result in higher values of u_* and vice-versa. Exceptions, however, can be noticed for conditions NM (2 points near to reattachment) and SS (top of the hill). For the rough wall conditions, RSA and RSB, no exception is recorded. Regarding the NM-conditions, both C_{est} and C_{par} follow the same trend and are close together (Fig. 7a). Upstream of the separation point, $C_{est} > C_{par}$ is satisfied. In the region of reversed flow this inequality is inverted. The crosses denote values of C calculated directly from Eq. (5) with values of u_* and $\partial_x p$ given by the DNS-simulations.

For flow over the smooth and rough hills, SS- RSA- and RSB-conditions, we have $C_{par} > C_{est}$. Only two exceptions

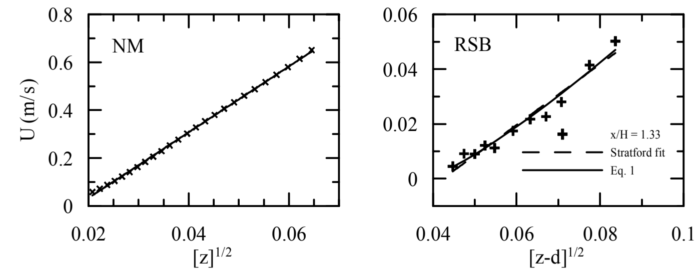

Figure 6. Stratford profiles. z is plotted in m; U in ms^{-1}.

upstream of the hill top are observed (Figs. 7b and 7d). Clearly, the higher the values of u_*, the less sensitive the results are to variations in C. For the SS-conditions, large variations of C on the upstream side of the hill and at the hill top are visible in the u_* predictions. For conditions RSA and RSB, large changes in C do not propagate significantly to u_*.

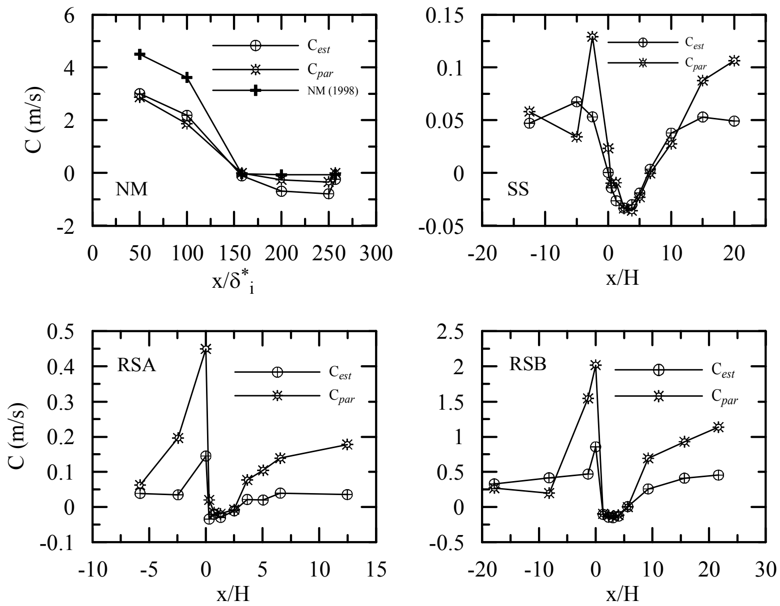

Figure 7. Behaviour of the parametrization function C(ms^{-1}).

5. Final remarks

The present work has investigated in detail the parametrization method proposed by Loureiro et al. (2008) to account for separation and roughness effects on the lower boundary condition for turbulent boundary layers. The overall agreement between the predictions of u_* furnished by Eqs. (5) and (8) and the 'exact' data of Na and Moin (1998) and of Loureiro et al. (2007a, 2007b, 2008) is very good, in particular, for conditions SS.

In general, values furnished by C_{par} are found to be larger than the estimated values, C_{est}. The differences, however, are not transmitted in proportion to predictions of u_*. This is noticed for all conditions, in special, RSA and RSB conditions. In fact, predictions of u_* for conditions RSB using C_{par} and C_{est} coincide almost exactly. An immediate implication is that C_{par} can be used to describe the lower boundary condition in all flow regions, including regions away or near to a separation point.

The use of wall functions to specify the lower boundary condition has been greatly criticized in the past in regard to its apparent lack of validity outside the conditions of equilibrium flow, particularly near flow separation. As an alternative approach, low-Reynolds number turbulence models have been developed. The success of the latter approach relies necessarily on the deployment of very fine grids, capable of resolving the near wall region to normal lengths of a fraction of ν/u_*. This requirement alone establishes such a large demand on computer cost so as to make any attempt at the numerical simulation of environmental or industrial flows an unfeasible endeavor.

For flows over rough surfaces, the above discussion is out of place: the wall function approach is the only viable option. It is then just natural that many authors have sought extensions of the standard law of the wall to rough walls. This is a very complex task on its own right. Very few authors, however, have dealt simultaneously with separation and roughness.

The present contribution helps to fill this gap. Together with Loureiro et al. (2008), this work forms a consistent theory on flow subject to separation and roughness, which might in the future be implemented in predictive numerical codes.

The present work has also provided valuable data on the mean properties of turbulent separated flows over rough hills – in particular, the wall shear stress – that can be used to validate numerical simulations of the problem.

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