EXERGETIC ANALYSIS OF A SOLAR COLLECTOR

Santiago del Rio Oliveira, santiagooliveira@yahoo.com.br
Luiz Fernando Milanez, milanez@fem.unicamp.br
Universidade Estadual de Campinas, Faculdade de Engenharia Mecânica

Abstract. *In the study of alternative energy sources, solar thermal energy is a promising source of great public and techno-scientific interest. The development of devices able to collect and store efficiently this energy is one of the great challenges of modern engineering. The performance of these devices can be optimized by means of thermodynamic analysis. A mathematical analysis of a flat plate solar collector is conducted to verify its thermal performance. This analysis is based on the first and second laws of thermodynamics considering the concept of exergy and its maximization. Optimum exit water temperature and optimum mass flow rate circulating through the solar collector as a function of the conditions of solar radiation are obtained. To exemplify the methodology, a case study is proposed and discussed.*

Keywords*: solar collector, optimization, exergy, second law of thermodynamics.*

1. INTRODUCTION

The thermodynamical analysis is an effective methodology to obtain precise informations about the energy efficiency and the losses due to the irreversibilities of a real device. Recently, emphasis has been given to the study of the minimization of the exergy destruction within a system, that is, the minimum loss of useful work, situation when the system operates with the minimum entropy generation. Thus, the procedure of exergy maximization or entropy generation minimization is employed to establish the theoretical limits of an ideal operation, very convenient to determine the better configuration for a specific application. This method was developed by Bejan (1982), and served as a basis to calculate the optimum parameters of a solar collector, as for example, the optimum temperature and mass flow rates of the working fluid inside the solar collector.

Due to the need to develop a solar collector integrated to a storage tank, Mohamad (1997) built a prototype with a thermal diode to avoid the reverse circulation during the night, and a mathematical model was idealized to evaluate the performance of the system. The simulation indicated that the thermal diode reduced significantly the heat losses at night. The exergy optimization of the working fluid in a solar collector and of the thermal energy storage system was analyzed by Aghbalou *et al.* (2006). Their study consisted of the analysis of a solar collector and a rectangular water storage tank containing phase change material (PCM), distributed in a set of plates in the upper part of the reservoir. An analytical solution for the melting process of the PCM together with a case study were presented and discussed. Another study, based on thermoeconomics, was presented by Ucar and Inalli (2007) with the objective to obtain the optimum solar collector area and the optimum volume of a thermal storage system, in order to develop an economically viable system.

The objective of this work is to perform a mathematical analysis of a flat plate solar collector to verify its thermal performance. This analysis is based on first and second laws of thermodynamics considering the concept of exergy maximization. Optimum exit water temperature and optimum water mass flow rate circulating through the solar collector are obtained as a function of solar radiation conditions. To exemplify this methodology a case study is proposed and discussed.

2. FORMULATION

Assuming unidimensional conduction and convection heat transfer, neglecting the viscous dissipation term in the energy equation due to the low velocity of the flow, the transient formulation for the water flow throught a solar collector, Fig. 1, results:

$$\rho c_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}\right)=k\frac{\partial^2 T}{\partial x^2}+\dot{q} \tag{1}$$

where $\dot{q}$ represents the radiation heat transfer rate per unit of volume of water in the solar collector. ρ, c_p, k and v_x are the density, the specific heat at constant pressure, the thermal conductivity of the water and the velocity component in x direction, respectively. Rewritting Eq. (1) in terms of $\partial T/\partial t$:

$$\frac{\partial T}{\partial t}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2}-v_x\frac{\partial T}{\partial x}+\frac{\dot{q}}{\rho c_p} \tag{2}$$

