THE AVERAGE MESO-SCALE BEHAVIOR OF A GAS-SOLID RISER FLOW

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Abstract. *Two-fluid modeling is currently the most practical approach for large cluster simulation (LCS) of gas-solid flows in risers. LCS requires sub-grid models for recovering the effects of non-resolved clusters. The literature presents sub-grid correlations generated from computational experiment also applying two-fluid modeling. The concerning simulations are developed in periodic domains, and an additional gas phase pressure gradient is introduced in the vertical direction to account for the flow driving force, which is chosen to exactly match the gravity acting on the gas-solid mixture. Even though the simulations give rise to low velocity gas-solid suspensions, the clustering mechanism is believed and generally assumed to be similar to that which prevails in riser flows. In this work we developed sub-grid simulation to investigate this assumption. We found that low velocity gas-solid suspensions can not completely describe the meso-scale behavior of a riser flow, so that more realistic gas-solid flow conditions are needed.*

Keywords*: gas-solid flow, two-fluid model, risers, sub-grid simulation*

1. INTRODUCTION

The current two-fluid simulations of real risers are necessarily performed in very coarse numerical meshes due to computational limitations, and only the macro-scales of the flow are resolved. Therefore, if accuracy is desired, the average effects of the spatial meso-scales filtered by the numerical meshes must be recovered. A simulation taking into account the meso-scale behavior may be addressed to as a large cluster simulation (LCS), in analogy to the large eddy simulation (LES) applied in turbulence modeling. While LES requires sub-grid models to recover the effects of non-resolved eddies, LCS requires sub-grid models to recover the effects of non-resolved clusters.

It is currently recognized that the most practical approach to modeling gas-solid flows in risers comes from the so called Eulerian or two-fluid models, and that computational experiment represents the most promising way for sub-grid investigation (Sundaresan, 2000). Sub-grid modeling and simulation requires the micro-scale of the flow to be properly described. The current state of the art does not include micro-scale experimental information of gas-solid flows, and the so called kinetic theory of granular flows (KTGF) is widely applied. This theory is an analogy with the kinetic theory of dense gases, which is modified to account for the inelastic particle collisions characteristic of gas-solid flows (Bagnold, 1954, Jenkins and Savage, 1983, Lun et al., 1984, Gidaspow, 1994).

Following the above, computational experiments have been performed by some researchers to provide sub-grid correlations to be used in LCS of gas-solid riser flows (Agrawal et al., 2001, Andrews IV et al., 2005). In general, the simulations were performed in small periodic domains which are thought to repeat themselves throughout the whole volume of a riser. As periodic boundaries are applied an additional gas phase pressure gradient is introduced in the vertical direction to account for the flow driving force. This additional term is chosen to exactly match the gravity acting on the gas-solid mixture, so that the simulations give rise to low velocity gas-solid suspensions. This assumption is brought from previous studies on the instabilities that develop in unforced granular materials when the inelasticity of the collisions among particles is accounted for, which ultimately leads to the formation of clusters (see, for instance, Goldhirsch et al., 1993, and Tan and Goldhirsch, 1997). Those studies clearly stand for quasi static regimes, where particulates arrange themselves in low velocity suspensions. It is believed, however, that the clustering mechanism that prevails is also relevant to rapid gas-solid flows (Tan and Goldhirsch, 1997).

In addition to the above assumptions, Agrawal et al. (2001) imposed macro-scale shear rates through opposed parallel vertical boundaries in their small scale periodic domains. From the predictions the authors determined meso-scale parameters of the flow that were analyzed as a function of the imposed macro-scale shear rates and average solid phase volumetric fractions. Andrews IV et al. (2005) further extended the analysis of Agrawal et al. (2001) by actually deriving expressions for the meso-scale parameters.

