NUMERICAL SOLUTION OF TIME DEPENDENT SETTLEMENTS IN HYDRAULIC FILLS

João Flávio Vieira de Vasconcellos, jflavio@iprj.uerj.br
Department of Mechanical Engineering and Energy
Instituto Politécnico, IPRJ, Rio de Janeiro State University, UERJ
P. O. 97.282, 28.601-970, Nova Friburgo, RJ, Brazil

Shailesh Singh, shailesh.singh@jcu.edu.au
Nagaratnam Sivakugan, siva.sivakugan@jcu.edu.au
Scholl of Engineering, James Cook University
Townsville, QLD 4811, Queensland, Australia

Abstract. *This paper reports some numerical investigation carried out on hydraulic fills. A methodology based on the numerical solution of the differential equation presented herein is introduced and tested. Typical dimensional performance curves are obtained and the influences of the governing parameters are discussed. A sensitivity analysis was performed in order to characterize the influence of the numerous parameters on the outputs of the present model.*

Keywords: *hydraulic fills, sensitivity analysis, finite volume method*

1. INTRODUCTION

Large underground voids are created in the process of removing ores from the mines. These underground voids are known as stopes and they can be approximated as rectangular prism with plan dimensions in the order of 20-60 m and heights as much as 200 m. Different types of backfills are used to fill the stopes and hydraulic fills are most popular backfill type used worldwide. Hydraulic fills are uncemented classified mine tailing or sand deposits mined off the site, with not more than 10% by weight of size less than 10 μm (Grice, 1988). Hydraulic backfill slurries are transported by gravity through boreholes and pipelines to the stopes. Slurry density is greater than 70% (solid by weight). The drainage is one of the main concerns in hydraulic backfills. Figure 1 shows the typical hydraulic fill stope. The barricades are made of free draining porous bricks and are used to block the horizontal access drives to retain the fill and prevent it from entering the drives. Hence, the free water is allowed to drain through the fill and the barricades into the empty drives. Mining accidents due to barricade failures are often catastrophic, resulting in sudden flow of wet slurry into the drives, trapping the miners and machinery in the vicinity. Most of the failures take place in the early hours of filling and are caused by the presence of excess water within the minefill. Several mechanisms such as piping and liquefaction have been suggested as the triggers (Bloss, 1998; Grice, 1988; Kuganathan, 2001).

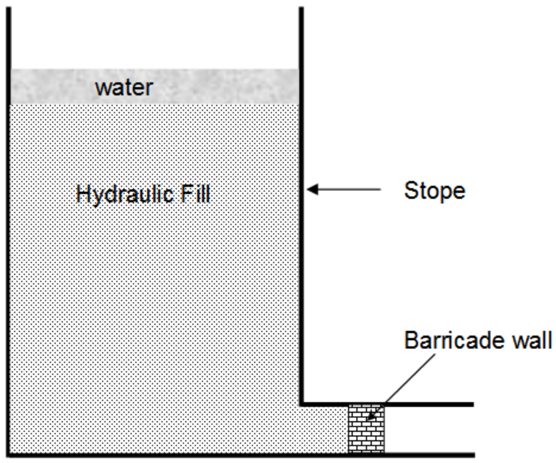

Figure 1 - Typical hydraulic fill stopes

Sivakugan *et al.* (2006) measured the permeability of hydraulic fills from different Australian Mines and porous barricade bricks. The permeability of barricade bricks are about two to three orders of magnitude of permeability of hydraulic fills. A numerical model and computer program was developed to study the pore water pressure development within two dimensional stopes by Issacs and Carter (1983). Traves and Issacs (1991) extended this model to three dimensions. Rankine *et al.* (2003) developed three dimensional numerical model using FLAC^{3D} to study the pore pressure development and drainage by considering the filling rate, hydraulic fill water content and fill characteristics.