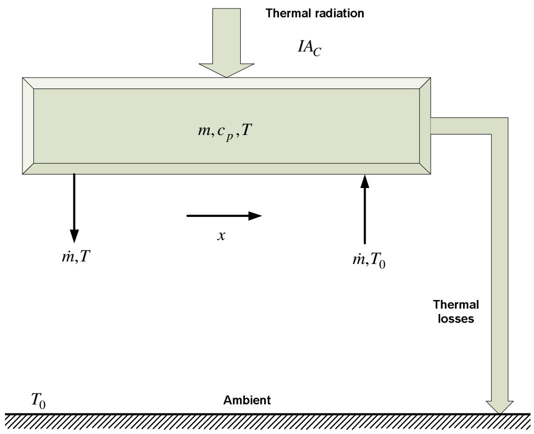


Figure 1. Scheme for a solar collector.

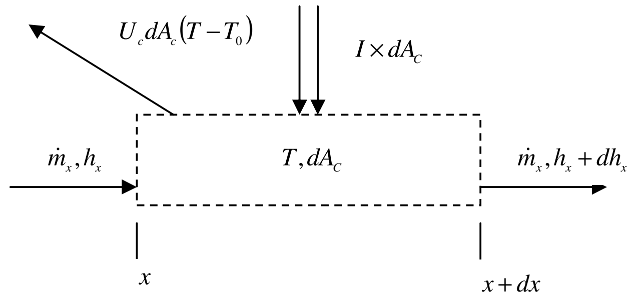


Figure 2. Differential control volume.

Utilizing $\dot{q} = \eta_c IA_c / V$ in Eq. (2):

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2} - v_x \frac{\partial T}{\partial x} + \frac{\eta_c I A_c}{\rho c_p V} \tag{3}$$

where η_c is the solar collector efficiency, V is the water volume inside the solar collector and I is the total flux of solar radiation. The thermal efficiency of a solar collector can be defined as the ratio of the energy absorbed by the water and the solar incident solar radiation rate, Howell et al. (1982), that is:

$$\eta_c = \frac{IA_c - U_c A_c (T - T_0)}{IA_c} \tag{4}$$

where U_c is the solar collector overall heat transfer coefficient. The mass *m* and the volume *V* of the water contained in interior the solar collector are related by the density:

$$\rho = m/V \tag{5}$$

The mean velocity of the water flow can be expressed as:

$$v_x = \dot{m}/\rho A_S \tag{6}$$

where $\dot{m}$ is the water mass flow rate and A_s is the cross sectional area of the ducts through which the water flows inside the solar collector. Mathematical expressions for the diffusive and convective terms of Eq. (3) can be obtained by means of two energy balances in the solar collector, one differential and another global. Fig. 2 illustrates a differential element of the solar collector with temperature T, length dx and unit depth. The first law of thermodynamics applied to this differential control volume indicates that the net heat transfer to the water accounts for the variation of the enthalpy *h* between positions x e $x + dx$, that is:

$$IdA_c - U_c dA_c (T - T_0) = \dot{m}_{x+dx} h_{x+dx} - \dot{m}_x h_x \tag{7}$$

From the mass conservation principle $\dot{m}_x = \dot{m}_{x+dx} = \dot{m}$ and for a differential area $dA_c = dx \times 1$, Eq. (7) can be expressed as:

$$Idx - U_c dx (T - T_0) = \dot{m}(h_{x+dx} - h_x) \tag{8}$$

Utilizing the Taylor expansion for h_{x+dx}, expressing the enthalpy variation of an incompressible fluid as funtion of its temperature variation, $dh_x = c_p dT$, and multiplying both sides of Eq. (8) by the solar collector length *L*, results:

$$IL - U_c L(T - T_0) = \dot{m} c_p L \frac{dT}{dx} \tag{9}$$

Utilizing a similar procedure, by means of a global energy balance for the entire solar collector, results:

$$IL - U_c L(T_{out} - T_0) = \dot{m} c_p (T_{out} - T_0) \tag{10}$$

where it was considered that the water inlet temperature into the solar collector is constant and equal to the ambient temperature T_0. T_{out} is the water temperature at the outlet of the collector. In the next equations T_{out} is written as T for simplicity. Subtracting Eq. (9) from Eq. (10) results:

$$\frac{dT}{dx} = \frac{(T - T_0)}{L} \tag{11}$$

Multiplying both sides of Eq. (11) by the mean velocity of the water flow v_x,

$$v_x \frac{dT}{dx} = v_x \frac{(T - T_0)}{L} \tag{12}$$

Dividing the expression $\dot{m} = \rho v_x A_s$ by $m = \rho V = \rho L A_s$, the result is $v_x / L = \dot{m}/m$. Substituting this result in Eq. (12), an expression for the convective term of Eq. (3) is obtained:

$$v_x \frac{dT}{dx} = \dot{m} \frac{(T - T_0)}{m} \tag{13}$$

The diffusive term of Eq. (3) can be obtained from Eqs. (6) and (13). Eq. (13) can be rewritten as:

$$\frac{dT}{dx} = \frac{\dot{m}}{m v_x} T - \frac{\dot{m}}{m v_x} T_0 \tag{14}$$

Deriving Eq. (14) with respect to *x*:

$$\frac{d^2 T}{dx^2} = \frac{\dot{m}}{m v_x} \frac{dT}{dx} \tag{15}$$

Substituting Eq. (14) into Eq. (15) and rearranging:

$$\frac{d^2 T}{dx^2} = \left(\frac{\dot{m}}{m v_x} \right)^2 (T - T_0) \tag{16}$$

Substituting the mass flow rate of Eq. (6) in Eq. (16) results in:

$$\frac{d^2 T}{dx^2} = \left(\frac{\rho A_s}{m} \right)^2 (T - T_0) \tag{17}$$

Substituting Eqs. (4), (13) and (17) in Eq. (3):

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \frac{\rho^2 A_s^2}{m^2} (T - T_0) - \dot{m} \frac{(T - T_0)}{m} + \frac{I A_c - U_c A_c (T - T_0)}{\rho c_p V} \tag{18}$$

Eq. (18) can be rearranged to be written as:

$$\dot{m} \frac{(T - T_0)}{m} = \frac{I A_c}{\rho c_p V} + \frac{k}{\rho c_p} \frac{\rho^2 A_s^2}{m^2} (T - T_0) - \frac{U_c A_c (T - T_0)}{\rho c_p V} - \frac{\partial T}{\partial t} \tag{19}$$

Inserting $m = \rho V$ in the first and third terms of the right side Eq. (19) results:

$$\dot{m}\frac{(T-T_0)}{m} = \frac{IA_c}{mc_p} + \frac{k}{c_p}\frac{\rho A_s^2}{m^2}(T-T_0) - \frac{U_c A_c (T-T_0)}{mc_p} - \frac{\partial T}{\partial t} \tag{20}$$

Multiplying both sides of Eq. (20) by mc_p results:

$$\dot{m}c_p(T-T_0) = IA_c + \frac{k\rho A_s^2}{m}(T-T_0) - U_c A_c (T-T_0) - mc_p\frac{\partial T}{\partial t} \tag{21}$$

Eq. (21) can be written dividing the temperature by T_0:

$$\dot{m}c_p T_0(T/T_0-1) = IA_c + \frac{k\rho A_s^2}{m}T_0(T/T_0-1) - U_c A_c T_0 (T/T_0-1) - mc_p T_0\frac{\partial}{\partial t}\left(\frac{T}{T_0}\right) \tag{22}$$

Eq. (22) can be rearranged to be written as:

$$\dot{m}c_p T_0 = \frac{IA_c}{(T/T_0-1)} + \frac{k\rho A_s^2}{m}T_0 - U_c A_c T_0 - \frac{mc_p T_0}{(T/T_0-1)}\frac{\partial}{\partial t}\left(\frac{T}{T_0}\right) \tag{23}$$

