HYSTERESIS LOOPS IN ISOTHERMAL CAVITATION

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Abstract. *This paper investigates the influence of the use of cubic equation of state (EOS) in the isothermal cavitaion of compressible fluids. To do so, a thermodynamic consistent cavitation model recently proposed has been used. This model is derived under the Thermodynamics of Irreversible Processes and takes into account the irreversible dissipative character of the phase change transformation. Numerical simulations carried out using linear and cubic EOS are presented and compared. The results obtained demonstrate that there is no significant difference between the responses of these two types of EOS. Hysteresis loop observed in both EOS are virtually the same, suggesting that metastable behavior, intrinsically present in cubic EOS, has little or none effect on the irreversible cavitaion phenomenon.*

Keywords*: Cavitation, Irreversible Thermodynamics, Cubic Equation of State*

1. INTRODUCTION

Cavitation is the formation of the vapor phase in a liquid. It is distinguished from boiling in the sense that the former is generated by pressure reduction, whereas the later refers to vapor generation as a result of heat addition. Internal as well as external flows may be subjected to cavitation under steady and unsteady regimes. Different features may be responsible for cavitation in liquid flows, such as boundary curvature of solid surfaces in contact with the fluid, vortices, turbulence and transient expansion waves (Arndt, 1981, Freitas Rachid and Costa Mattos, 1998).

There are several practical engineering problems in which cavitation plays an important role and so must be regarded as a design consideration. Whatever the applications and the physical features responsible for cavitation appearance are, it is in general approached by the models under the traditional thermostatic basis. The pressure is assumed to remain constant at the saturated vapor pressure and the phase change process is considered to take place without presenting any kind of dissipation. This behavior is unrealistic form the thermodynamic point of view and, in fact, gives erroneous pressure responses in phase and in magnitude (Utturkar et al., 2005).

To overcome this problem, a consistent thermodynamic cavitation model has been recently proposed by Freitas Rachid (2003). In that paper a model developed under the light of the Thermodynamic of Irreversible Processes was presented to describe cavitation in isothermal flows of compressible liquids. Constitutive relationships were derived from two thermodynamic potentials - the Helmholtz free energy and a pseudo-potential of dissipation – providing state laws and evolution equations for the mass rate of phase change and the vapor volume fraction, in such a way the entropy production inequality was unconditionally satisfied whatsoever the initial and boundary conditions. Dissipative phase change transformation in cavitating flows has been demonstrated to be an intermediate case of two reversible cases. One in which the phase change took place at constant pressure (the saturation pressure) and the other in which the vapor expanded and contracted without transforming into liquid. To simplify the analysis, linear equation of state (EOS) has been used for both the liquid and the vapor.

On the other hand, it is well known that linear EOS may fail in properly describing cavitation since it does not take into account metastable behaviors intrinsically incorporated in more complex and realistic EOS. So, question arises as to the suitability of using linear EOS in cavitation models. To investigate it in detail, the analysis carried out in (Freitas Rachid, 2003) is repeated in this paper by employing cubic EOS proposed by Shamsudar and Lienhard (1993). The results obtained herein show no significant difference between the responses obtained with cubic and linear EOS. The hysteresis loops obtained in the simulation with linear EOS are virtually the same when the cubic EOS are considered. This result suggests that metastable behavior intrinsically present in the cubic EOS has little or none influence on the irreversible phase change transformations.

2. MECHANICAL MODEL

In contrast to general two-phase fluid flows where the phases can assume very different geometrical configurations throughout the flow region (Arndt, 1981; Amromin, 2000; Ishii, 1975), cavitation is a localized phenomenon which takes place at discrete and small regions of the fluid flow. Based on it, it is reasonable to assume that there exists no significant relative motion or slip between the phases, what is equivalent to consider that the liquid and vapor phases have the same velocities. In addition, if we assume that both phases have always the same temperature during liquid–vapor phase transformations, it suffices to consider the balance equations (mass, momentum and energy) for the mixture

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as a whole, instead of doing it for each phase separately (Rajagopal, 1995). These assumptions are the fundamental basis of the cavitation model proposed in (Freitas Rachid, 2003), which is summarized in the next sections.

2.1 – Balance Equations

Based on the past hypothesis, the fluid can be regarded as a pseudo-mixture of the two phases (liquid and vapor) with average properties. Under suitable regularity assumptions and also by restricting the analysis to isothermal transformations ($\dot\theta = 0$ and $\nabla\theta = 0$; being the absolute temperature), only the following classical forms of the balance equations and a local version of the second law are required to describe the problem (Freitas Rachid, 2003):

$$\dot\rho + \rho\nabla\cdot\mathbf{u} = 0 \tag{1}$$

$$\rho\dot{v} = -\nabla p + \nabla\cdot\mathbf{S} + \rho\mathbf{g} \tag{2}$$

$$d = -\left(p + \Psi\right)\nabla\cdot\mathbf{u} + \mathbf{S}\cdot\mathbf{D} - \dot\Psi \geq 0 \tag{3}$$

The above equations, in Eulerian coordinates, represent the balance of mass, the balance of linear momentum and the Clausius–Duhen inequality, respectively. As usual, the superimposed dot stands for the material time derivative, ρ is the mass density of the mixture, $\mathbf{u}$ is the spatial velocity field, p is the thermodynamic pressure, $\mathbf{S} = \mathbf{S}^T$ is the extra stress tensor due to motion, $\mathbf{g}$ is the external body force per unit mass, $\mathbf{D}$ is the rate of deformation tensor and Ψ is the Helmholtz free energy of the mixture per unit volume. Eq. (3) is a local version of the Second Law of the Thermodynamics and requires that the rate of energy dissipation, d, be non-negative. It establishes a distinction between possible ($d \geq 0$) processes and impossible ($d < 0$) ones. Processes that do not violate the second law are classed as reversible or non-dissipative ($d = 0$) and irreversible or dissipative ($d > 0$).

