SOLUTION OF THE MPCP OPTIMIZATION PROBLEM USING THE MATHEMATICA PROGRAM

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Abstract. *The well-known CGAM problem was formulated in 1994 to serve as a benchmark for comparison of different thermoeconomic optimization methodologies. The CGAM cogeneration plant produced 30 MW of power and 14 kg/s of saturated vapor at 20 bar. The objective function consisted in a total cost rate related to thermodynamic variables and installation costs. Because the CGAM problem originates from an essentially academic viewpoint, its models do not correspond to the industrial reality of energy systems, and do not contemplate important operational and technological restrictions. Recently, an alternative cogeneration-system optimization problem has been formulated, denoted the MPCP problem – Maximum Profit Cogeneration Plant (in Portuguese, PCLM), which attempts to preserve the simplicity of the CGAM, while coupling modern economic concepts to current technologies. In the MPCP problem, the configuration, the efficiencies of the involved equipment, the investment and operation costs, the obtained revenues, and the imposed physical limitations lead to an objective function, which represents the net present value (NPV) of the monetary gain for the period of plant operation examined. A noteworthy feature of the MPCP problem is that the presented costs are compatible with those practiced in the national industry. In this article, the optimum (i.e., maximum) NPV value is obtained, using an optimization toolbox of the Mathematica® program, appropriate for multivariable nonlinear functions subject to constraints. The optimal values of the decision variables indicate that the plant should operate at the physical limits allowed for the main equipment, namely, at the maximum energy efficiency of the gas turbine generator set, and the minimum temperature difference inside the heat recovery steam generator.*

Keywords*: thermoeconomics, optimization, NPV, cogeneration, MPCP*

1. INTRODUCTION

Thermoeconomics started during the sixties, when several researchers began pioneering studies in analysis, optimization, and design of thermal systems, associating thermodynamic and economic concepts, in order to improve efficiency and reduce environmental impacts. However, it was not until the end of the eighties, that a systematic approach with new methodologies, nomenclature and definitions began to be developed. This effort persists today, with the continued improvement of the methodologies, and the search for new applications. In the nineties, the researchers C. Frangopoulos, G. Tsatsaronis, A. Valero and M. von Spakovsky compared their methodologies through application to a simple and predefined problem: the CGAM problem (Tsatsaronis, 1994; Bejan *et al.*, 1996). Therein, the optimization of a hypothetic cogeneration plant with five individual components is proposed, which produces 30 MW of electrical power in a regenerative cycle and 14 kg/s of saturated vapor at 20 bar. The CGAM cogeneration system consists of an air compressor, a combustion chamber, a gas turbine, a heat recovery steam generator, which produces the saturated vapor at the required process conditions, and an air preheater, located at the compressor exit in order to recover some thermal energy of the turbine exhaust. In the problem formulation, the physical, thermodynamic, and economic models are defined, as well as the objective function, the decision variables, and the constraints. Several simplifications are assumed, in order to ensure the applicability of the CGAM problem to compare different optimization techniques. As a consequence, the modeling of the performance of the CGAM components is essentially theoretical, and does not correspond to the industrial reality of actual systems. In other words, the models do not contemplate important operational and technological restrictions, so that the CGAM problem is incomplete from an engineering perspective.

The purpose of this article is to present and solve an optimization problem, that is an alternative to the CGAM problem. The novel proposal is entitled the MPCP problem – Maximum Profit Cogeneration Plant (Costa, 2008; Costa *et al.*, 2008), which keeps the original simplicity of the CGAM, but introduces modern economic concepts and knowledge of technologies and physical limitations of actual power plants. A new configuration for the cogeneration plant is established, and the formulation of the physical, thermodynamic, and economic models lead to the objective function, which is the net present value (*NPV*) of the monetary gain for a given period of plant operation. The optimization problem thus consists in the maximization of the net present value.

There are two main aspects of the MPCP problem, which make it particularly suitable to the Brazilian reality: (i) only the production of electrical power is fixed, while the production of saturated vapor is free to vary from a prescribed minimum value, and (ii) the cost equation for the gas turbine generator set is obtained through an statistical analysis, which encompasses a database of prices, converted to the national market, of a population of generators with similar capacities and constructive types commercialized by international manufacturers. In the present approach, to obtain the optimal *NPV* value of the MPCP problem, an optimization toolbox of the Mathematica® program (Wolfram, 1999) is employed, appropriate for multivariable nonlinear functions subject to constraints. The optimal values of the objective function and decision variables obtained here serve as reference values for other studies, which might involve the MPCP problem.

It is hoped that the MPCP problem will contribute to the practice of ecoefficiency, through an optimization paradigm which attempts to conciliate the goal of profit maximization of gas and energy enterprises with environmental impact mitigation. This important matter is routinely present in current debates by corporations and universities about the implementation of new energy projects.