Following the community effort, we have also performed two-fluid sub-grid simulations of gas-solid flows in periodic domains (Milioli and Milioli, 2007a,b). All the previous literature works considered a particulate size typical of high density catalytic cracking fluidized bed reactors (75 μm, with density of 1500 kg/m3), while our simulations included also a particulate size typical of low density circulating fluidized bed combustors (520 μm, with density of 2620 kg/m3). We have also imposed a flow driving force through an additional gas phase pressure gradient chosen to balance the gravity over the mixture and, therefore, like in the previous literature works, we have produced predictions of low velocity gas-solid suspensions.

It is not known, however, how representative such low velocity fields are regarding the flow in real high velocity risers. In this article we propose to investigate this particular issue. In order to fulfill such a task we propose, differently from all the previous works, to apply an additional gas phase pressure gradient in excess to that required to match the gravity acting on the gas-solid mixture. At first, it was thought that by applying this procedure the flow would accelerate and find a new statistical steady state regime at a higher velocity level. Different extra gas phase pressure gradients would cause the flow to stabilize at different levels of velocity. The above supposition was tested and failed since no significant counter effect developed in the flow so as to compensate the imposed acceleration. We think, however, that even though the flow never finds its statistical steady state regime, a dynamical pattern is found that still allows for the desired analysis by considering instantaneous predictions at suitable gas phase velocities inside the range typical of circulating fluidized beds. In the present work we applied this procedure, and a discussion is advanced of the transient results that have been found.

2. MODELING

Multiphase flow two-fluid models stand on the major hypothesis of continuum for all of the phases, no matter fluid or particulate. The phases are treated as inter-penetrating dispersed continua in thermodynamic equilibrium. The theory of two-fluid models has been developed by many researchers. Some classical reference works on this matter are those due to Anderson and Jackson (1967), Ishii (1975), Drew (1983), Gidaspow (1994) and Enwald et al. (1996). The two-fluid models comprise a basic set of mass and momentum averaged conservative equations plus closure laws for viscous stress tensors, viscosities, pressures and drag. The description of solid phase properties requires, in addition, the determination of granular temperatures from a pseudo thermal energy balance provided by the kinetic theory of granular flows. Also, as periodic boundaries are to be applied, an additional gas phase pressure gradient must be introduced in the vertical direction to account for the flow driving force.

A formulation of the two-fluid model including closure laws based on the KTGF (Gidaspow, 1994; Syamlal et al., 1993, Agrawal et al., 2001), and including gravity compensation for applying periodic boundary conditions, comprises:

Gas phase continuity

$$\frac{\partial}{\partial t}\left(\rho_g \alpha_g\right)+\vec{\nabla}\cdot\left(\rho_g \alpha_g \vec{U}_g\right)=0 \tag{1}$$

Solid phase continuity

$$\frac{\partial}{\partial t}\left(\rho_s \alpha_s\right)+\vec{\nabla}\cdot\left(\rho_s \alpha_s \vec{U}_s\right)=0 \tag{2}$$

Gas phase momentum

$$\frac{\partial}{\partial t}\left(\rho_g \alpha_g \vec{U}_g\right)+\vec{\nabla}\cdot\left(\rho_g \alpha_g \vec{U}_g \vec{U}_g\right)=-\alpha_g\left(\vec{\nabla}P_g+\psi\vec{\nabla}P_g^*\right)+\vec{\nabla}\cdot\left(\alpha_g \overline{\overline{\tau}}_g\right)+\rho_g \alpha_g \vec{g}+\beta\left(\vec{U}_s-\vec{U}_g\right) \tag{3}$$

Solid phase momentum

$$\frac{\partial}{\partial t}\left(\rho_s \alpha_s \vec{U}_s\right)+\vec{\nabla}\cdot\left(\rho_s \alpha_s \vec{U}_s \vec{U}_s\right)=-\alpha_s\left(\vec{\nabla}P_g+\psi\vec{\nabla}P_g^*\right)-\vec{\nabla}\left(P_s\right)+\vec{\nabla}\cdot\left(\alpha_s \overline{\overline{\tau}}_s\right)+\rho_s \alpha_s \vec{g}-\beta\left(\vec{U}_s-\vec{U}_g\right) \tag{4}$$