The balance equations presented so far are valid for the mixture as whole. The mixture is considered to be made up of liquid and vapor phases which can or cannot coexist at a same material point and time instant. To take it into account, we consider an internal variable α which denotes the vapor volume fraction. The vapor volume fraction α, $\alpha \in [0,1]$ is defined as being the ratio of the volume of vapor and the total volume of the mixture. Thus, the mass density of the mixture can be expressed as

$$\rho = (1-\alpha)\rho_l + \alpha\rho_v \tag{4}$$

in which ρ_l and ρ_v stand for the mass densities of the liquid and vapor phases, respectively. Since our objective is to describe the liquid–vapor transformations, it is convenient to split the mass conservation of the whole mixture Eq. (1) as:

$$\alpha\dot\rho_v + \rho_v\dot\alpha + \alpha\rho_v\nabla\cdot\mathbf{u} - \Gamma = 0 \tag{5}$$

$$(1-\alpha)\dot\rho_l - \rho_l\dot\alpha + (1-\alpha)\rho_l\nabla\cdot\mathbf{u} + \Gamma = 0 \tag{6}$$

in which Γ is a source term which represents the mass rate of phase change per unit volume. When $\Gamma > 0$, liquid is transformed into vapor. On the other hand, if $\Gamma < 0$, then vapor is transformed into liquid. Finally, if $\Gamma = 0$, then there is no phase change and, consequently, the liquid and vapor phases are conserved independent from each other.

To complete the problem description, we must add to the foregoing equations the constitutive relationships for the mixture in such a way that inequality Eq. (3) be satisfied no matter the external actions, initial and boundary conditions.

2.2 – Constitutive Equations

The constitutive relations describing the macroscopic mechanical behavior of the mixture are derived in the framework of the Thermodynamics of Irreversible Processes. In this theory, once the local state of the material has been characterized by means of an appropriate choice of a set of state variables, two thermodynamical potentials – the Helmholtz free energy and a pseudo-potential of dissipation–are sufficient to derive a complete set of constitutive equations. For this particular problem, we choose as state variables the local mass densities of the liquid ρ_l and the vapor ρ_v, the vapor volume fraction α and the absolute temperature θ. This specific choice restricts the validity of the forthcoming analysis to a particular class of fluids (generalized Newtonian fluids), since the list of state variables might incorporate other variables not considered herein as in the case of many non-Newtonian fluids. The restriction associated with α is treated in this model as a physical property in the constitutive equations.

Helmholtz free energy – State Laws Following the classic assumption of the Thermodynamics of Irreversible Processes, the free energy per total unit volume Ψ is supposed to be a function of the state variables. Since the fluid is regarded as a mixture of the two phases, its behavior is supposed to comprise a combination of the liquid and vapor thermo-mechanical properties. Thus, the following form is proposed:

$$\Psi(\rho_l, \rho_v, \alpha, \theta) = (1-\alpha)\rho_l \Psi_l(\rho_v, \theta) + \alpha\rho_v \Psi_v(\rho_v, \theta) + \mathrm{I}(\alpha) \tag{7}$$

with

$$\mathrm{I}(\alpha) = \begin{cases} 0 & , \alpha \in [0,1] \\ +\infty, & \text{otherwise} \end{cases} \tag{8}$$

in which, Ψ_l and Ψ_v represent the free energies per unit mass of liquid and vapor, respectively, and I (α) is the indicator function of the convex set $[0,1] \subset \Re$. It is included in the free energy potential to take the internal constraint of the mixture into account as a constitutive assumption. In other words, the term I(α) prevents α from getting out of its admissible interval since it would be required an infinite amount of energy to do this. The state laws, relating the reversible components of the thermodynamic forces to the state variables, are obtained from the free energy potential as follows:

$$\mathrm{B}^{\rho_l} = \frac{\partial\Psi}{\partial\rho_l} = (1-\alpha)\left(\Psi_l + \rho_l \frac{\partial\Psi_l}{\partial\rho_l}\right) = (1-\alpha)\mathrm{g}_l \tag{9}$$

$$\mathrm{B}^{\rho_v} = \frac{\partial\Psi}{\partial\rho_v} = \alpha\left(\Psi_v + \rho_v \frac{\partial\Psi_v}{\partial\rho_v}\right) = \alpha\mathrm{g}_v \tag{10}$$

$$\mathrm{B}^{\alpha} \in \partial\Psi = \mathrm{g}_v\Psi_v - \mathrm{g}_l\Psi_l + \partial\mathrm{I}(\alpha) \tag{11}$$

In the above equations, g_l and g_v stand for the Gibbs specific free energies of the liquid and vapor phases, respectively, and the term $\partial\mathrm{I}(\alpha)$ is the subdifferential of the indicator function I (α) at α (Moreau, 1988). It is such that $\partial\mathrm{I}(\alpha) = \emptyset$ if $\alpha \in [0,1]$; $\partial\mathrm{I}(\alpha = 0) = \{\mathrm{h} \in \mathrm{R};\, \mathrm{h} \le 0\}$, $\partial\mathrm{I}(\alpha = 1) = \{\mathrm{h} \in \mathrm{R};\, \mathrm{h} \ge 0\}$ and $\partial\mathrm{I}(0 < \alpha < 1) = \{0\}$.