2. BRIEF SUMMARY OF THE CGAM PROBLEM

The CGAM problem has been extensively documented in the literature (Tsatsaronis, 1994; Bejan *et al.*, 1996; Vieira, 2003; Costa *et al.*, 2008), thus only a brief summary shall be included here. The formulation of the CGAM problem includes the equations, that describe the cogeneration system behavior (physical model), the state equations used to calculate the thermodynamic properties of the mass streams (thermodynamic model), and the equations employed to calculate the capital, fuel, and operation and maintenance costs for the system (economic model).

To simplify the physical and thermodynamic models, the following assumptions are made: (i) the air and combustion gases behave as ideal gases with constant specific heats; (ii) the fuel is considered to be pure methane, and its combustion is complete; (iii) all the components, except the combustion chamber, are adiabatic. Environmental physical references are further prescribed, so that the temperature, pressure, and relative humidity of the atmospheric air are T_0 = 298.15 K (25 ^oC), P_0 = 1.013 bar, and 60%, respectively. The chemical composition of the air is specified by the following molar fractions: 0.2059 of oxygen, 0.77489 of nitrogen, 0.0003 of carbon dioxide, and 0.0190 of water.

The equation for the capital cost rate (in US$/s) of the system is written as a function of the purchased-equipment costs of the components (in US$), the annual capital recovery factor (%), the number of hours of plant operation per year, and a nondimensional coefficient to account for the operation and maintenance costs. The economic model further establishes that the total cost rate of the CGAM system is the sum of the capital cost rate and the fuel cost rate; the latter is proportional to the mass flow rate and lower heating value of the fuel.

The optimization problem consists in the minimization of the system total cost rate, which is the objective function, assuming fixed production amounts of electrical power and process steam. The objective function has five degrees of freedom, represented by the selected decision variables, namely, the air compressor pressure ratio, the isentropic efficiencies of the air compressor and gas turbine, the temperature of the air at the preheater exit, and the temperature of the combustion gases at the gas turbine inlet.

3. MPCP PROBLEM: AN ALTERNATIVE TO THE CGAM PROBLEM

In the MPCP problem (Costa, 2008; Costa *et al.*, 2008), a new cogeneration system is conceived, which retains the simplicity and power range of the CGAM system, however it reflects the actual and current technologies and engineering practices in modern cogeneration projects. The formulation of the MPCP problem is presented in sections 4 and 5.

After a detailed account of the drawbacks of the CGAM system, Costa *et al.* (2008) propose the MPCP system with the following distinguishing characteristics: (i) the air compressor (AC), combustion chamber (CC), and gas turbine (GT) are integrated in one single equipment, the gas turbine generator set (GTG), whose cost is obtained through an statistical analysis; (ii) there is no air preheater; (iii) the inequality constraint $\Delta T_{pp} \geq 10$ ^oC is imposed for the temperature difference at the pinch point inside the heat recovery steam generator (HRSG); (iv) natural gas, rather than methane, is the fuel to be burned in the new system; (v) 100 ºC is the minimum permissible value for the stack gas temperature; (vi) the method employed to compute the properties of the streams across the gas turbine generator set leads to realistic values for the pressure ratio in the compressor; (vii) realistic values for the energy losses of the gas turbine generator set are obtained through statistical correlations of simulated data; (viii) only the exergetic efficiency of the gas turbine generator set and the mass flow rate of process steam exported by the HRSG are selected as decision variables; (ix) optimization is effected for the profits of the cogeneration plant, keeping the electrical power demand fixed at 30 MW, while letting the steam production free to vary from a minimum value of 12 kg/s; (x) instead of using the capital recovery method (as in the CGAM problem) or the cost of energy method, the modernly used economic indicator *NPV* (net present value) is adopted as the objective function in the MPCP problem, discounting future cashflows at the capital cost of today; (xi) equipment costs are obtained through statistical regressions of real cost and performance data; (xii) a proper technique is utilized to calculate the temperature of the exhaust gases, based on an statistical analysis of a population of gas turbines with similar characteristics; (xiii) the rate of energy consumption by the auxiliary equipment (BOP – Balance of Plant) is considered to amount to 2.5% of the power produced by the plant, and the capital cost of the auxiliary equipment is taken into account in the economic model; the pumping power demand for the feedwater is calculated separately, since it depends on the process steam mass flow rate; (xiv) there is a deaerator in the MPCP system; (xv) a specification commonly used in refineries is prescribed for the process steam, namely, 14 bar and 285 ºC; therefore, because the steam is superheated, the HRSG is split into four different sections, water preheater (WPH), economizer (ECO), evaporator (EVAP), and superheater (SH); (xvi) the approach, i.e., the temperature difference between the water at the exit of the economizer and the saturated steam in the evaporator is $\Delta T_{appr} = 5$ º C; (xvii) the blow-down of the evaporator amounts to 1% of the total feedwater mass flow rate entering the HRSG; in addition, the blow-down water is expanded in a flash tank, to generate steam for the deaerating process; (xviii) the water-side pressure drops inside the HRSG are accounted for, as percentages of the inlet pressure in each HRSG section.

