A SIMPLIFIED MODEL OF THE TYMPANIC MEMBRANE BEHAVIOR IN FLIGHT CONDITIONS

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Abstract. *The variation of aircraft cabin pressure is one of the major factors that yields discomfort for passengers and crew. In this work a mathematical model was developed to describe the tympanic membrane behavior in function of cabin pressure changes. The tympanic cavity was modeled as an air cavity separated from the outside environment by the tympanic membrane and the Eustachian tube. The tympanic membrane was modeled as a rigid circular plate with negligible mass, connected to a linear spring. The Eustachian tube was modeled as a tube with airflow resistance. Parametric analysis for the elastic constant of the tympanic membrane and the airflow resistance of the Eustachian tube were conducted because it was found that these parameters are not well established in the literature. The simulation results for typical cabin pressure variations showed that people with higher airflow resistances of the Eustachian tube suffer higher deformations in their tympanic membranes. On the other hand, higher values of the elastic constant imply in lower deformation of the tympanic membrane but imply in higher pressure difference. Simulations also showed that the rate of pressure variation employed affect the tympanic membrane deformation; higher rates imply in higher deformations.*

Keywords*: tympanic membrane, middle ear, cabin pressure, comfort, mathematical modeling.*

1. INTRODUCTION

It is well known that the atmospheric pressure decreases with the increasing altitude. Hence, airplanes, when in flight, need to have the cabin pressure reduced in such a way as to approach the external atmospheric pressure. This is necessary so the airplane structure does not need to be too much heavier to support greater pressures gradients. So, the airplane internal pressure is reduced in the ascent stage, while it is increased in the descent stage of the flight. However, the cabin pressure variation provokes some discomforts to the crew and passengers. The tympanic membrane is directly affected by this pressure variation.

The human ear is divided in three parts, external ear, middle ear and internal ear (Dângelo e Fattini, 2000). The middle ear is the most important part for this study, because it contains the tympanic membrane, tympanic cavity, human ear ossicles, mastoid air cell system and it is connected to the bone part of the Eustachian tube. The tympanic cavity is an air cavity separated from the external medium by the tympanic membrane and the Eustachian tube. The Eustachian tube is a physiologic tube extending from the middle ear to the nasopharynx. It is formed of bone, cartilage and fibrous tissue (Armstrong and Heim, 1934). The cartilaginous part of the Eustachian tube is normally collapsed by the compression of external tissues. It is opened actively by the tensor palati muscle during swallowing, yawning, or movements of the mandible. It opens for 0.20 to 0.25 seconds once every 1 to 2 minutes (Sadé e Ar, 1997; Kanick e Doyle, 2005). Thus, it is noted that the tympanic cavity remains the most part of the time isolated from the external environment. Therefore the tympanic cavity pressure is often different from the cabin pressure. Thus, the tympanic membrane, which separates both parts, is deformed by the action of the pressure difference.

The deformation suffered by the tympanic membrane is one of the main reasons of discomfort in airplane passengers and crew. Thus, in this work a mathematical model was developed in order to demonstrate how the tympanic membrane distorts according to the cabin pressure variation.

1.1. The Human Ear

The human ear is basically formed by three parts: external ear, middle ear and inner ear (Dângelo e Fattini, 2000) (fig. 1). Among the three parts that forms the human ear, the middle ear is the one of most interest in this study. It consists of two air cavities physically connected but physiologically separated. The anterior part, tympanic cavity, contains the ossicles of human ear. The posterior part is formed of a system of air cells, also called mastoid air cell system (Kanick e Doyle, 2005).

The tympanic cavity is connected to the nasopharynx through the Eustachian tube. The posterior part of the Eustachian tube is bony and rigid and is physically an extension of the tympanic cavity. The tympanic membrane separates the tympanic cavity from the external environment. Thus the tympanic membrane is the most affected structure because of the cabin pressure variation. For that reason, it is the indicator of discomfort and pain to be modeled.

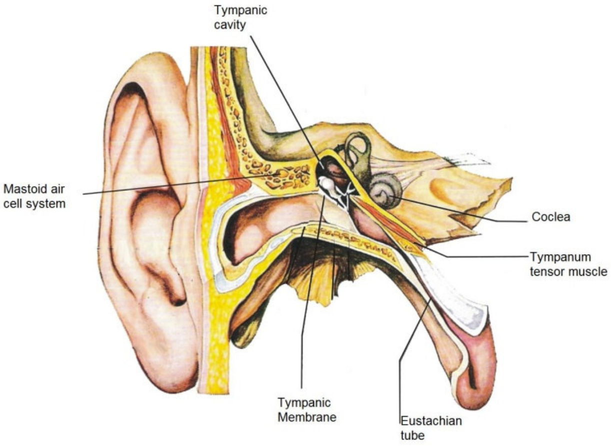

Figure 1: Human ear anatomy.