Pseudo-potencial of dissipation – Complementary Laws To assign a dissipative character to the mixture, and also to ensure that the second law is always satisfied, we assume that the pseudo-potential of dissipation Φ is an objective, convex and differentiable function of **D**, Γ and $\dot\alpha$, i.e. $\Phi(\mathbf{D}, \Gamma, \dot\alpha; \alpha)$, where α is taken as an independent parameter, with the following properties:

$$\Phi(\mathbf{D}, \Gamma, \dot\alpha; \alpha) \ge 0, \quad \forall(\mathbf{D}, \Gamma, \dot\alpha) \text{ and } \Phi(0, 0, 0, \alpha) = 0 \tag{12}$$

The additional information associated with the dissipative behavior can be obtained from Φ through the following complementary laws:

$$\mathbf{S} = \frac{\partial\Phi}{\partial\mathbf{D}}, \qquad \mathrm{B}^{\Gamma} = \frac{\partial\Phi}{\partial\Gamma}, \qquad \mathrm{B}^{\dot\alpha} = \frac{\partial\Phi}{\partial\dot\alpha} \tag{13}$$

in which B^{Γ} and $\mathrm{B}^{\dot\alpha}$ are the thermodynamic forces associated with the phase change transformation and with the evolution of the vapor volume fraction. In addition, if we choose the rate of the energy dissipation. d as being given by

$$\hat{\mathrm{d}} = \mathbf{S} \cdot \mathbf{D} + \mathrm{B}^{\Gamma}\Gamma + \mathrm{B}^{\alpha}\dot\alpha - \mathrm{I}(\alpha)\mathbf{D} \cdot \mathbf{I} \ge 0 \tag{14}$$

for any actual evolution, with **I** being the second-order identity tensor, then we get from the convexity property of Φ that:

$$\mathrm{d} \ge \hat{\mathrm{d}} = \mathbf{S} \cdot \mathbf{D} + \mathrm{B}^{\Gamma}\Gamma + \mathrm{B}^{\alpha}\dot\alpha - \mathrm{I}(\alpha)\mathbf{D} \cdot \mathbf{I} \ge \Phi(\mathbf{D}, \Gamma, \dot\alpha; \alpha) - \Phi(\mathbf{0}, 0, 0, \alpha) \ge 0 \quad \alpha \in [0,1] \tag{15}$$

In view of Eq. (15), it is easy to see that $\hat{d} \geq 0$ for any actual evolution and so the Second Law of Thermodynamics Eq. (3) is always satisfied. It is worth noting that the last term in the right-hand side of Eq. (14) was introduced to give the model a thermodynamically consistent character. If α could be out of the interval [0,1], then the dissipation would become negative what is not possible. It is important to remark that Φ does not need to be a convex function. The convexity property is only a sufficient condition in order to satisfy Eq. (15). From the mechanical viewpoint, Eq. (15) establishes that the rate of energy dissipation is the sum of three parcels. The first parcel in the right-hand side of Eq. (15) is associated with viscous effects due to the motion of the mixture as whole. The other two parcels reflect internal changes in the mixture. The second and third terms express the dissipative mechanisms associated with the evaporation-condensation process and with viscous dissipation in the liquid as a result of the expansional-compressional motion of the vapor phase. Without loosing generality, if we assume the mixture behaves as a Newtonian fluid one possible choice for Φ is:

$$\Phi(D,\Gamma,\dot{\alpha};\alpha) = \frac{\lambda}{2}(\mathbf{D}\cdot\mathbf{I})^2 + \mu\mathbf{D}\cdot\mathbf{D} + \frac{1}{2\beta}\Gamma^2 + \frac{\varepsilon}{2}\dot{\alpha}^2 \tag{16}$$

where $\beta = \hat{\beta}(\alpha)$ and $\varepsilon = \hat{\varepsilon}(\alpha)$ are positive generic functions of α. For this specific choice, the state and complementary constitutive laws become:

$$p = (1-\alpha)p_l + \alpha p_v \tag{17}$$

$$\mathbf{S} = \lambda\mathbf{D}\cdot\mathbf{I} + 2\mu\mathbf{D} \tag{18}$$

$$\Gamma = \beta B^{\Gamma} \tag{19}$$

$$\varepsilon\dot{\alpha} = B^{\dot{\alpha}} \tag{20}$$

$$\alpha(1-\alpha)B^{\Gamma} = \alpha(1-\alpha)(g_l - g_v) \tag{21}$$

$$B^{\alpha} + B^{\dot{\alpha}} + (\rho_l - \rho_v)(\alpha g_v + (1-\alpha)g_v) + B^{\Gamma}(\alpha\rho_l + (1-\alpha)\rho_v) = 0 \tag{22}$$