4. MPCP PROBLEM: THERMODYNAMIC AND PHYSICAL MODELS

The configuration of the cogeneration plant for the MPCP problem consists of the following main components, according to the propositions listed in section 3: a gas turbine generator set (GTG), a heat recovery steam generator (HRSG) with four sections (WPH, ECO, EVAP, SH), a deaerator vessel (DA), a flash tank for fluid expansion (FT), and a feedwater pump (FWP). A schematic flow diagram for the MPCP system is shown in Fig. 1, with the indication of the principal equipment, and the relevant stream markers for the mass and energy balances.

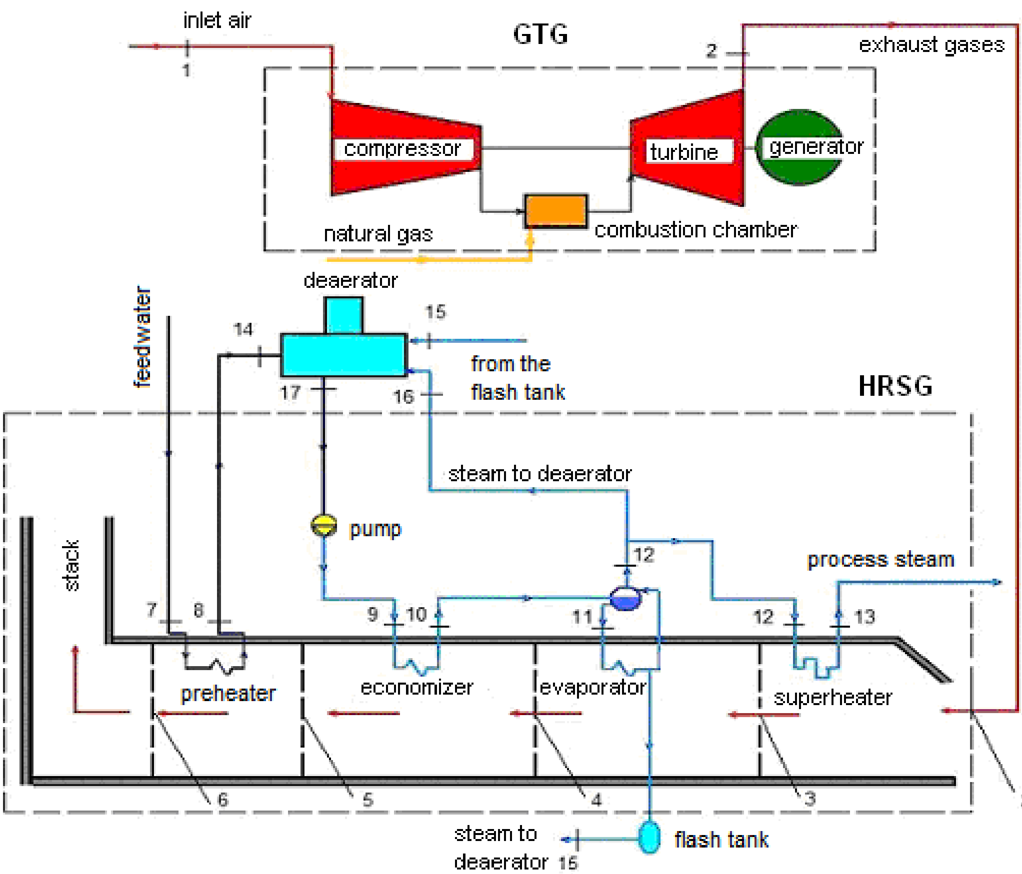

Figure 1. Proposed configuration for the cogeneration system of the MPCP problem.