Volumetric continuity

$$\alpha_g+\alpha_s=1 \tag{5}$$

Interface drag function

$$\beta=150\frac{\alpha_s^2 \mu_g}{\alpha_g (d_p \varphi_s)^2}+1.75\frac{\rho_g \alpha_s \left|\vec{U}_g-\vec{U}_s\right|}{(d_p \varphi_s)} \quad \text{for } \alpha_s > 0.2 \qquad \text{(Ergun, 1952)} \tag{6}$$

$$\beta=\frac{3}{4}C_{Ds}\frac{\rho_g \alpha_s \alpha_g \left|\vec{U}_g-\vec{U}_s\right|}{(d_p \varphi_s)}\alpha_g^{-2.65} \qquad \text{for } \alpha_s \le 0.2 \qquad \text{(Wen and Yu, 1966)} \tag{7}$$

$$\text{where } C_{Ds}=\begin{cases}\dfrac{24}{Re_P}\left(1+0.15\cdot Re_p^{0.687}\right) & for \quad Re_p < 1000\\ 0.44 & for \quad Re_p \ge 1000\end{cases} \qquad \text{(Rowe,1961)} \tag{8}$$

$$Re_p = \frac{\left|\vec{U}_g - \vec{U}_s\right| d_p \rho_g \alpha_g}{\mu_g} \tag{9}$$

Granular temperature (Syamlal et al., 1993)

$$\Theta = \left(\frac{-K_1 \alpha_s tr\left(\overline{\overline{D}}_s\right) + \sqrt{K_1^2 tr^2\left(\overline{\overline{D}}_s\right)\alpha_s^2 + 4K_4\alpha_s\left[K_2 tr^2\left(\overline{\overline{D}}_s\right) + 2K_3 tr\left(\overline{\overline{D}}_s^2\right)\right]}}{2\alpha_s K_4} \right)^2 \tag{10}$$

where $\overline{\overline{D}}_s = \frac{1}{2}\left[\vec{\nabla}\vec{U}_s + \left(\vec{\nabla}\vec{U}_s\right)^T\right]$ (11)

$$K_1 = 2(1+e)\rho_s g_0 \tag{12}$$

$$K_2 = \frac{4 d_p \rho_s (1+e)\alpha_s g_0}{3\sqrt{\pi}} - \frac{2}{3}K_3 \tag{13}$$

$$K_3 = \frac{d_p \rho_s}{2}\left\{\frac{\sqrt{\pi}}{3(3-e)}\left[1 + 0.4(1+e)(3e-1)\alpha_s g_0\right] + \frac{8\alpha_s g_0 (1+e)}{5\sqrt{\pi}}\right\} \tag{14}$$

$$K_4 = \frac{12\left(1-e^2\right)\rho_s g_0}{d_p\sqrt{\pi}} \tag{15}$$

$$g_0 = \frac{3}{5}\left[1 - \left(\frac{\alpha_s}{\alpha_{s,max}}\right)^{1/3}\right]^{-1} \tag{16}$$

Viscous stress tensor ($k = s, g$)

$$\overline{\overline{\tau}}_k = \mu_k\left[\vec{\nabla}\vec{U}_k + \left(\vec{\nabla}\vec{U}_k\right)^T\right] + \left(\lambda_k - \tfrac{2}{3}\mu_k\right)\left(\vec{\nabla}\cdot\vec{U}_k\right)\overline{\overline{I}} \tag{17}$$

where $\mu_g = \text{constant}$ (18)

$$\lambda_g = 0 \tag{19}$$

$$\mu_s = \frac{4}{5}\alpha_s \rho_s d_p g_0 (1+e)\left(\frac{\Theta}{\pi}\right)^{1/2} \tag{20}$$

$$\lambda_s = \frac{4}{3}\alpha_s \rho_s d_p g_0 (1+e)\left(\frac{\Theta}{\pi}\right)^{1/2} \tag{21}$$