in which

$$p_l = \rho_l^2\frac{\partial\Psi_l}{\partial\rho_l} \quad \text{and} \quad p_v = \rho_v^2\frac{\partial\Psi_v}{\partial\rho_v} \tag{23}$$

stand for the thermodynamic pressures of the liquid and the vapor and the material parameters $\lambda = \hat{\lambda}(\alpha)$ and $\mu = \hat{\mu}(\alpha)$ are average properties between, respectively, λ_l and λ_v and μ_l and μ_v having α as a weighting factor, such that $\lambda(\alpha) = 0 = \lambda_l$ and $\lambda(\alpha = 1) = \lambda_V$, $\mu(\alpha = 0) = \mu_l$ and $\mu(\alpha = 1) = \mu_V$. These material parameters must be such that $\mu \geq 0$ and $\lambda + \frac{2}{3}\mu \geq 0$ in order to satisfy Eq. (15). The equations (19-22) should be analyzed for three distinct situations; $\alpha \in [0,1]$, $\alpha = 0$ and $\alpha = 1$, which correspond to the existence of liquid along with vapor, pure liquid and pure vapor, respectively.

For $\alpha \in [0,1]$, it is possible to prove (Freitas Rachid, 2003) that Eqs. (19-22) are reduce to:

$$\Gamma = \beta(g_l - g_v) \tag{24}$$

$$\varepsilon\dot{\alpha} = p_v - p_l \tag{25}$$

On the other hand, for both $\alpha = 0$ and $\alpha = 1$, it is possible to show that $\Gamma = 0$ and $\dot{\alpha} = 0$ and Eq. (22) is reduced to (Freitas Rachid, 2003):

$$p_l - p_v - \rho_v\left(g_l - g_v\right) \geq 0, \quad \text{if} \quad \alpha = 0 \tag{26}$$

$$p_l - p_v - \rho_l\left(g_l - g_v\right) \leq 0, \quad \text{if} \quad \alpha = 1 \tag{27}$$

These two last inequalities provide the admissible values for p_v and p_l in the pseudo-fluid consisting of the liquid phase and vapor only. For $p_v = p_{sv}$, then (26) requires that $p = p_l \geq p_{sv}$. Similarly, for $p_l = p_{sv}$, then Eq. (27) requires that $p = p_v \leq p_{sv}$, as it would be expected.

Eqs. (2),(3),(5),(6) along with Eqs. (17-18) and (23-27) completed by suitable boundary and initial conditions are sufficient to describe the isothermal momentum-driven cavitation of a compressible Newtonian fluid flow, as long as specific forms of the Helmholtz free energy for the liquid Ψ_l and for the vapor Ψ_v are provided.

4 – HELMHOLTZ FREE ENERGIES OF THE LIQUID AND OF VAPOR PHASES

Once the Helmholtz free energies of the liquid and vapor phases have been specified, the equations of state (EOS) relating the pressure and mass density, as well as the Gibbs free energy difference between the liquid and the vapor, are readily available. In this paper, linear and cubic equations of state will be considered, so that the model predictions regarding liquid-vapor transformations based on these EOS can be compared.

4.1 – Linear EOS

For the linear equations of state for both the liquid and vapor phases, the following Helmholtz free energies are chosen (Freitas Rachid, 2003):

$$\Psi_l(\rho_l,\theta) = -\,c_l\theta\log(\theta) + c_l\log(\rho_l) + M/\rho_l \tag{28}$$

$$\Psi_v(\rho_v,\theta) = -c_v\theta\log(\theta) + K_v\log(\rho_l) + L \tag{29}$$

in which c_l and c_v are the specific heats at constant volume of the liquid and vapor phases, respectively. The parameters $K_l = \hat{K}_l(\theta)$, $K_v = \hat{K}_v(\theta)$, $M = \hat{M}(\theta)$ and $L = \hat{L}(\theta)$ are positive material functions which depend on the temperature. The function $\hat{L}$ is associated with the phase change latent heat and the functions $\hat{K}_l$ e $\hat{K}_v$ and are related to the isothermal compressibility of the phases. Since we have implicitly assumed that the vapor behaves as a perfect gas, then $\hat{K}_v(\theta) = R_v\theta$, R_v being the gas constant. By virtue of Eq. (23) the partial pressures of the liquid and of the vapor are:

$$p_l = K_l\rho_l - M \tag{30}$$

$$p_v = K_v\rho_v \tag{31}$$

By taking into account that the Gibbs free energies of the liquid and the vapor are equal when the phases are in equilibrium at the saturation pressure p_{sv} at any temperature θ, it can be shown that:

$$g_l - g_v = K_l\log\left(\frac{\rho_l}{\rho_{sl}}\right) + K_v\log\left(\frac{\rho_{sv}}{\rho_v}\right) + M\left(\frac{1}{\rho_{sl}} - \frac{1}{\rho_l}\right) \tag{32}$$

in which ρ_{sl} and ρ_{sv} stand for the liquid and vapor saturated mass densities, respectively, associated with the saturation pressure p_{sv}.