The following parameters and constraints are adopted in the MPCP problem (Costa *et al.*, 2008): (i) the same environmental references as those of the CGAM (see section 2), with $MW_a = 28.648$ kg/kmol for the molecular weight of the air; (ii) $MW_{ng} = 18.75$ kg/kmol for the molecular weight of the natural gas, according to its chemical composition set by Costa (2008); (iii) $P_{11} = P_{12} = 14.5$ bar; (iv) $T_{11} = T_{12} = T_{sat,EVAP}$, with $T_{sat,EVAP} = 196.7$ °C; (v) $T_4 = T_{11} + \Delta T_{pp}$, where $\Delta T_{pp} \geq 10$ °C; (vi) $T_{10} = T_{11} - \Delta T_{appr}$, where $\Delta T_{appr} = 5$ °C; (vii) the temperature T_9 is equal to the saturation temperature of the steam in the deaerator increased by 1 °C due to pumping, $T_9 = T_{sat,DA} + 1$; (viii) the temperature T_7 is equal to the ambient temperature increased by 1 °C due to pumping, $T_7 = T_0 + 1$; (ix) $P_7 = 1.213$ bar; (x) $P_8 = P_{DA} = 1.113$ bar; (xi) $T_6 \geq 100$ °C; (xii) $T_{13} = 285$ °C and $P_{13} = 14$ bar, as specified for the process steam; and (xiii) the mixture quality in the flash tank is 0.1811. It is further assumed in the MPCP problem, that the air and the combustion gases behave as ideal gases with constant specific heats, denoted respectively by $c_{p,a}$ and $c_{p,g}$.

4.1. Equations for the gas turbine generator set

In Costa *et al.* (2008) and Costa (2008), a method to calculate the real exhaust gas temperature, T_2 (see Fig. 1), at the exit of the GTG component is presented in detail. The basis of the method is a thermodynamic formulation, which makes use of a database obtained from manufacturers of gas turbines. Here, only an outline of the method is given, for completeness of the paper. Two GTG quantities are considered given, the gross power capacity, $\dot{W}_{gr}$ (kW), and the heat rate, *HR*, related to the total energy rate supplied to the GTG, $\dot{E}_{Q,f}$, by

$$\dot{E}_{Q,f} = \dot{m}_f \, LHV = \dot{W}_{gr} \, HR \, , \tag{1}$$

where $\dot{m}_f$ is the mass flow rate of fuel, in kg/s. It is remarked that $\dot{W}_{gr}$ is not equal to the net power exported by the plant to the grid, $\dot{W}_{net}$, because of the power consumption by the auxiliary equipment.

Having established a population sample of fourteen aeroderivative GTG models from international suppliers, all with ISO-conditions capacities similar to the one used in this work (30 MW ± 6 MW), simulations have then been conducted using the GT PRO® software (Thermoflow, Inc., 2004), to obtain average values for the following quantities: the loss rates, $\dot{W}_L$, inside the GTG due to entropy generation; the power consumption by the air compressor, $\dot{W}_{AC}$; and the air/fuel mass ratio, $R_{a/f,m}$. The results are:

$$\dot{W}_{L,n} = \kappa_n \, \dot{E}_{Q,f} \, , \tag{2}$$

$$\dot{W}_{AC} = \dot{m}_a c_{p,a} (T_{AC,e} - T_{AC,i}) = 36.41\% \, \dot{E}_{Q,f} \, , \tag{3}$$

$$R_{a/f,m} = \frac{\dot{m}_a}{\dot{m}_f} = 55.49 \, . \tag{4}$$

In Eq. (2), n identifies the type of turbine generator loss; the appropriate percentages are given in Costa (2008), κ_{AC} = 5.12, κ_{GT} = 8.38, κ_{mec} = 0.92, and κ_{el} = 0.74 for the air compression, gas expansion, mechanical, and electrical losses, respectively. In Eq. (3), $\dot{m}_a$ is the air mass flow rate, and the subscripts i and e denote inlet and exit, respectively. It is important to note that the GTG works with very high excess of air for combustion, since the air flow also works to cool the equipment. Therefore, there is no risk of obtaining extremely high temperature values in the model, relative to those actually reached in GTGs of the main manufacturers.

Denoting by $\dot{E}_{Q,g}$ the energy rate carried away by the exhaust gases, and $\dot{W}_{GT}$ the gas turbine power, the energy balance equations for the gas turbine generator set are:

$$\dot{W}_{gr} = \dot{E}_{Q,f} - (\dot{W}_{AC} + \dot{W}_{L,GT} + \dot{W}_{L,mec} + \dot{W}_{L,el} + \dot{E}_{Q,g}) \, , \tag{5}$$

$$\dot{W}_{GT} = \dot{W}_{gr} + \dot{W}_{AC} + \dot{W}_{L,el} + \dot{W}_{L,mec} \, ; \tag{6}$$

note that $\dot{W}_{AC}$ and $\dot{W}_{GT}$ already include $\dot{W}_{L,AC}$ and $\dot{W}_{L,GT}$, respectively. The exhaust temperature T_2 results from the combination of Eqs. (5) and (6), so that

$$T_2 = T_{cc,e} - \frac{\dot{W}_{GT}}{\dot{m}_g \, c_{p,g}} = T_{AC,e} + \frac{\dot{E}_{Q,f} - \dot{W}_{GT}}{\dot{m}_g \, c_{p,g}} \, , \tag{7}$$

where $T_{cc,e}$ is the combustion chamber exit temperature, and the mass flow rate of gases is $\dot{m}_g = \dot{m}_a + \dot{m}_f$.