According to Fay et al. (2005), the tympanic membrane is composed of a series of layers and two of these layers contain collagen fibers. One has fibers that run in a radial pattern while the adjacent layer's fibers run in the circumferential direction. According to Cheng et al. (2007), on average the tympanic membrane is 10 mm diameter and 0.08 mm thick. Several experiments were conducted with the purpose of obtaining the mechanical properties of the tympanic membrane. It was observed that the membranes have an elastic behavior but there were great differences in the results obtained by many authors (Cheng et al., 2007; Fay et al., 2005).

1.2. Gas exchange mechanisms

Just as the cabin pressure changes, the tympanic cavity pressure also changes. However, these changes are different in most cases. Therefore, the cabin pressure and the tympanic cavity pressure might be different. The human ear has some mechanisms that balance the air pressure and composition among the tympanic cavity and adjacent environments. The main known mechanisms are the gas exchange through the Eustachian tube, through gas diffusion with the blood and gas exchange with the mastoid air cell system.

1.2.1. Gas exchange through the Eustachian tube

The main tympanic cavity gas exchange mechanism is through the Eustachian tube, also known as ventilation or inhalation. According to SAE ARP1270 (2000), the bony part of the Eustachian tube, next to the tympanic cavity, is normally opened. Whilst the cartilaginous part, next to the nasopharynx, is normally closed. Sadé e Ar (1997) say that the Eustachian tube is 3 to 4 cm long, formed by two cone like structures fused together by a narrow ring, the isthmus, which is a ring 1 to 2 mm long and 0.6 to 1.2 mm in diameter. When the pressure difference between the tympanic cavity and the nasopharynx is greater than 3,4 kPa, it is enough to open the Eustachian tube passively. This may happen when the airplane is in the ascent stage of the flight. In the descent stage of the flight, the necessary pressure difference between the tympanic cavity and the nasopharynx must be greater than 5,9 kPa to open passively the Eustachian tube. This value is greater because of the physicals characteristics of the Eustachian tube.

2. MATHEMATICAL MODEL

The tympanic cavity was modeled as an air reservoir with volume V. The Eustachian tube, which connects this air reservoir with the nasopharynx, was modeled as a tube with an airflow resistance R_v (fig. 2). The tympanic membrane was modeled as a rigid circular plate with negligible mass, connected to a linear spring with elastic modulus K. Some of these parameters are well established in the literature but others disagree to each other in more than one order of magnitude. The aim of this model is to establish the relationship between the tympanic membrane deformation and the cabin pressure variation.

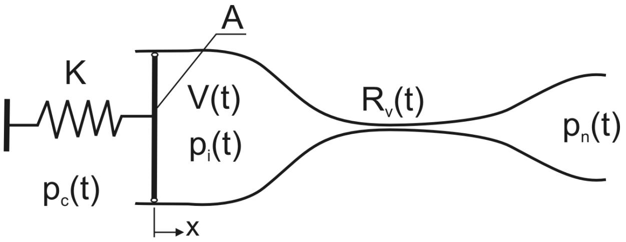

Figure 2: Simplified model of the middle ear.

The final model equation was developed in such a way to obtain the tympanic membrane deformation (x) as a function of the cabin pressure (p_c), cabin pressure change rate (dp_c/dt) and Eustachian tube airflow resistance (R_v).

The conservation of mass equation indicates that the rate of change of mass within the tympanic cavity is equal to the mass flow through the Eustachian tube.

$$\rho(t)\cdot q(t)=\frac{dm(t)}{dt} \tag{1}$$

where ρ – density of the air, [kg/m^3];
q – volumetric air flow through the Eustachian tube, [m^3/s];
m – mass of air in the tympanic cavity, [kg].

The airflow is related to the pressure difference between the nasopharynx and the tympanic cavity and the Eustachian tube airflow resistance by the following equation:

$$q(t)=\frac{\Delta p(t)}{R_v(t)}=\frac{p_n(t)-p_i(t)}{R_v(t)} \tag{2}$$

where p_i – tympanic cavity pressure, [Pa];
p_n – nasopharynx pressure, [Pa];
R_v – Eustachian tube air flow resistance, [Pa.s/m^3].