4.2 – Cubic EOS

For the cubic equations of state for both the liquid and vapor phases, the following Helmholtz free energies are chosen (Shamsudar and Lienhard, 1993):

$$\Psi_l(\rho_l,\theta) = \int_{1/\rho_l}^{R\theta/p_r} p\,dv + \int_{\theta_r}^{\theta} c_{po}(\hat{\theta})\left(\frac{\theta}{\hat{\theta}} - 1\right)d\hat{\theta} - R(\theta - \theta_r) \tag{33}$$

$$\Psi_v(\rho_v,\theta) = \int_{1/\rho_v}^{R\theta/p_r} p\,dv + \int_{\theta_r}^{\theta} c_{po}(\hat{\theta})\left(\frac{\theta}{\hat{\theta}} - 1\right)d\hat{\theta} - R(\theta - \theta_r) \tag{34}$$

in which

$$p = p_{sv}\left[1 - \frac{(v - v_{sl})(v - v_m)(v - v_{sg})}{(v - b)(v^2 + 2cv + d)}\right] \tag{35}$$

In the past expressions, R represents the universal gas constant, p_r and θ_r stand for a pressure and temperature of reference, c_{po} is the specific heat at constant and low pressure, p_{sv} is the saturation pressure of fluid at any temperature θ; $v_{sl} = (1/\rho_{sl})$ and $v_{sv} = (1/\rho_{sv})$ are the specific volumes of saturated liquid and saturated vapor at a temperature θ; and v_m, b, c, d are constitutive parameters which depend on the temperature θ. By virtue of Eq.(23) the partial pressures of the liquid and of the vapor are:

$$p_l = p_{sv}\left[1 - \frac{(v_l - v_{sl})(v_l - v_m)(v_l - v_{sv})}{(v_l - b)(v_l^2 + 2cv_l + d)}\right] \tag{36}$$

$$p_v = p_{sv}\left[1 - \frac{(v_v - v_{sl})(v_v - v_m)(v_v - v_{sv})}{(v_v - b)(v_v^2 + 2cv_v + d)}\right] \tag{37}$$

in which $v_l=(1/\rho_l)$ and $v_v=(1/\rho_v)$ are respectively specific volumes of liquid and vapor phases.

By taking into account that the Gibbs free energies of the liquid and the vapor are equal when the phases are in equilibrium at the saturation pressure p_{sv} at any temperature θ, it can be shown that:

$$g_l - g_v = \int_{1/\rho_l}^{1/\rho_v} p\,dv + \frac{p_l}{\rho_l} - \frac{p_v}{\rho_v} \tag{38}$$

5 – MODEL PREDICTIONS FOR LINEAR AND CUBIC EOS

The dynamics of the phase-change transformations is now investigated from a theoretical point of view, in order to compare the predictions of phase-change taking into account the linear and the cubic EOS. The capability of the model in describing typical situations is illustrated by a simple quantitative example. For this purpose, consider a sample of a liquid–vapor mixture of pure water in equilibrium at a saturation pressure of p_{sv} = 2.34 kPa (θ = 293 K) having, initially at t = 0, a vapor volume fraction of α_0= 0.2. To simplify the interpretation of the results and to reduce the number of material parameters to be specified, we shall consider that the phase-change transformation is the only dissipative mechanism present, so that ε = 0. The constitutive constants of the model for water at 293 K are taken as: K_v = 135 kJ/kg, K_l = 990 kJ/kg, M = 988 MPa for the liner EOS and v_{sl}=0.0010018 m^3/kg , v_{sg}= 57.79 m^3/kg , v_m= 0.029335 m^3/kg , b=0.000728601 m^3/kg , c=0.00303702 m^3/kg , d= -7.06179x10^{-7} $(m^3/kg)^2$ for the cubic EOS . The value of the coefficient β responsible for the dissipative phenomenon of phase change is assumed to be β = 10^{-8} kg^2/m^3J s. As it has been demonstrated in (Freitas Rachid, 2003), this value is the one for which dissipative effects in cavitation takes place.

For water at this temperature, it has been shown in (Freitas Rachid, 2003) that for β ≥ 10^{-6} kg^2/m^3Js the phase change takes place in a reversible way (with no dissipation) and the fluid pressure remains constant and equal to the saturation pressure p_{sv}. On the other hand, for β ≤ 10^{-12} kg^2/m^3Js the phase change takes place also in a reversible way (with no dissipation) and the fluid pressure fluctuates to accommodate the expansion and contraction of the vapor bubbles in the mixture. The extreme values of β represent adequate upper and lower bounds for describing peculiar situations with no rate of energy dissipation.

The mixture is confined into a device and can have its original volume expanded or contracted along the time by means of external actions. It is further assumed that, at anytime instant, the mixture is almost everywhere homogeneous throughout its volume, so that the convective terms of the balance equations of mass can be neglected. Under these circumstances, the time rate of the volume change per unit volume of the whole mixture is described by the term $\nabla \cdot u$, which is assumed to be a prescribed input function of the time as represented in Fig. 1. Since $\alpha \in (0,1)$ for this input function, the equations governing the behavior of the mixture are given by Eqs. (5),(6),(24) along with (30-32) whether the linear EOS is considered or along with (33-38) whether the cubic EOS is assumed. By virtue of the choice ε = 0, we can see through (25) that p = p_l= p_v. By combining the past equations we can arrive at an initial value non-linear problem of differential equations in the independent variable t for the unknowns α and ρ_l and ρ_v:

$$\dot{\alpha} = \frac{\alpha(1-\alpha)\rho_v\rho_l\nabla\cdot u(\rho_v - \rho_l\xi) + \beta(g_l - g_v)(\alpha\rho_v^2 + (1-\alpha)\rho_l^2\xi)}{\alpha\rho_l\rho_v^2 + (1-\alpha)\rho_v\rho_l^2\xi} \tag{39}$$

$$\dot{\rho}_l = \frac{\dot{\alpha}\rho_l - \beta(g_l - g_v) - (1-\alpha)\rho_l\nabla\cdot u}{1-\alpha} \tag{40}$$

$$\dot{\rho}_v = \frac{\beta(g_l - g_v) - \dot{\alpha}\rho_v - \alpha\rho_v\nabla\cdot u}{\alpha} \tag{41}$$

where $\xi = \frac{\rho_v^2 K_v}{\rho_l^2 K_l}$ if the linear EOS is considered and

$$\xi = \frac{\left[\left(v_v^3 + Dv_v^2 + Ev_v + F\right)\left(3v_v^2 + 2Av_v + B\right) - \left(3v_v^2 + 2Dv_v + E\right)\left(v_v^3 + Av_v^2 + Bv_v + C\right)\right]\left(v_l^3 + Av_l^2 + Bv_l + C\right)^2}{\left[\left(v_l^3 + Dv_l^2 + Ev_l + F\right)\left(3v_l^2 + 2Av_l + B\right) - \left(3v_l^2 + 2Dv_l + E\right)\left(v_l^3 + Av_l^2 + Bv_l + C\right)\right]\left(v_v^3 + Av_v^2 + Bv_v + C\right)^2}$$

if the cubic EOS is considered, in which $A = -v_{sl} - v_m - v_{sv}$, $B = v_{sl}v_{sv} + v_{sl}v_m + v_m v_{sv}$, $C = -v_{sl}v_m v_{sv}$, $D = 2c - b$, $E = d - 2bc$ and $F = -bd$. This problem is numerically approximated by using a fourth-order Runge–Kutta method. Typical time-steps used in the simulation are of the order of 10^{-3} s.

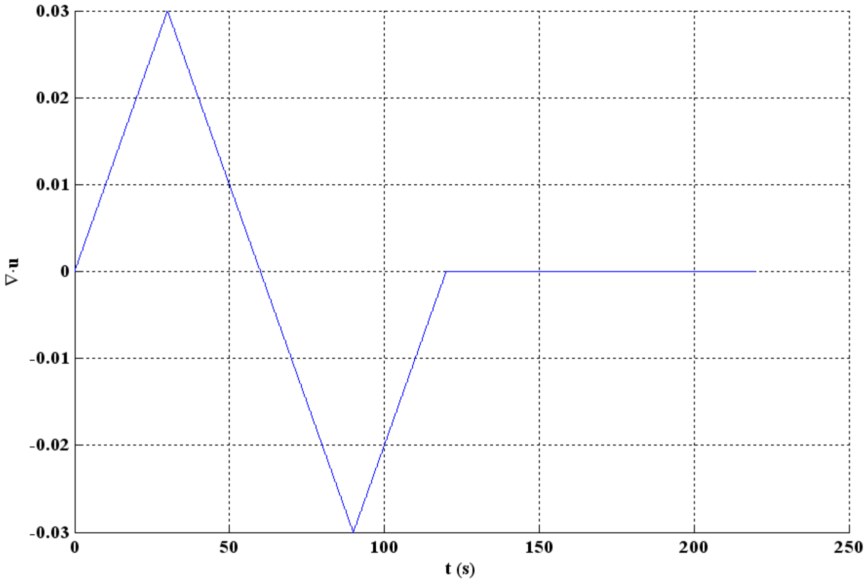

Figura 1 – Input excitation $\nabla \cdot u$ as a function of time

In what follows, the predictions of the model taking into account the linear EOS and the cubic EOS are compared in order to investigate its influence on the phase change process.

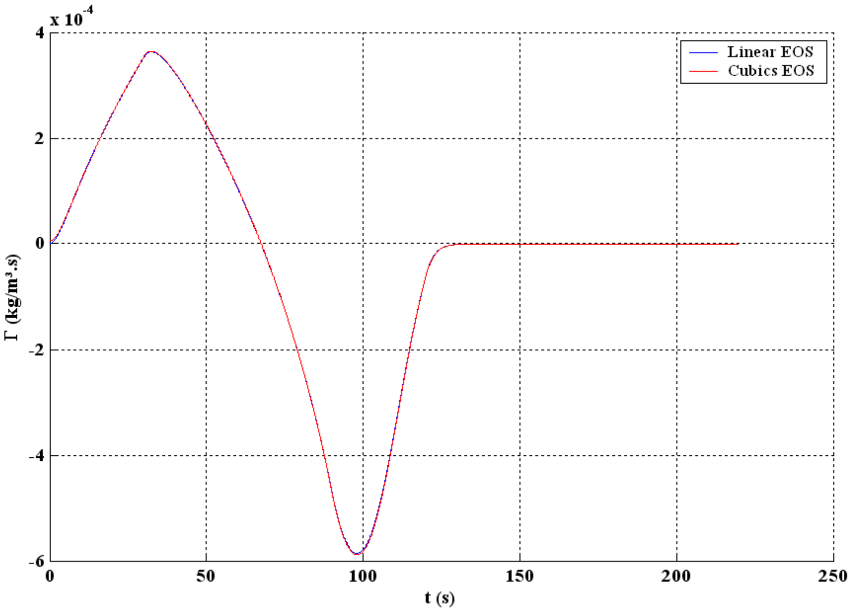

Figura 2 – Mass rate of phase change as a function of time for linear and cubic EOS