Solid phase pressure

$$P_s = \alpha_s \rho_s \Theta\left[1 + 2(1+e)g_0\alpha_s\right] \tag{22}$$

Gravity compensation (for periodic boundaries)

$$\vec{\nabla}P_g^* = \left(\rho_s\alpha_s + \rho_g\alpha_g\right)\vec{g} \tag{23}$$

3. SIMULATION

In the present simulation a 2cm x 2cm wide and 8cm tall vertical hexahedral domain is considered, applying a 1mm x 1mm x 1mm uniform hexahedral numerical mesh. The flow enters the domain through the bottom and exits at the top, while walls are considered in all of the lateral boundaries. A time step of $5\text{x}10^{-5}$ seconds is applied which is suitable for sub-grid simulations. The expected time scale of clusters of the order of 10^{-2} seconds (following Sharma et al., 2000) is expected to be fully captured. A particulate size of 520 μm is applied so that the smaller clusters on the flow are expected not to be larger than 5.2 mm (following Agrawal et al., 2001). Therefore, regarding the solid phase, both the spatial and temporal meshes which are applied are suitable for sub-grid simulations.

A flow driving force, i.e. an additional gas phase pressure gradient was applied in excess to that required to match the gravity acting on the gas-solid mixture. Different values of the driving force factor (ψ) were considered, and in all of the situations the statistical steady state regime was never reached. It was found that no significant counter effect develop in the flow so as to compensate the imposed acceleration. The results of simulation presented in this work were generated applying a ψ of 1.5, which allowed to go through a range of gas axial velocities typical circulating fluidized beds in reasonable computing times.

The initial condition for the simulation was a time step taken inside the statistical steady state regime found by applying the additional gas phase pressure gradient which exactly matches the gravity acting on the gas-solid mixture (i.e. $\psi = 1$). Therefore, the simulation occurs in two stages, the first for $\psi = 1$ and the second for $\psi = 1.5$. Only the second stage is to be analyzed in this work. The first stage has already been considered in a previous work (Milioli and Milioli, 2007a). Table 1 shows the operating and numerical conditions applied in the simulation.

Table 1. Operating and numerical conditions.

Domain 2 cm x 2 cm x 8cm Mesh 32000 cubic cells (1mm x 1mm x 1mm) 35721 nodes Time step 5 x 10^{-5} s rms for convergence 1 x 10^{-5} Properties d_p = 520 μm ρ_s = 2620 kg/m^3 v_t = 2.6457 m/s ρ_g = 1.1614 kg/m^3 μ_g = 1.82 x 10^{-5} Pa.s e = 0.9	Driving force $\psi = 1$ (first stage) $\psi = 1.5$ (second stage) (*) Boundary conditions Entrance/exit: periodic Walls: free slip Initial conditions (first stage) $u_g = v_g = w_g =$ 0 m/s $u_s = v_s = w_s =$ 0 m/s $\alpha_s =$ 0.05 m$_s^3$/m^3 Initial conditions (second stage) A time step inside the statistical steady state regime taken from the simulation for the first stage (with $\psi = 1$).

(*) the value of ψ was chosen to ensure a stable evolution of the iterative procedure through a suitable range of gas phase velocities in a reasonable computing time.

4. RESULTS

Figure 1 shows the time development of the phase's volume averaged axial velocities. Departing from the initial conditions defined in Table 1, under a unitary driving force factor (i.e. an additional gas phase pressure gradient to exactly match the gravity on the suspension), the flow develops and reaches a statistical steady state regime. Then the driving force coefficient is turned to 1.5, and the flow becomes accelerated. Notice that the volume averaged solid phase axial velocity results negative in the statistical steady state regime, in the stage of the simulation where the gravity acting on the suspension is exactly matched. This is a clear consequence of the clustering that develops in the flow, since the gravity does not simply act on a bunch of individual particles, but on a relatively heavier bunch of clusters of particles.