Sivakugan *et al.* (2006) studied the pore pressure developments within the two dimensional stopes using FLAC using method of fragments. In all previous studies, pore water pressure development within the stopes has been estimated assuming the constant value of permeability. Singh *et al.* (2008) and Singh and Sivakugan (2008), observed the permeability of hydraulic fills is a logarithmic function of effective vertical stress.

The sensitivity analysis is the study of how the variation in the output of a model is influenced, qualitatively or quantitatively, to different sources of variation. It has been used as an important tool in many areas of knowledge (Porous media flow (Iglesias and Dawson, 2007), environmental flow (Khemka *et al.*, 2006); life insurance contracts (Christiansen, 2008); heat transfer (Parente Jr and de Sousa Jr, 2008) and mass transfer (Vasconcellos *et al.*, 2003)). Sensitivity analysis is the first and the most important step in the optimization problems, because it yields the information about the increment or decrement tendency of the design objective function with respect to the design parameter. Therefore, sensitivity analysis plays an important role in determining which parameter of the process should be modified for effective improvement (Sarigul and Secgin, 2004).

In this paper, pore water pressure development within the stopes has been estimated by using the reduced total stress and permeability variation with height. An analytical analysis has been done using flow equation to estimate pore water pressure distribution within the stopes at different time. Finite Volume Method (FVM) (Patankar, 1980; Ferziger and Peric, 2002; Maliska, 1995) has been used to solve the partial differential equations. Numerical examples are presented to validate the proposed methodology and to assess the sensitivity analysis for eight parameters.

2. TIME DEPENDENT SETTLEMENT IN HYDRAULIC FILLS EQUATION

Following assumption were made to study the pore water pressure development of hydraulic fills within the stopes.

1. Hydraulic fills are saturated and homogenous;
2. There is no decant water above phreatic surface;
3. Water and hydraulic fill particles are incompressible;
4. Flow of water is in one direction;
5. Permeability of hydraulic fill is not constant within the stopes (Singh *et al.*, 2008);
6. Darcy's law is valid; and,
7. Total stress reduces due to arching (K. Pirapakaran and Sivakugan, 2007).

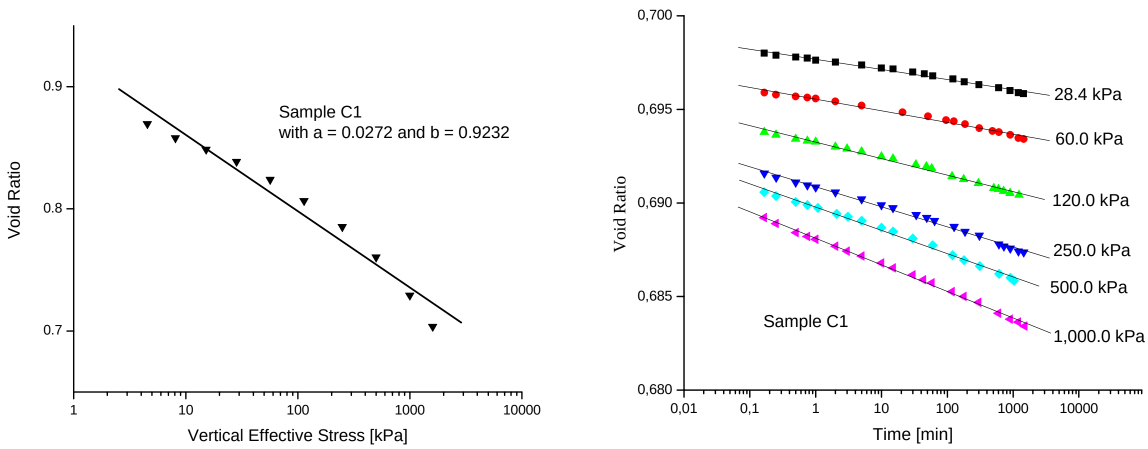

Figure 2 – For sample C1, void ratio versus vertical effective stress (left), adapted from Singh *et al.* (2008), and void ratio versus time (right), adapted from Singh and Sivakugan (2008).