The mass rate phase change per unit volume, Γ, is plotted against the time in Fig. 2 for the linear and cubic EOS. As it can be seen, the behavior of the responses is virtually the same no matter the use of the EOS type. Due to the expansion in the first-half of the cycle generated by $\nabla \cdot \mathrm{u}$, liquid is transformed into vapor until t=60s. On the other hand, vapor is converted into liquid due to the contraction of the mixture induced the external action in the second-half of the cycle. Since the areas enclosed by this curve in these two periods are not the same, the vapor generated in the first period is not fully transformed into liquid in the second one.

The pressure histories for the linear and cubic EOS are shown in Fig. 3. The pressure of the mixture decreases during the first-half period and reaches values two times greater than the saturation pressure in the second-half of the cycle. In this situation, it can be noticed the presence of a relaxation phenomenon since the pressure takes ~ 20 s to come to its equilibrium value after the rate of volume change has vanished. Once again, the responses observed with the linear and with the cubic EOS are almost the same, suggesting that simpler EOS can be used in the model without compromising the overall dissipative phenomenon.

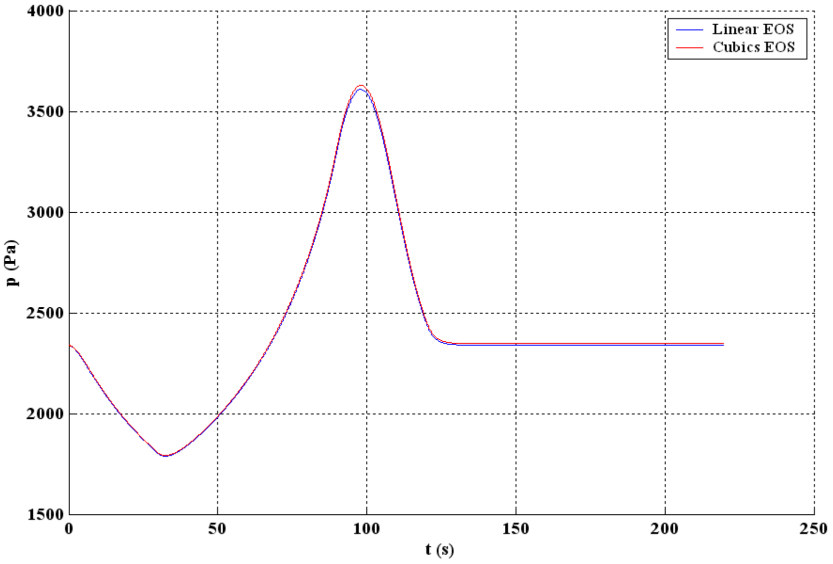

Figura 3 – Pressure as a function of time for linear and cubic EOS

Fig. 4 presents the plot of the rate of energy dissipation, $E_g=B^{\Gamma}\Gamma$, as a function of time for the linear and cubic EOS. No significant difference is noted whether linear or cubic EOS is employed. As it has been anticipated, it is clear in this figure that there exists energy dissipation during the phase change process in which vapor becomes liquid and vice-versa. Since the energy dissipation represents the area below the curve, it becomes clear that the dissipation is greater in the second-half of the cycle than that observed in the first-half. It can be explained by noting in Fig. 2 that the amount of liquid formed in the second period is greater than that of vapor generated in the first one. A simple calculation reveals that the overall energy dissipation is ~ 0.75 kJ/m^3.

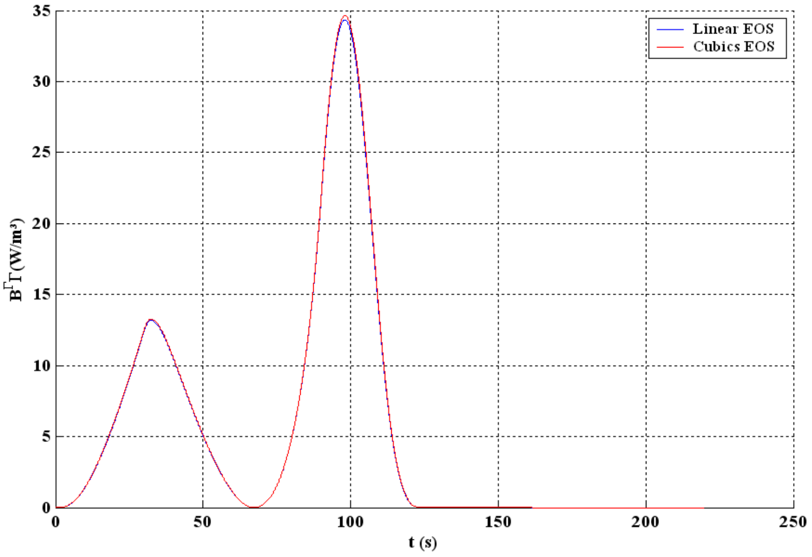

Figura 4 – Rate of energy dissipation as a function of time for linear and cubic EOS

The evolution of the vapor volume fraction in time is presented in Fig. 5 for the linear and cubic EOS. No noticeable difference is observed in Fig. 5 between the responses. The excess of liquid generated in the second-half of the cycle reported in the past paragraphs is responsible for an absolute difference between the final and initial values of the vapor volume fraction of about 6.27×10^{-10}. It corresponds to an increase in the final mass density of the mixture of 1.57×10^{-9} % in relation to its initial value.