Figure 1 also shoes that, while both the phase axial velocities grow in time, the slip velocity is kept approximately constant. This suggests that the clustering topology of the flow is not significantly affected by the increasing gas axial velocities. This is a first evidence of the validity of the assumption of Tan and Goldhirsch (1997), that the clustering mechanism in low velocity suspensions also prevails in rapid gas-solid flows.

Figures 2 to 4 show the time evolution of instantaneous volume averaged predictions as a function of a modified domain width based Froude's number, defined as $Fr_h^* = \left(\langle v_s \rangle \cdot \left| \langle v_s \rangle \right| \right) / \left(|g| \cdot h \right)$, where $\langle v_s \rangle$ is the domain volume averaged axial velocity of the solid phase. It should be noted that this number is a variable in the simulation, and is negative as the solid flows downwards, and positive as the solid flows upwards. Also, the modified Froude's number increases as the gas axial velocity increases, since the solid axial velocity increases as well. It should be noted that the definition of the conventional Froude's number $\left(Fr_h = v_t^2 / \left(|g| h \right) \right)$ is not useful in the present analysis since the domain width and the particle terminal velocity are not varied.

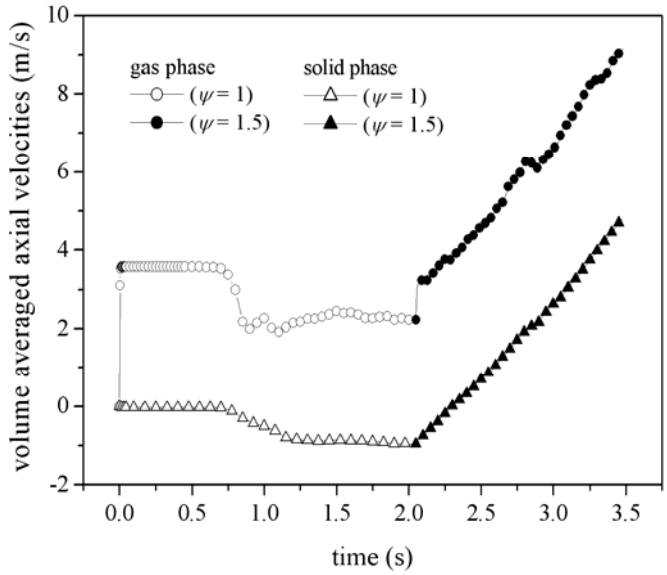

Figure 1. Time development of the phase's volume averaged axial velocities.

Figure 2 presents the variation of the volume averaged axial velocities against the modified Froude's number. It is seen that both the phase velocities increase with the increasing modified Froude's number. However, the velocities increasing rates decrease in time, suggesting an asymptotic behavior towards the independence of the axial velocities with the modified Froude's number. Such an independence, however, is possibly attained only in the pneumatic transport regime. Once more, the averaged slip velocity is kept approximately constant as the flow conditions are changed, reinforcing the correctness of the Tan and Goldhirsch's assumption.