Substituting Eq. (2) in Eq. (1), the following equation is obtained

$$\frac{dm(t)}{dt}=\rho(t)\cdot\frac{p_n(t)-p_i(t)}{R_v(t)}=\frac{m(t)}{V(t)}\cdot\frac{(p_n(t)-p_i(t))}{R_v(t)} \tag{3}$$

where V – volume of the tympanic cavity, [m^3];

A forth equation is obtained considering the air as a perfect gas, thus its state equation is given by the Eq. (4).

$$m(t)=\frac{p_i(t)V(t)}{RT_i} \tag{4}$$

where R – air constant, [J/kg.K];
T_i – air temperature, [K].

Substituting Eq. (4) in Eq. (3),

$$\frac{p_i(t)}{RT_i}\cdot\frac{(p_n(t)-p_i(t))}{R_v(t)}=\frac{d}{dt}\left(\frac{p_i(t)\cdot V(t)}{RT_i}\right)=\frac{1}{RT_i}\left(V(t)\cdot\frac{dp_i(t)}{dt}+p_i(t)\cdot\frac{dV(t)}{dt}\right) \tag{5}$$

The tympanic cavity volume can be written in the following form,

$$V(t) = V_0 - A(x(t) - x_0) \tag{6}$$

$$dV(t) = -A\,dx(t) \tag{7}$$

where A – tympanic membrane surface area, [m^2];
x – tympanic membrane deformation, [m].

The force balance between the two sides of the tympanic membrane results in Eq. (8).

$$p_i(t)A - p_c(t)A = -K(x(t) - x_0) \tag{8}$$

Thus,

$$p_i(t) = p_c(t) - \frac{K}{A}(x(t) - x_0) \tag{9}$$

$$dp_i(t) = dp_c(t) - \frac{K}{A}dx(t) \tag{10}$$

Substituting Eq. (6), (7), (9) and (10) in Eq. (5), and considering that $p_n(t)=p_c(t)$, the following ordinary differential equation is obtained:

$$\frac{dx(t)}{dt} = \frac{\frac{K}{A \cdot R_V(t)} \cdot \left(\frac{A \cdot p_c(t)}{K} - (x(t) - x_0) \right) \cdot (x(t) - x_0) - (V_0 - A \cdot (x(t) - x_0)) \cdot \frac{dp_c(t)}{dt}}{2 \cdot K \cdot (x(t) - x_0) - A \cdot p_c(t) - \frac{V_0 \cdot K}{A}} \tag{11}$$

Equation (11) relates the tympanic membrane deformation to the cabin pressure, cabin pressure change rate and Eustachian tube airflow resistance. The forth order Runge-Kutta method was used to solve Eq. (11).

3. RESULTS

Some of the model parameters were not found in the literature, so parametric analyses were conducted varying those mathematical model parameters. Table 1 shows the average values of the model parameters used in the simulations.

Table 1: Average values of model parameters used in simulation.

Parameter	Mean	Units	Parameter	Mean	Units
V_0	8.75	ml	Eustachian tube collapsed period	75	s
A	0.6	cm^2	Eustachian tube opening duration	0.25	s
K	500	N/m	Ascent passively open Eustachian tube pressure	3.4	kPa
Rv closed	1.10^{10}	Pa.s/m^3	Descent passively open Eustachian tube pressure	5.9	kPa
Rv open	1000	Pa.s/m^3			

Two typical airplane cabin pressure curves were used in the model simulations. Those curves are presented in Fig. 3 and Fig. 4. They provide model entrance data for cabin pressure and cabin pressure change rate.

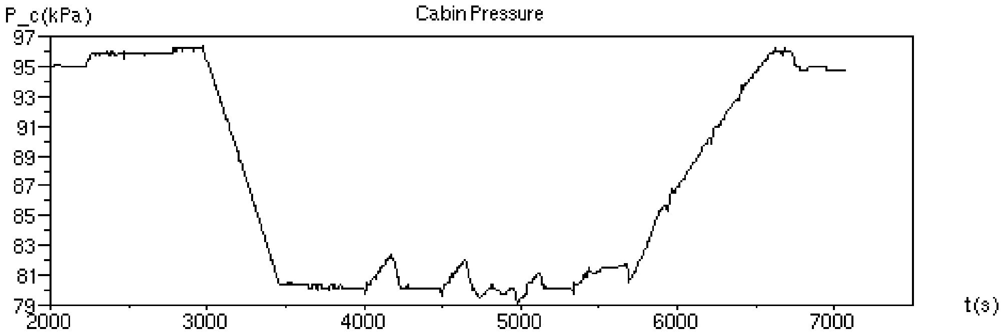

Figure 3: First typical cabin pressure curve.