The GTG exergetic efficiency, ε_{GTG} , is given by (Bejan *et al.*, 1996; Costa, 2008)

$$\varepsilon_{GTG} = \frac{\dot{W}_{gr}}{\dot{m}_f \, e_f} \, , \tag{8}$$

where e_f = 49552.61 kJ/kg is the specific exergy of the fuel, encompassing the physical and chemical components. The relationship between the exergetic efficiency and the heat rate is

$$HR = \frac{LHV}{\varepsilon_{\mathrm{GTG}}\, e_{\mathrm{f}}} . \tag{9}$$

4.2. Equations for the HRSG and other vessels

The equations that represent the mass and energy balances for the HRSG, deaerator, and flash vessels of the MPCP system are given here; for details, the reader is referred to Costa *et al.* (2008) and Costa (2008). The HRSG has four sections: the feedwater preheater (WPH), the economizer (ECO), the evaporator (EVAP), and the superheater (SH). The water-side pressure drops inside the HRSG are given as percentage values of the inlet pressure in each section: 8.24% for the WPH (0.1 bar), 3.33% for the ECO (0.5 bar), and 3.45% for the SH (0.5 bar). The gas-side pressure drops inside the HRSG are given as 0.48% for each section (0.05 bar), relative to the pressure at the entrance to the HRSG; relative to this same pressure, the drop in the stack is 0.67% (0.007 bar).

Denoting by $\dot{m}_{\mathrm{s,EVAP}}$, $\dot{m}_{\mathrm{s,PR}}$, $\dot{m}_{\mathrm{s,DA}}$, $\dot{m}_{\mathrm{w,BD}}$, $\dot{m}_{\mathrm{s,FT}}$, and $\dot{m}_{\mathrm{w,EVAP}}$ the mass flow rates of, respectively, steam produced in the evaporator (total), process steam to be exported, steam for deaeration, blow-down water from the evaporator, steam obtained from the expansion in the flash tank, and water in the evaporator (total), the following mass relations apply in the MPCP problem:

$$\dot{m}_{\mathrm{s,EVAP}} = \dot{m}_{\mathrm{s,PR}} + \dot{m}_{\mathrm{s,DA}} = 0.99\,\dot{m}_{\mathrm{w,EVAP}} , \tag{10}$$

$$\dot{m}_{\mathrm{w,BD}} = 0.01\,\dot{m}_{\mathrm{w,EVAP}} , \tag{11}$$

$$\dot{m}_{\mathrm{s,FT}} = 0.1811 \cdot (0.01\,\dot{m}_{\mathrm{w,EVAP}}) = 0.001811\,\dot{m}_{\mathrm{w,EVAP}} . \tag{12}$$

In each section j (j = WPH, ECO, SH) and for each non-saturated fluid l (gas, water) in the HRSG, the relation

$$Q_j = UA_j\, LMTD_j = \dot{m}_l c_{p,l} (T_{\mathrm{H}} - T_{\mathrm{L}}) \tag{13}$$

applies, where T_{H} and T_{L} are, respectively, the high and low temperatures of the fluid l, and $LMTD$ is the log mean temperature difference, $LMTD = (\Delta T_{\max} - \Delta T_{\min})/\ln(\Delta T_{\max}/\Delta T_{\min})$.

Finally, the energy balances for the evaporator and deaerator are given, respectively, by

$$\dot{Q}_{\mathrm{EVAP}} = UA_{\mathrm{EVAP}}\, LMTD_{\mathrm{EVAP}} = \dot{m}_{\mathrm{w,EVAP}} \cdot \left[(h_{11} - h_{10}) + 0.99\,(h_{12} - h_{11}) \right], \tag{14}$$

$$\dot{Q}_{\mathrm{DA}} = \dot{m}_{\mathrm{s,FT}} \cdot (h_{g,\mathrm{DA}} - h_{f,\mathrm{DA}}) + \dot{m}_{\mathrm{s,DA}} \cdot (h_{\mathrm{DA}} - h_{f,\mathrm{DA}}) , \tag{15}$$

where h_f and h_g are the enthalpies of saturated water in the liquid and vapor phases, respectively, and h_{DA} is the enthalpy of the steam for deaeration after expansion at the deaerator entrance.