The consolidation equation can be derived as following, on the basis of aforementioned assumptions. Similar analysis has been performed by Das (2005) or Terzaghi *et al.* (1996).

$$\frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right)-\frac{\gamma}{\left(1+e_o\right)}\left[\frac{a'}{t}+\frac{a}{\sigma^a-u}\xi\right]=0 \qquad (1)$$

where u is pore water pressure [Kpa], k is the permeability of hydraulic fills [mm/hr], e_o is the initial void ratio, γ is the unit weight of tailing [kN/m3], σ^a is the arching stress [kPa] and t is the time [min]. Values of a are obtained from experimental data, Figure 2 (left), and values of a' are obtained from experimental data, Figure 2 (right).

σ^a can be expressed as following (K. Pirapakaran and Sivakugan, 2007)

$$\sigma^a = \frac{\gamma}{2K\tan\delta}\left(\frac{wl}{l+w}\right)\left[1-e^{-2\left(\frac{l+w}{wl}\right)yK\tan\delta}\right] \tag{2}$$

where w is the width of stopes, l is the length of stopes, K is the Earth pressure coefficient, and δ is the wall .

Singh *et al.* (2008) measured the permeability of hydraulic fills under surcharge. The permeability of hydraulic fills varies linearly with logarithmic of vertical effective stress. The permeability variation with vertical effective stress of one hydraulic fill sample is shown in Figure 3. Using Eq. (3) the permeability variation can be expressed as following.

$$k = d - c\ln\left(\sigma^a - u\right) \tag{3}$$

where c and d are parameters which are functions of load pressure. Details about this experimental procedure can be seen at Singh *et al.* (2008).

The backfilling in the stopes is done in batches. In this analysis, it has been assumed that the load increment due to batching is constant; hence the rate of load increment is given as following

$$\frac{\partial\sigma}{\partial t} = \xi \text{, with } \xi > 0 \tag{4}$$

where ξ is a constant and given value.

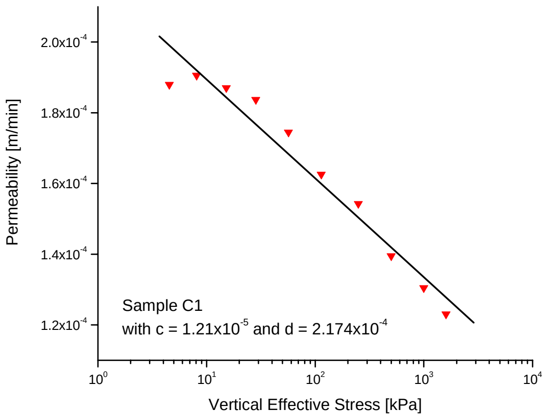

Figure 3 – Permeability versus Vertical effective stress (right), adapted from Singh *et al.* (2008).

Two sets of different boundary conditions are analyzed in this work; the first is used to model this phenomenon for pervious barricade wall,

$$u = 0 \text{ at } y = 0 \text{ (top)} \tag{5}$$

$$u = 0 \text{ at } y = l \text{ (bottom)} \tag{6}$$

and, the other set is used to model for impervious barricade walls,

$$u = 0 \text{ at } y = 0 \text{ (top)} \tag{7}$$

$$\frac{du}{dy} = 0 \text{ at } y = l \text{ (bottom)} \tag{8}$$

Actually we are not modeling the problem shown in Figure 1; Eq. (1) is one-dimensional while the real problem shown in Figure 1 is two-dimensional. The numerical model presented herein will be used to simulate an experimental apparatus (one-dimensional) which has been built to understand some aspects of the problem shown in Figure 1.