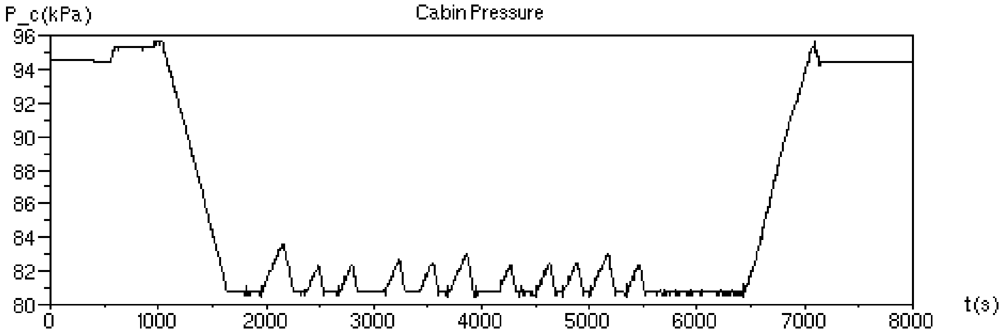

Figure 4: Second typical cabin pressure curve.

The analyses of the above pressures curves show the three main stages of a flight. The first stage is the ascent stage represented by the decreasing of the cabin pressure. The second stage is the cruising stage, represented by the stability of the cabin pressure in a lower pressure. The third stage is the descent stage, represented by the increasing cabin pressure.

Maximum deformations of the tympanic membrane are expected during ascent and descent stages, because in these stages there are higher cabin pressure changes rates. It is noted that in the cruising stage there are also pressure changes, which may occur at high change rates provoking great deformations of the tympanic membrane.

Using Tab. 1 contents in the model, tympanic membrane deformation for both cabin pressure curves shown above were simulated. The results are shown in Fig. 5.

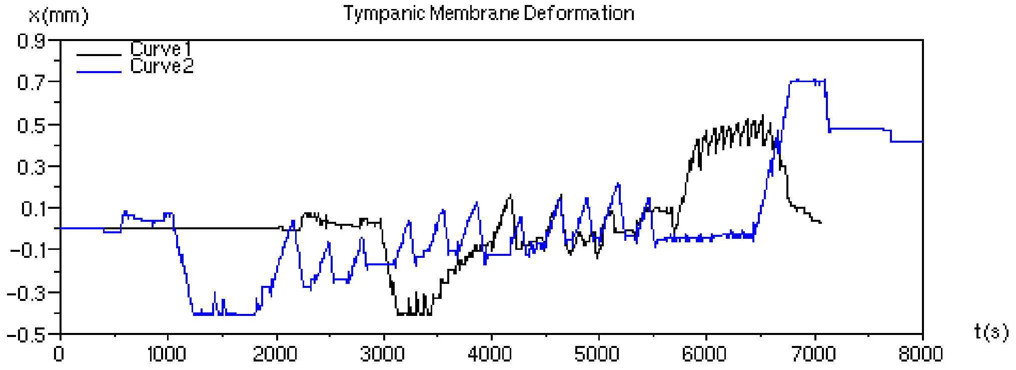

Figure 5: Tympanic membrane deformation for the first and second cabin pressure curves.

The analyses of the figures above show that the tympanic membrane deformation is larger for the stages of ascent and descent of the airplane flight just as it was expected. Also, it is noted that the tympanic membrane deformation reaches a maximum value during the ascent stage. This happens because the pressure difference between the tympanic cavity and the nasopharynx is enough to open the Eustachian tube passively. For the descent stage, it is noted that the tympanic membrane deformation is greater in the second curve than in the first curve. This happens because the cabin pressure change rate is greater in the descent stage of the second curve than in the first curve (Fig.3 e Fig.4). For this reason the tympanic membrane deformation reaches a maximum in the descent stage in the second curve.