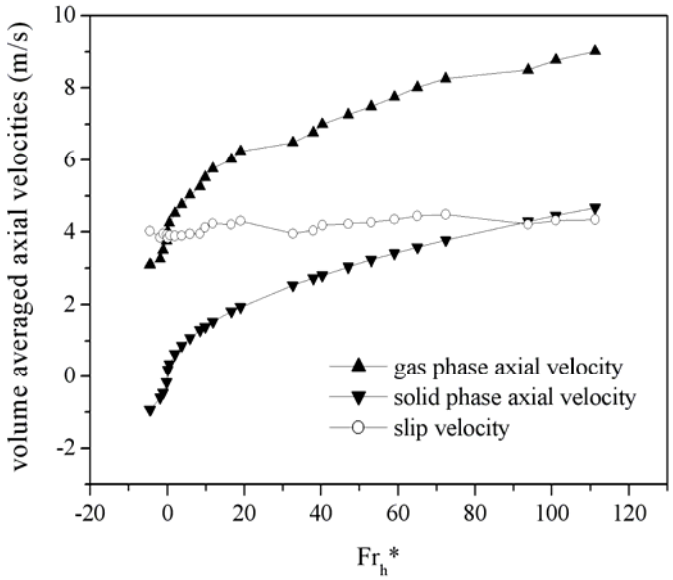

Figure 2. Volume averaged axial velocities against the modified Froude's number.

Figure 3 presents the variation of the volume averaged solid phase volume fraction against the modified Froude's number. It is seen that the averaged solid fraction is kept constant independently of the flow conditions, which is reasonable in view of the application of periodic boundaries at entrance and exit. The fact that the solid fraction of the initial suspension is kept constant for any modified Froude's number, or throughout all the range of gas axial velocities, is another indication of the correctness of the Tan and Goldhirsch's assumption.

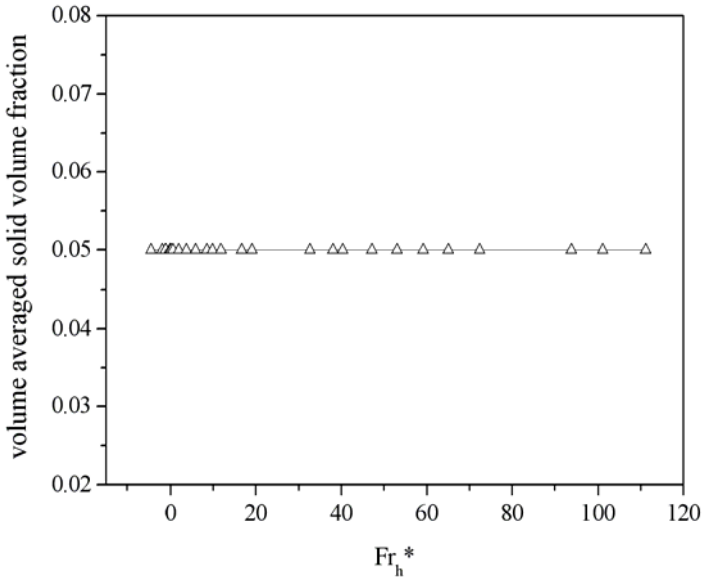

Figure 3. Volume averaged solid phase volume fraction against the modified Froude's number.

Figure 4 presents the variation of the volume averaged solid phase shear strain rate and solid phase pressure against the modified Froude's number. Both the parameters decrease at higher modified Froude's numbers, or higher gas axial velocities, also showing the asymptotic behavior already seen in Figure 2. Lower shear strain rates are a consequence of lower velocity gradients, so that at higher velocities the flow tends to become more homogeneous. While this is an indication that the pneumatic transport regime is approaching, it is also a feature that contradicts the Tan and Goldhirsch's assumption. It seems that the increasing uniformity of the flow also contributes to lower solid phase pressures.