3. SOLUTION METHODOLOGY

In the present work, equations Eq. (1) was solved numerically using the finite volume method (Patankar, 1980; Ferziger and Peric, 2002; Maliska, 1995). In this methodology, the first step is to divide the solution domain in small non-overlapping control volumes whose faces are aligned with the coordinate lines, as shown in Figure 4. Next, the equation is integrated along each one of these control volumes yielding a set of algebraic equation. For any internal finite volume, the integration of Eq. (1) is:

$$\int_{y_s}^{y_n}\left[\frac{\partial}{\partial y}\left(k\frac{\partial u}{\partial y}\right)-\frac{\gamma}{(1+e_o)}\left[\frac{a'}{t}+\frac{a}{\sigma^a-u}\xi\right]\right]dy = \tag{9}$$
$$k\frac{\partial u}{\partial y}\bigg|_{y_s}^{y_n}-\frac{\gamma}{(1+e_o)}\left[\frac{a'}{t}+\frac{a}{\sigma_P^a-u_P}\xi\right]\Delta y_P=0$$

The pressure derivatives along the faces of the control volumes are approximated using piecewise linear profiles, then Eq. (9) can be rewritten as,

$$k_n\left(\frac{u_N-u_P}{\delta y_n}\right)-k_s\left(\frac{u_P-u_S}{\delta y_s}\right)-\frac{\gamma}{(1+e_o)}\left[\frac{a'}{t}+\frac{a}{\sigma_P^a-u_P}\xi\right]\Delta y_P=0 \tag{10}$$

where k_n and k_s are respectively the values of k at interface north and south of each volume. Their values are calculated using the geometric average as proposed by Patankar (1980),

$$k_n=\frac{k_Pk_N\left(\Delta y_N+\Delta y_P\right)}{k_P\Delta y_N+k_N\Delta y_P} \text{ and } k_s=\frac{k_Pk_S\left(\Delta y_S+\Delta y_P\right)}{k_P\Delta y_S+k_S\Delta y_P} \tag{11}$$

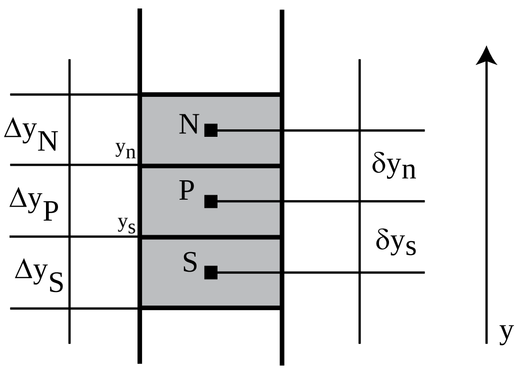

Figure 4 – Schematic diagram of the control volume used to integrate Eq. (9).

The subscript P refers to the control volume properly which neighbors are at **N**orth and **S**outh. Equation (10) is non-linear because the coefficients "k", which are related with Eq. (3), depend upon the pore water pressure itself. After rearrangements all algebraic equations are cast into the following formula

$$A_pu_p=A_nu_n+A_su_s+S_P \tag{12}$$

For a given iteration, the coefficients were evaluated using the pressure value from the previous iteration. The solution was considered to be converged when, for all grid points, the difference in pressure between two consecutive interactions divided by the difference between the highest and the lowest pressure was less than 10^{-8}.

4. RESULTS

Figure 5 shows pore water pressure versus height for different times, considering two different types of boundary conditions. All lines in the left plot of this figure have the behavior expected for this problem (Das, 2005).