5. MPCP PROBLEM: ECONOMIC MODEL AND OBJECTIVE FUNCTION

In the proposed problem, the goal is to optimize (in fact, maximize) the profits of the plant as a whole, by varying the GTG specification and the production of process steam. Thus, instead of using the capital recovery method in order to minimize the total system cost, herein the objective function is identified with the financial index *NPV*, the net present value for the plant investment, using discounted financial flows. The decision variables selected in the MPCP problem are only two: the GTG exergetic efficiency, $\varepsilon_{\mathrm{GTG}}$ (related to *HR* thru Eq. (9)), and the process steam mass flow rate, $\dot{m}_{\mathrm{s,PR}}$.

The purchased-equipment costs are obtained from statistical analyses (Costa, 2008). In the case of the gas turbine generators, in particular, their costs are related to the power capacities and efficiencies. From a set of points generated with the individual data (cost, power, heat rate) for each GTG in the selected sample, the behavior of the cost Z_{GTG} (in 10^3 US$) as a function of power, $\dot{W}_{\mathrm{gr}}$ (in kW), and heat rate, *HR*, is obtained through a nonlinear regression (Costa, 2008), such that

$$Z_{\mathrm{GTG}}\left(\dot{W}_{\mathrm{gr}},\ HR\right)=\frac{9181.9\,\dot{W}_{\mathrm{gr}}}{1+0.5589\,\dot{W}_{\mathrm{gr}}+0.7208\,HR}\,. \tag{16}$$

By inserting the desired power, 30000 kW, in Eq. (16), the GTG cost as a function of *HR* only is obtained,

$$Z_{\mathrm{GTG}}\left(30000\ \mathrm{kW},\ HR\right)=\frac{2.75457\cdot 10^{8}}{16768+0.7208\,HR}\,. \tag{17}$$

The cost of the HRSG is equal to the sum of the costs of its individual sections, i.e., $Z_{\mathrm{HRSG}} = Z_{\mathrm{WPH}} + Z_{\mathrm{ECO}} + Z_{\mathrm{EVAP}} + Z_{\mathrm{SH}}$ (in US$). It is shown in Costa *et al.* (2008), that the following relations apply:

$$Z_{\mathrm{WPH}}=2080.7\cdot UA_{\mathrm{WPH}}-64128\,, \tag{18}$$

$$Z_{\mathrm{ECO}}=2080.7\cdot UA_{\mathrm{ECO}}-64128\,, \tag{19}$$

$$Z_{\mathrm{EVAP}}=1301.5\cdot UA_{\mathrm{EVAP}}+230759\,, \tag{20}$$

$$Z_{\mathrm{SH}}=2173\cdot UA_{\mathrm{SH}}-1468.3\,, \tag{21}$$

where U is the global heat transfer coefficient (in kW/m$^2\cdot{}^{\circ}$C), and A is the heat transfer area (in m^2).

The costs due to taxes and other expenses related to importation of equipment are also considered in the economic formulation. These costs are classified as internalization costs, and, for one given component, they are taken into account through a nondimensional factor, ζ_{IC}, which multiplies the purchased-equipment cost, Z, of the component. For the gas turbine generator set and heat recovery steam generator, Costa (2008) calculates the internalization cost factors as $\zeta_{\mathrm{IC,GTG}} = 1.661$ and $\zeta_{\mathrm{IC,HRSG}} = 1.803$, respectively. The total investment cost for the MPCP problem is finally written as

$$Z_{\mathrm{MPCP}}=\beta\left(\varsigma_{\mathrm{IC,GTG}}Z_{\mathrm{GTG}}+\varsigma_{\mathrm{IC,HRSG}}Z_{\mathrm{HRSG}}\right), \tag{22}$$

where the nondimensional multiplier β is applied to account for the other costs of the plant, namely, the deaerator, flash tank, pump, other accessories, and engineering costs. In practice, the value of β is 2.5 on average (Costa, 2008).

An economic analysis can then be performed, to determine the *NPV* (in R$) of the cogeneration plant project, discounting the monetary gain of the several cash flows with a prescribed hurdle rate. The revenues are obtained by selling electrical power and process steam. The expenses arise from the purchased-equipment costs, engineering (construction and assembly), fuel consumption, and operation and maintenance. The plant is considered to operate during twenty six years. The general equation which expresses the *NPV* function is

$$NPV=\tau\left[R_{\mathrm{EE}}+R_{s,\mathrm{PR}}-\left(Z_{\mathrm{MPCP}}+E_{\mathrm{f}}+E_{\mathrm{O\&M}}\right)\right], \tag{23}$$

where τ is the exchange rate (τ = 2.50 R$/US$). In Eq. (23), R_{EE} (in US$) is the revenue obtained by selling electrical energy, given by