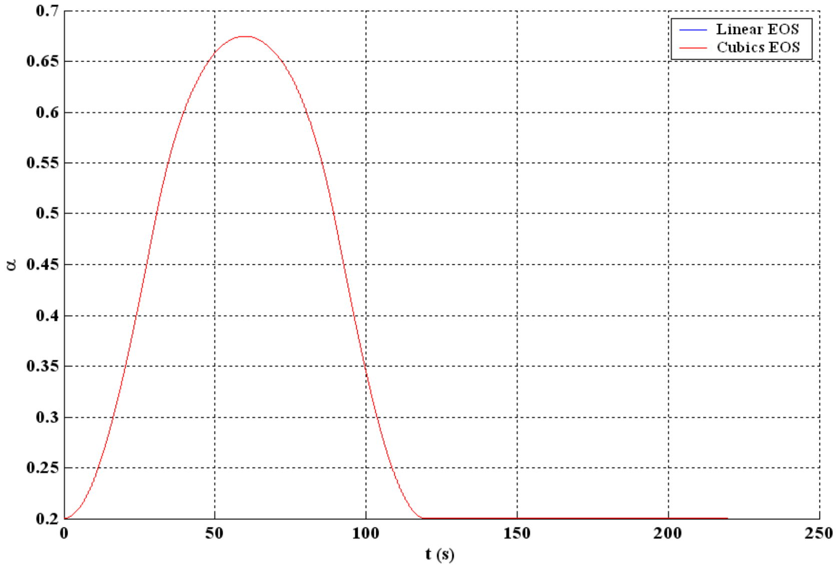

Figura 5 – Vapor volume fraction as a function of time for linear and cubic EOS

Finally, the physical situations analyzed herein can be better visualized by plotting the pressure against the vapor volume fraction as shown in Fig. 6 for both the liner and the cubic EOS. Although the energy dissipation in the total process is in fact small, the model predictions of isothermal phase change transformation clearly show the existence of a hysteresis loop, typical of dissipative phenomena. Although a small difference between the linear and cubic EOS response can be noticed, it has no significance for practical purposes.

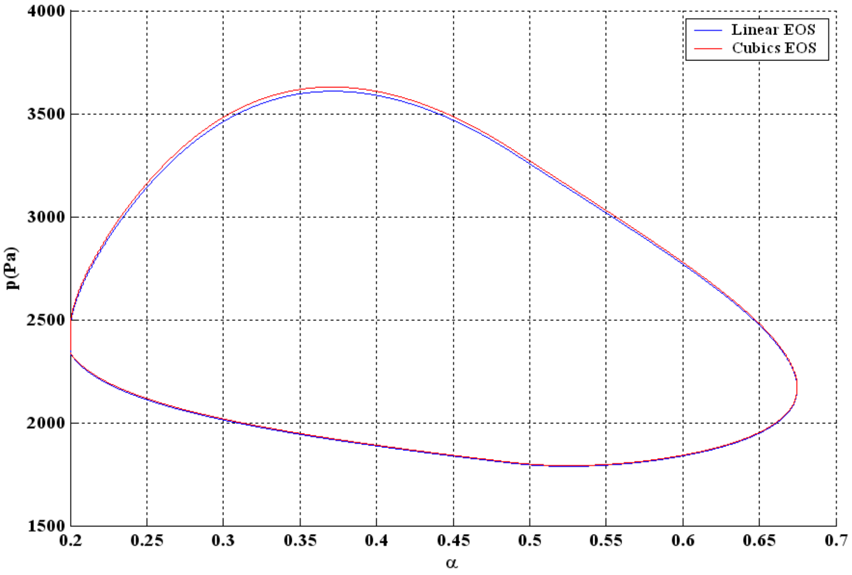

Figura 6 – Pressure against vapor volume fraction for linear and cubic EOS

The good performance exhibited by the linear EOS when compared with the cubic EOS allow us to conclude that the inner metasable behaviors of the cubic EOS does not play an important role in the phase change process. This fact can be justified by noting the fluid pressure oscillates around the saturation pressure (see Fig. 3) and the linear EOS provides a good approximation of the cubic EOS in the neighborhood of the saturated liquid and vapor regions.

6. CONCLUDING REMARKS

The influence of the use of more accurate and realistic equations of sate (EOS), such as the cubic EOS, in the irreversible cavitation model presented in (Freitas Rachid, 2003) has been investigated in this paper. Numerical simulations for a simple problem in which a liquid-vapor mixture of water is dynamically expanded and contracted were carried out by taking into account the cubic EOS proposed by Shamsudar and Lienhard (1993). The results obtained were compared with that obtained by using linear EOS, for both the liquid and vapor phases. For the cases analyzed herein it has been virtually noted no difference between the responses obtained with cubic and linear EOS. This result suggests that metastable behavior intrinsically present in the cubic EOS has little or none influence on the irreversible phase change transformations.

7. ACKNOWLEDGEMENTS

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