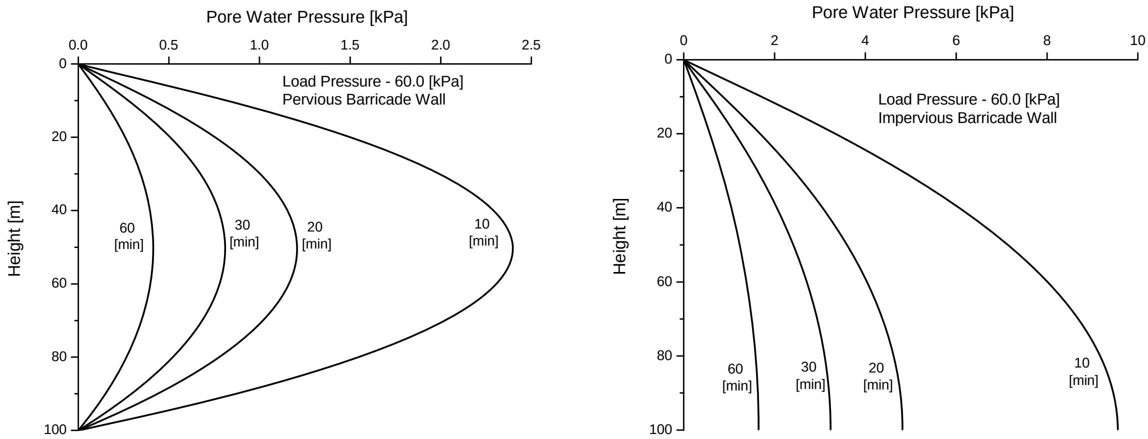

Figure 5 – Height versus Pore water pressure for different times (Porous barricade wall) and pervious barricade wall (left plot) and impervious barricade wall (right plot)

5. SENSITIVITY ANALYSIS

The scope of sensitivity analysis is to rank the importance of the various parameters of the mathematical model. In the present work, we analyze the scaled sensitivity coefficients, which are defined as

$$X_s(t) = \beta_s \frac{\partial u(t)}{\partial \beta_s} \tag{13}$$

where β_s are the parameters used in the present sensitivity analysis and may be one of these parameters: $\{a; a'; c; d; \delta; e_0; \gamma; \xi\}$. As it can be observed in Eq. (13), the scaled sensitivity coefficients have all the same units of u, therefore a direct comparison between each sensitivity coefficients is then possible.

Figure 6 shows the sensitivity of a and ξ parameters. It can be seen that any increasing of a or ξ due to extremely low sensitivity of $u(t)$ with respect to these parameters one could say that the water porous pressure is not affected these parameters.

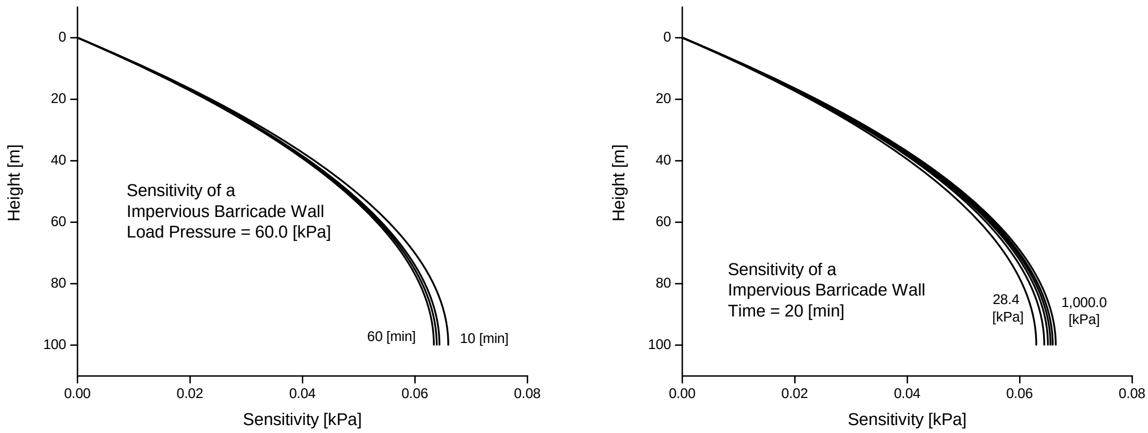

Figure 6 – Sensitivity of Coefficients a and ξ with respect to time and position (height) (left) and with respect to load pressure and position (height) (right)

Figure 7 shows the sensitivity of the parameter a'. For these values of sensitivity, ones may infer that a' plays an important role in this model and for that reason this value should be evaluated carefully. Small errors on a' evaluation will lead to errors on $u(t)$ solutions. A very similar behavior has been found for $\{a'; c; d; e_0; \gamma\}$ parameters; therefore these plots will not be presented herein.