$$R_{\mathrm{EE}}=\sum_{t=0}^{25}\frac{24\cdot 365\cdot \Pi_{\mathrm{EE}}\cdot 1.062^{t}\cdot \dot{W}_{\mathrm{net}}}{\left(1+H\right)^{t}}\,, \tag{24}$$

where t is the period of operation in years (t = 26), Π_{EE} is the price of electrical energy in the first year (Π_{EE} = 89.95 US$/MWh), the factor 1.062 projects a rate of energy price increase of 6.2% per year due to natural gas usage, and H is the hurdle rate (H = 12% per year). The revenue obtained by selling process steam, $R_{s,\mathrm{PR}}$ (in US$), is given by

$$R_{s,\mathrm{PR}}=\sum_{t=0}^{25}\frac{3600\cdot 24\cdot 365\cdot \Pi_{s,\mathrm{PR}}\cdot 1.062^{t}\,\dot{m}_{\mathrm{s,PR}}}{(1+H)^{t}}\,, \tag{25}$$

where $\Pi_{s,\mathrm{PR}}$ is the price of process steam in the first year ($\Pi_{s,\mathrm{PR}}$ = 0.012 US$/kg). The total expenses due to fuel consumption, E_{f}, and operation and maintenance, $E_{\mathrm{O\&M}}$, are given by

$$E_{\mathrm{f}} = \sum_{t=0}^{25} \frac{3600 \cdot 24 \cdot 365 \cdot c_{\mathrm{f,m}} \cdot 1.062^{t}\, \dot{m}_{\mathrm{f}}}{(1+H)^{t}}, \tag{26}$$

$$E_{\mathrm{O\&M}} = \sum_{t=0}^{25} \frac{24 \cdot 365 \cdot c_{\mathrm{O\&M}}}{(1+H)^{t}}, \tag{27}$$

where $c_{\mathrm{f,m}}$ is the cost of fuel in the first year on a mass basis ($c_{\mathrm{f,m}}$ = 0.334 US\$/kg), and $c_{\mathrm{O\&M}}$ is the cost of operation and maintenance ($c_{\mathrm{O\&M}}$ = 100 US\$/h).

Developing Eq. (23) for the objective function in the Mathematica® program (Wolfram, 1999), *NPV* can finally be expressed as a function of the two selected decision variables for the MPCP problem (Costa, 2008; Costa *et al.*, 2008); the appropriate general equation with the problem parameters is

$$\begin{aligned}
NPV\left(\varepsilon_{\mathrm{GTG}},\, \dot{m}_{\mathrm{s,PR}}\right) = \tau\Big[& 3.8013 \cdot 10^{6}\, \Pi_{\mathrm{EE}} - 77465.9 \cdot c_{\mathrm{O\&M}} + 4.56156 \cdot 10^{8} \cdot \dot{m}_{\mathrm{s,PR}} \cdot \Pi_{\mathrm{s,PR}} \cdot \frac{c_{\mathrm{f,m}} \cdot \left(-2.83071 \cdot 10^{8} - 15914.9 \cdot \dot{m}_{\mathrm{s,PR}}\right)}{\varepsilon_{\mathrm{GTG}}} - \\
& \beta \cdot \Bigg[\frac{2.75457 \cdot 10^{11} \cdot \zeta_{\mathrm{IC,GTG}} \cdot \varepsilon_{\mathrm{GTG}}}{0.663245 + 16768 \cdot \varepsilon_{\mathrm{GTG}}} + 1.33333 \cdot \zeta_{\mathrm{IC,HRSG}} \cdot \Big[101035 + \\
& \frac{1.7968 \cdot \left(30750 + 1.72883 \cdot \dot{m}_{\mathrm{s,PR}}\right) \cdot \ln\left[\dfrac{2.71523 \cdot 10^{6} + 152.656 \dot{m}_{\mathrm{s,PR}}}{1.62601 \cdot 10^{7} + 914.179 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 1.75986 \cdot 10^{6} \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}\right]}{\varepsilon_{\mathrm{GTG}}} + \\
& \frac{847236 \dot{m}_{\mathrm{s,PR}} \cdot \left(30750 + 1.72883 \dot{m}_{\mathrm{s,PR}}\right) \cdot \ln\left[\dfrac{2.15092 \cdot 10^{7} + 1209.29 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 2.33587 \cdot 10^{6} \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}{1.85449 \cdot 10^{7} + 1042.63 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 2.04093 \cdot 10^{6} \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}\right]}{2.9643 \cdot 10^{6} + 166.659 \cdot \dot{m}_{\mathrm{s,PR}} - 294943 \dot{m}_{\mathrm{s,PR}} \cdot \varepsilon_{\mathrm{GTG}}} - \\
& \frac{807375 \dot{m}_{\mathrm{s,PR}} \cdot \left(30750 + 1.72883 \dot{m}_{\mathrm{s,PR}}\right) \cdot \ln\left[\dfrac{1.9123 \cdot 10^{7} + 1075.13 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 2.04093 \cdot 10^{6} \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}{1.64139 \cdot 10^{7} + 922.823 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 1.75986 \cdot 10^{6} \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}\right]}{2.70908 \cdot 10^{6} + 152.31 \cdot \dot{m}_{\mathrm{s,PR}} - 281067 \dot{m}_{\mathrm{s,PR}} \cdot \varepsilon_{\mathrm{GTG}}} - \\
& \frac{803193 \dot{m}_{\mathrm{s,PR}} \cdot \left(30750 + 1.72883 \dot{m}_{\mathrm{s,PR}}\right) \cdot \ln\left[\dfrac{1.62601 \cdot 10^{7} + 914.179 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 268987 \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}{1.35449 \cdot 10^{7} + 761.523 \dot{m}_{\mathrm{s,PR}} + (-2.22735 \cdot 10^{7} - 1252.27 \dot{m}_{\mathrm{s,PR}}) \varepsilon_{\mathrm{GTG}}}\right]}{2.71522 \cdot 10^{6} + 152.656 \dot{m}_{\mathrm{s,PR}} - 267734 \dot{m}_{\mathrm{s,PR}} \cdot \varepsilon_{\mathrm{GTG}}} \Big]\Bigg]\Big].
\end{aligned} \tag{28}$$