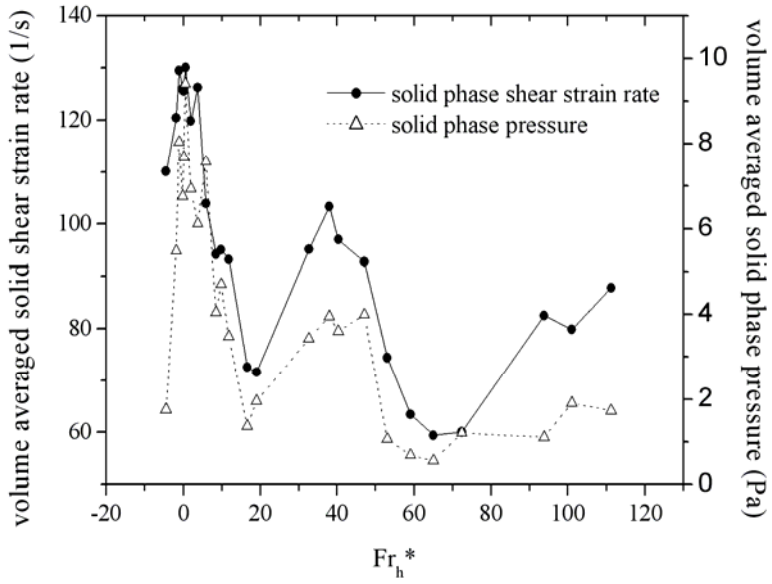

Figure 4. Volume averaged solid phase shear strain rate and solid phase pressure against the modified Froude's number.

5. CONCLUSIONS

The main proposition of this article was to discuss whether low velocity gas-solid suspensions can actually be used to describe the meso-scale behavior of rapid gas-solid riser flows. We considered, in particular, the proposition of Tan and Goldhirsch (1997) that the clustering mechanism in low velocity suspensions also prevails in rapid gas-solid flows. Through a computer experiment, we found arguments in both senses, in favor and against the proposition.

We found evidence that the clustering topology of the flow is not significantly affected by increasing gas axial velocities, which supports the Tan and Goldhirsch's assumption. In addition, the solid volume fraction of the initial suspension resulted constant throughout the whole range of gas axial velocities, which also supports the proposition. But we also found evidence that the gas-solid flow becomes more homogeneous at higher velocities, which is in contradiction with Tan and Goldhirsch's assumption.

In view of the present results we conclude that low velocity gas-solid suspensions can not completely describe the meso-scale behavior of a riser flow. Therefore, more realistic gas-solid flow conditions must be produced if accurate sub-grid correlations are desired.

6. ACKNOWLEDGEMENTS

This work was supported by The State of São Paulo Research Foundation (FAPESP) and The National Council for Scientific and Technological Development (CNPq).

7. LIST OF SYMBOLS

C_D - drag coefficient, non-dimensional
d_p - particle diameter, m
e - solid phase restitution coefficient, non-dimensional
Fr_h - Froude´s number, non-dimensional
$\vec{g}$ - gravity acceleration, m/s^2
g_0 - radial distribution function, non-dimensional
$\overline{\overline{I}}$ - unit tensor
P - pressure, N/m^2
$\vec{\nabla} P_g^*$ - additional gas phase pressure gradient to exactly match the gravity acting on the gas-solid mixture, N/m^3
Re_p - Reynolds number, non-dimensional
t - time, s
$\vec{U}$ - average velocity vector, m/s
u, v, w - velocity components in the x, y, z directions, m/s
v_t - particle terminal velocity, m/s

Greek

α - volume fraction, m_k^3/m^3
$\alpha_{s,max}$ - solid volume fraction at packing, m_s^3/m^3
β - gas-solid friction coefficient, kg/m^{3}s
Θ - granular temperature, m^2/s^2
λ - bulk viscosity, Ns/m^2
μ - dynamic viscosity, Ns/m^2
ρ - density, kg/m^3
$\overline{\overline{\tau}}$ - viscous stress tensor, N/m^2
φ - particle sphericity, non-dimensional
ψ - driving force factor ($\psi = 1$ for exactly matching the gravity acting on the mixture), non-dimensional

Subscripts

g - gas phase
k - either gas or solid phases
s - solid phase

8. REFERENCES

Agrawal, K., Loezos, P. N., Syamlal, M. and Sundaresan, S., 2001, "The role of meso-scale structures in rapid gas-solid flows", Journal of Fluid Mechanics, Vol. 445, pp. 151-185.