Writing Eq. (23) in terms of the expression $A_c T_0 k\rho A_s/m$ in the second and third terms of the right hand side:

$$\dot{m}c_p T_0 = \frac{IA_c}{(T/T_0-1)} - \frac{A_c T_0 k\rho A_s}{m}\left(\frac{U_c m}{k\rho A_s} - \frac{A_s}{A_c}\right) - \frac{mc_p T_0}{(T/T_0-1)}\frac{\partial}{\partial t}\left(\frac{T}{T_0}\right) \tag{24}$$

Defining the dimensionless length parameter $l_c = m/\rho A_s$:

$$\dot{m}c_p T_0 = \frac{IA_c}{(T/T_0-1)} - \frac{A_c T_0 k}{l}\left(\frac{U_c l}{k} - \frac{A_s}{A_c}\right) - \frac{mc_p T_0}{(T/T_0-1)}\frac{\partial}{\partial t}\left(\frac{T}{T_0}\right) \tag{25}$$

Finally, utilizing the following variables in Eq. (25):

$$\theta = \frac{T}{T_0}, \quad i = IA_c, \quad \mathrm{Bi}_c = \frac{U_c l_c}{k}, \quad K = \frac{A_c T_0 k}{l_c}\left(\mathrm{Bi}_c - \frac{A_s}{A_c}\right), \quad \beta = mc_p T_0 \tag{26}$$

an expression for the energy conservation of the water flow inside the solar collector can be obtained, rewritten as:

$$\dot{m}c_p T_0 = \frac{i}{(\theta-1)} - K - \frac{\beta}{(\theta-1)}\frac{\partial\theta}{\partial t} \tag{27}$$

where θ is the dimensionless temperature based on the ambient temperature, i is the incident solar radiation rate and Bi is the Biot number based on the overall heat transfer coefficient U_c. The units of K and β are, respectively, Watt and Joule. The water specific exergy ϕ between states of temperatures T_0 and T, can be written as:

$$\phi = (u-u_0) + p_0(v-v_0) - T_0(s-s_0) + v_x^2/2 + gZ \tag{28}$$

where u_0, v_0 and s_0 are, respectively, the specific internal energy, specific volume and specific entropy of the water evaluated at temperature T_0 and pressure p_0. Neglecting the terms of kinetic and potential energy in Eq. (28) results:

$$\phi = (u + p_0 v) - (u_0 + p_0 v_0) - T_0(s-s_0) \tag{29}$$

Utilizing the definition of enthalpy, $h = u + pv$, in Eq. (29):

$$\phi = (h - h_0) - T_0 (s - s_0) \tag{30}$$

For incompressible fluid flow, $h - h_0 = c_p (T - T_0)$ and $s - s_0 = c_p \ln(T/T_0)$. Thus, from Eq. (30):

$$\phi = c_p (T - T_0) - c_p T_0 \ln(T/T_0) \tag{31}$$

Eq. (31) can be written as:

$$\phi = c_p T_0 (T/T_0 - 1) - c_p T_0 \ln(T/T_0) \tag{32}$$

Utilizing the dimensionless temperature $\theta = T/T_0$ in Eq. (32) and rearranging:

$$\phi = c_p T_0 (\theta - 1 - \ln\theta) \tag{33}$$

Total water exergy Φ, corresponding to a given mass flow rate $\dot{m}$, during a time interval $(0, t)$ can be written as:

$$\Phi = \int_0^t \dot{m}(t) \times \phi(t) dt \tag{34}$$

Eq. (27) can be rewritten in terms of the water mass flow rate in the form:

$$\dot{m}(t) = \frac{1}{c_p T_0} \left[\frac{i}{(\theta - 1)} - K - \frac{\beta}{(\theta - 1)} \frac{\partial \theta}{\partial t} \right] \tag{35}$$