6. OPTIMIZATION PROCESS AND RESULTS

On substituting the prescribed values for τ, β, $\zeta_{\mathrm{IC,GTG}}$, $\zeta_{\mathrm{IC,HRSG}}$, Π_{EE}, $c_{\mathrm{O\&M}}$, $\Pi_{\mathrm{s,PR}}$, and $c_{\mathrm{f,m}}$ in Eq. (28), the objective function *NPV* may thus be expressed solely in terms of $\varepsilon_{\mathrm{GTG}}$ and $\dot{m}_{\mathrm{s,PR}}$. The MPCP problem may then be solved, i.e., the optimal values $\varepsilon^{*}_{\mathrm{GTG}}$ and $\dot{m}^{*}_{\mathrm{s,PR}}$, as well as the global maximum value NPV^{*} of the objective function, may be obtained through an optimization process. Here, the NMaximize tool of the Mathematica® Optimization ToolBox is used, which selects automatically the Nelder&Mead Simplex optimization algorithm. The allowable interval of variation for the exergetic efficiency is 29% ≤ $\varepsilon_{\mathrm{GTG}}$ ≤ 35%, to guarantee realistic values for the efficiency, compatible with those of manufacturers of gas turbine generators. Also, the allowable interval of variation for the mass flow rate of process steam is 12 kg/s ≤ $\dot{m}_{\mathrm{s,PR}}$ ≤ 20 kg/s, to be consistent with the interval for $\varepsilon_{\mathrm{GTG}}$, and with the constraints $\Delta T_{\mathrm{pp}} \geq$ 10 °C and $T_6 \geq$ 100 °C. On executing the NMaximize optimization tool, it returns the following optimal values: NPV^{*} = R\$ 177,770,000.00, $\varepsilon^{*}_{\mathrm{GTG}}$ = 35%, and $\dot{m}^{*}_{\mathrm{s,PR}}$ = 13.579 kg/s. The corresponding values of the constrained variables are ΔT_{pp} =10 °C and T_6 = 114.6 °C.

7. CONCLUSIONS

The modeling and solution of the MPCP optimization problem exposed in this paper successfully lead to a relatively simple objective function formulation and a profit-optimal cogeneration system, which are compatible with the Brazilian industrial reality. It is hoped that the thermodynamic and economic modeling will be useful to designers of real cogeneration systems of similar sizes. Two important aspects of the MPCP problem should be noted. First, the products of the plant (electricity and process steam) are not both fixed, so that the optimum search process may vary the specification of the equipment to reach the maximum *NPV*. Second, statistical analyses have been carried out, to obtain the equipment costs practiced in the national market. The problem formulation is simple enough to serve as an application to different types of optimization algorithms.

The resulting optimal values show that, for the established selling prices of electrical energy and process steam, the maximum *NPV* corresponds to the maximum allowable value for $\varepsilon_{\mathrm{GTG}}$, and to a process steam flow rate which leads to the minimum value of ΔT_{pp} in the HRSG. Clearly, it pays to design the cogeneration plant with maximum general efficiency. This result makes one realize, that it is possible to conciliate the goal of maximum economic gain with the reduction of greenhouse gases emissions and conservation of natural resources. The next step in the course of this work will be to test another optimization algorithm, and to perform sensitivity studies near the optimum point.

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