Anderson, T. B. and Jackson, R., 1967, "Fluid mechanical description of fluidized beds. Equations of motion", Indust. Engng. Chem. Fundam., Vol. 6, pp. 527-539.

Andrews IV, A.T., Loezos, P. N. and Sundaresan, S., 2005, "Coarse-grid simulation of gas-particle flows in vertical risers", Ind. Eng. Chem. Res., Vol. 44, No. 16, pp. 6022-6037.

Bagnold, R.A., 1954, "Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear", Proceedings of the Royal Society, Vol. A225, pp. 49-63.

CFX, 2004, "Discretization and solution theory", in Solver theory manual, Ed. Ansys Canada Ltda, pp. 227-250.

Drew, D. A., 1971, "Averaged field equations for two-phase media", Studies in Appl. Meth., Vol. 1, No. 2, pp.133-136.

Enwald, H., Peirano, E. and Almstedt, A. -E., 1996, "Eulerian two-phase flow theory applied to fluidization", Int. J. of Multiphase Flow, Vol. 22, pp. 21-66.

Ergun, S., 1952, "Fluid flow through packed columns", Chem. Engng. Prog., Vol. 48, No. 2, pp. 89-94.

Gidaspow, D., 1994, "Multiphase flow and fluidization", Academic Press, San Diego, CA, 407 p.

Goldhirsch, I., Tan, M. -L. and Zanetti, G., 1993, "A molecular dynamical study of granular fluids I: the unforced granular gas in two dimensions", Journal of Scientific Computing, Vol. 8, No. 1, pp. 1-40.

Ishii, M., 1975, "Thermo-fluid dynamic theory of two-phase flow", Eyrolles, Paris, 248 p.

Jenkins, J.T. and Savage, S.B., 1983, "A Theory for the Rapid Flow of Identical, Smooth, Nearly Elastic Spherical Particles", J. Fluid. Mech., Vol. 130, pp. 187-202.

Lun, C. K. K., Savage, S. B., Jeffrey, D. J. and Chepurniy, N., 1984, "Kinetic theories for granular flows: inelastic particles in Couette flow and singly inelastic particles in a general flow field", J. Fluid. Mech., Vol. 140, pp. 223-256.

Milioli, C.C., Rotava, E. and Milioli, F.E., 2007a, "On the sub-grid two-fluid simulation of gas-solid riser flows", XIX Congresso Brasileiro de Engenharia Mecânica - COBEM 2007, Brasília-DP, Brasil.

Milioli, C.C., Rotava, E. and Milioli, F.E., 2007b, "A sub-grid two-fluid simulation of a typical gas-solid riser flow", XIX Congresso Brasileiro de Engenharia Mecânica - COBEM 2007, Brasília-DP, Brasil.

Rowe, P. N., 1961, "Drag forces in a hydraulic model of a fluidized bed. Part II", Trans. IChemE, Vol. 39, pp. 175-180.

Sharma, A. K., Tuzla, K., Matsen, J. and Chen, J. C., 2000, "Parametric effects of particle size and gas velocity on cluster characteristics in fast fluidized beds", Powder Technology, Vol. 111, pp. 114-122.

Sundaresan, S., 2000, "Modeling the hydrodynamics of multiphase flow reactors: current status and challenges", AIChE J., Vol. 46, No. 6, pp. 1102-1105.

Syamlal, M, Rogers, W. and O´Brien, T, 1993, "MFIX documentation theory guide", U.S. Department of Energy, West Virginia.

Tan, M. -L. and Goldhirsch, I., 1997, "Intercluster interactions in rapid granular shear flows", Physics of Fluids, Vol. 9, No. 4, pp. 856-869.

Wen, C. Y. and Yu, Y. U., 1966, "Mechanics of fluidization", Chem. Engng. Prog. Symp. Series, Vol. 62, pp. 100-111.

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