With Eq. (33), the product $\dot{m}(t) \times \phi(t)$ is then rewritten as:

$$\dot{m}(t) \times \phi(t) = \frac{1}{c_p T_0} \left[\frac{i}{(\theta - 1)} - K - \frac{\beta}{(\theta - 1)} \frac{\partial \theta}{\partial t} \right] \times c_p T_0 (\theta - 1 - \ln\theta) \tag{36}$$

Simplifying and rearranging Eq. (36):

$$\dot{m}(t) \times \phi(t) = \left(\frac{i - \beta \times \partial\theta/\partial t}{\theta - 1} - K \right) (\theta - 1 - \ln\theta) \tag{37}$$

Finally, substituting Eq. (37) in Eq. (34) results:

$$\Phi = \int_0^t \left(\frac{i - \beta \times \partial\theta/\partial t}{\theta - 1} - K \right) (\theta - 1 - \ln\theta) dt \tag{38}$$

3. EXERGETIC OPTIMIZATION

One expression for the dimensionless temperature θ of the water at the exit of the solar collector that maximizes the integral of Eq. (38), that is, the temperature that maximizes the exergy delivered by the water can be obtained utilizing tools of variational calculus. Eq. (38) has the form $\Phi = \int_0^t f(t, \theta, \theta') dt$. The function $f(t, \theta, \theta')$ in which θ reaches a maximum satisfies Euler equation:

$$\frac{\partial f}{\partial \theta} - \frac{\partial}{\partial t} \left(\frac{\partial f}{\partial \theta'} \right) = 0 \tag{39}$$

Partial derivatives of the function $f(t,\theta,\theta')$ with respect to θ', θ and t are written as:

$$\frac{\partial f}{\partial \theta'}=\frac{\beta(1-\theta+\ln\theta)}{\theta-1} \tag{40}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial \theta'}\right)=\frac{\beta(-\theta'+\theta'/\theta)(\theta-1)-\beta\theta'(1-\theta+\ln\theta)}{(\theta-1)^2} \tag{41}$$

$$\frac{\partial f}{\partial \theta}=\frac{\left[\beta\theta'-i-K(\theta-1)^2\right](\theta-1)+(i-\beta\theta')\theta\ln\theta}{\theta(\theta-1)^2} \tag{42}$$

Solving Euler equation, the dimensionless temperature θ_{opt} of the water is written in the form:

$$\frac{(\theta_{opt}-1)^3}{\theta_{opt}\ln\theta_{opt}-\theta_{opt}+1}=\frac{i(t)}{K} \tag{43}$$

Eq. (43) is a transcendental equation whose solution can be obtained numerically. Its solution depends on the knowledge of the incident solar radiation flux, the thermal conductivity and density of the water, the area of the solar collector, the water flow duct cross sectional area, the water mass contained in the solar collector and of the overall de heat transfer coefficient. Substituting the results obtained from the solution of Eq. (43) in Eq. (35), the optimum mass flow rate $\dot{m}_{opt}$ of the water circulating through the solar collector as function of time can be calculated.

It may be noticed in Eq. (35) that it is necessary to calculate the optimum temperature variation rate. A good approximation for this derivative can be obtained utilizing results from the graph of the optimum temperatures as function of time, obtained from Eq. (43). With the numerical values of $\partial\theta/\partial t$ and Eq. (35) it is possible to estimate the optimum water mass flow rate circulating in the solar collector as a function of the daily condition of the solar radiation.

4. RESULTS AND DISCUSSION

An idealized flat plate solar collector is positioned in a recommended inclination of 32°. This inclination corresponds to the local latitude of the city of Bauru-SP, which is 22°, plus 10° to compensate the winter period. Physical parameters utilized in the calculations of the solar collector can be visualized in Tab. 1.

Table 1. Calculation parameters for the solar collector.