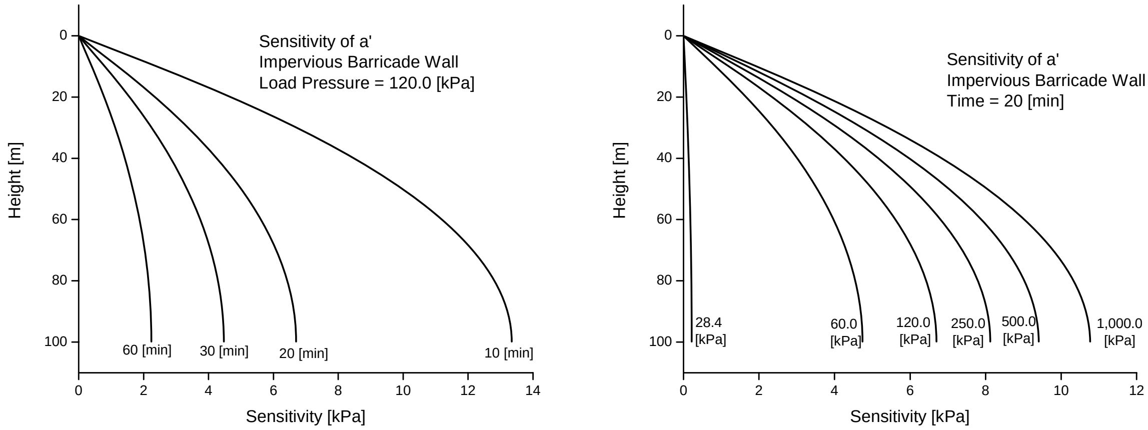

Figure 7 – Sensitivity of Coefficients a and ξ with respect to time and position (height) (left) and with respect to load pressure and position (height) (right)

Figure 8 illustrates the evolution in time of porous water pressure sensitivities with respect to parameters $\{a;a';c;d;\delta;e_0;\gamma;\xi\}$ at $y=66.6\ [m]$. Looking at Figure 8 it is possible to say if one parameter is more or less sensitivity than other. An important requirement in parameter estimation is that the sensitivity coefficients should not be of small magnitude, and when two or more parameters are estimated simultaneously, their sensitivity coefficients must be linearly independent over the experimental time domain (Beck *et al.*, 1985). Similar shapes (time dependence) of sensitivity coefficients for two different parameters indicate that their effects on the model response are similar, being, therefore, impossible to tell them apart. Larger sensitivity coefficients are related to better chances of obtaining good estimates. The sensitivity analysis performed herein shows that only $\{a';c;d\}$ are linearly independent for an experimental time lower than 10 minutes, therefore just these parameters might be estimated simultaneously

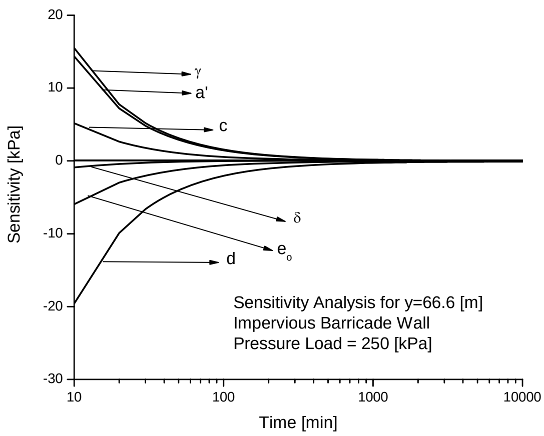

Figure 8 – Sensitivity analysis for $y=66.6\ [m]$

6. CONCLUSIONS

Numerical simulation of time dependent settlements in hydraulic fills is presented in this paper. The application of sensitivity calculations with respect to process variables permits a quantitative determination of how process variables govern performance, and allows us to identify the parameters with higher influence on the model. In this study, it was shown that these parameters $\{a';c;d\}$ are the most sensivitiy and important in the model presented herein.

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