A_c = 2.0 m^2	A_s = 0.007 m^2	U_c = 4.5 W/m^{2}K	k = 0.613 W/m.K	T_0 = 300 K	m = 4 kg
c_p = 4179 J/kgK	l_c = 0.5714 m	$\mu = 855\times10^{-6}$ kg/m.s	Bi_c = 4.1948	K = 2697.7 W	β = 4999800 J

Inalli et al. (1997) suggested a value of 4.5 W/m^{2}K for the overall heat transfer coefficient of the solar collector. This value was adopted for the solar collector calculations in this work. The numerical values of k, ρ, μ and c_p were obtained from the text book of Incropera and DeWitt (2002) for saturated water at temperature of 300 K. Parameters A_c, A_s and m were adopted. A representative day for the solar radiation flux was chosen to calculate the design and optimization parameters. The chosen day was 01/12/2008 and the data were obtained from Institute of Metheorological Researches in the city of Bauru – IPMET. The solar radiation flux in the period between 9 h and 18 h can be seen in Fig. 3.

The first step in the calculation procedure consists in the optimization of the water mass flow rate through the solar collector in order to extract the maximum work (exergy) from the water heated by solar energy. In order to do this, it is necessary to calculate the solar collector optimum exit temperature, θ_{opt}, as function of the daily solar radiation rate $i(t)=I(t)\times A_c$, according to Eq. (43). This equation is a transcendental equation and its solution was obtained numerically. Calculation parameters for this solution are the thermal radiation rate $i(t)$ and the parameter K. With the numerical solution of this equation, numerical values for the dimensionless optimum temperature θ_{opt} are obtained.

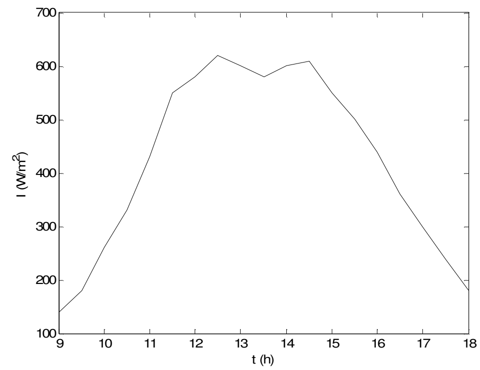
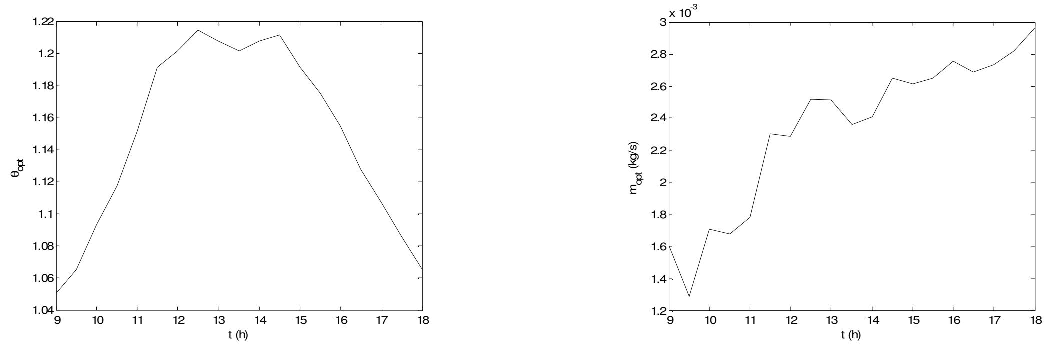


Figure 3. Solar radiation flux in the period between 9 h and 18 h.

Fig. 4a shows the results for the dimensionless optimum temperature of the water exiting the solar collector as a function of time. It may be noticed that the higher values of the optimum temperature occur between 11 h and 15 h and the optimum temperature of the water exiting the solar collector reaches a maximum value of approximately 364 K between 12 h and 13 h. Furthermore, by comparing Fig. 3 with Fig. 4a it may be verified that the form of the optimum temperature profiles as function of time is similar to the form of the solar radiation rate profiles as function of time, a trend also observed by Bejan (1982). Therefore, it is possible to conclude that the optimum temperature of the water exiting the solar collector must vary according to the solar radiation rate $i(t)$ for the maximum supply of exergy by the water.



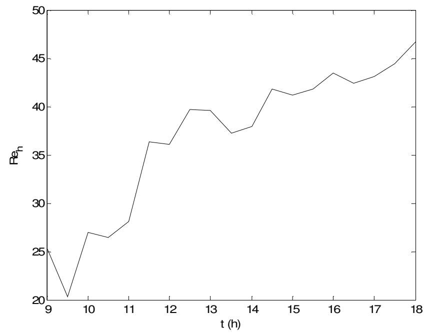
(4a) (4b)

Figure 4. Optimum exiting temperature and mass flow rate of the water in the solar collector as a function of time.

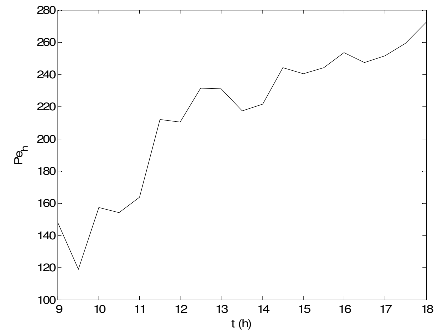
Once the numerical values of θ_{opt} as function of time are obtained, the optimum water mass flow rate $\dot{m}_{opt}$ that should circulate through the solar collector can be calculated as a function of time using Eq. (35), where the variation rate $\partial\theta_{opt}/\partial t$ can be estimated from Fig. 4a for each time interval. Furthermore, the numerical values of the parameters c_p, T_0, K and β can be visualized in Tab. 1. The results obtained are shown in Fig. 4b. With the water density, the water mass flow rate of kg/s can be converted into volume flow rate.

From Fig. 4b it may be verified that the water mass flow rate is a function that increase with time, reaching a maximum of 0.003 kg/s near 18 h. Furthermore, from Fig. 4b, pronounced variations of the numerical values obtained may be noticed, requiring a mechanical pump programmed to attend these different values. From a practical and also economical point of view a design of this kind may not be viable. Thus, it is necessary to verify if the utilization of the optimized values obtained are economically acceptable. One simplified alternative is the utilization of a mean value for the optimum water volume flow rate. For the present study, this value is 8.4 l/h.

With the results of Fig. 4b and the thermophysical properties of Tab. 1 the Reynolds number of the water flow inside the ducts of the solar collector as function of time can be calculated. The results can be visualized in Fig. 5a. With the thermophysical properties of Tab.1 the Prandtl number for the water flow was evaluated as being 5.8. Thus, the Peclet number, defined as the product of the Reynolds number by the Prandtl number was also calculated. The values of the Peclet number as function of time can be visualized in Fig. 5b. When the Peclet number is higher than 100, the axial conduction can be neglected. The Peclet number is a measure of the relative importance between axial convection and axial conduction. Thus, in the solar collector mathematical modeling, the diffusive term has little influence on the physical problem being analyzed, because during the time interval considered, from 9h to 18 h the values of the Peclet number where always above 100.



(a)



(b)

Figure 5. Reynolds and Peclet numbers of water as function of time.

5. CONCLUSIONS

Based on the results obtained, the following conclusions can be drawn:

1) The utilization of the second law of thermodynamics in the analysis of processes and equipments proved to be a useful tool and of great importance. Several informations, as design parameters, were obtained and evaluated by the application the second law together with the first law of thermodynamics.

2) The optimum temperature of the water exiting the solar collector as well as its optimum mass flow rate were obtained through the exergy concept.

3) The water should circulate with variable mass flow rate inside the solar collector, being a function of the water temperature inside the collector, both variable with time.

4) The water mass flow rate that should circulate inside the solar collector is a function of the incident solar radiation flux on the surface of the solar collector.

5) This method is limited to this simple collector configuration. More complex models should be analyzed by numerical and experimental techniques, due to the mathematical limitations of the analytical solutions.

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