Figure 5 also shows that in the cruising stage of the flight, those pressure variations provoke deformations at high rates in the tympanic membrane. Besides, it shows that the tympanic membrane return to the equilibrium condition is very slow during the cruising stage for the parameters values adopted in the simulation.

The Eustachian tube airflow resistance and the tympanic membrane elastic constant are parameters that were found to be in great disagreement among different authors. So, parametric analyses were conducted for these two parameters with the purpose of obtaining the model sensitivity for them.

The first parametric analysis consists in simulating the model for different values of the Eustachian tube airflow resistance. Those values are 100, 500, 1000 and 4000 Pa.s/m^3. Figure 6 shows the graphic of the tympanic membrane deformation for this parametric analysis considering the first typical cabin pressure curve (Fig. 3).

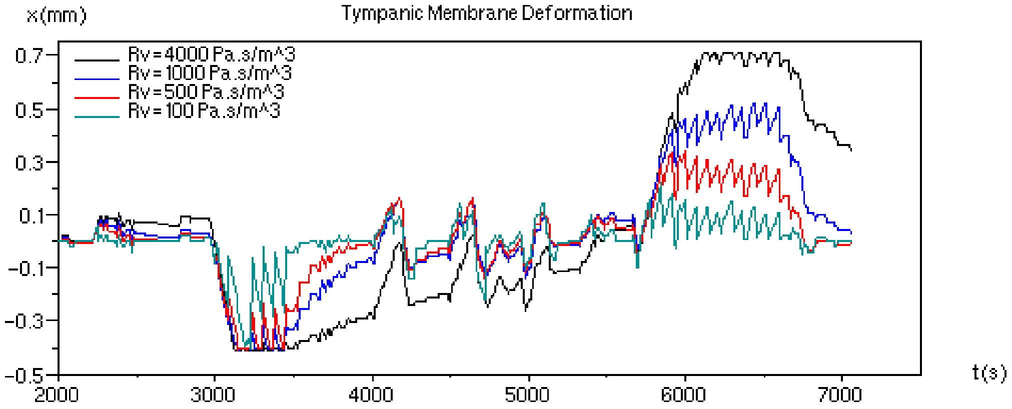

Figure 6: Tympanic membrane deformation for several values of the Eustachian tube airflow resistance, relative to the first cabin pressure curve.

The result observed in Fig. 6 was the expected and means that those people who present higher Eustachian tube airflow resistance will suffer more pain in the tympanic membrane during the flight. However, people with healthy Eustachian tubes with no mucus, that means lower values of airflow resistance, will be more comfortable during the flight because they will suffer lower deformations of the tympanic membrane.

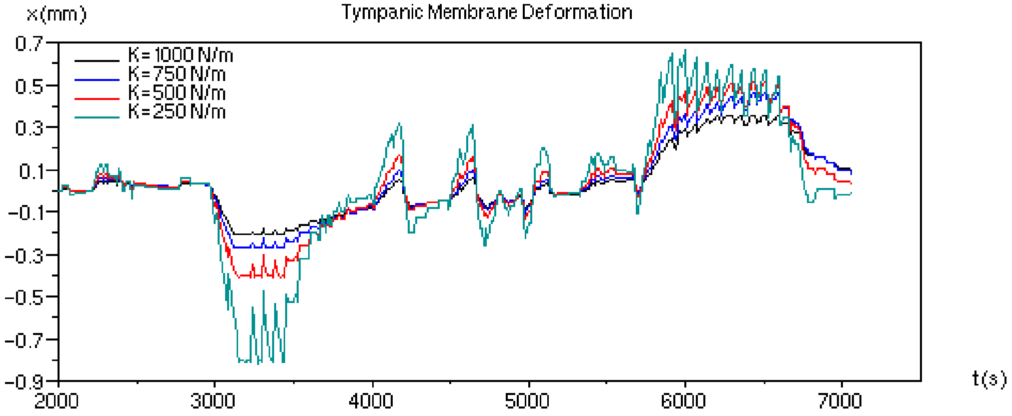

Figure 7: Tympanic membrane deformation for several values of the tympanic membrane elastic constant, relative to the first cabin pressure curve.

The second parametric analysis consists in simulating the model for different values of the tympanic membrane elastic constant. The values adopted in the simulation are 250, 500, 750 and 1000 N/m. Figures 7 and 8 show respectively the tympanic membrane deformation and the tympanic membrane pressure difference relatively for the first cabin pressure curve (Fig. 3).

Figure 7 shows that the tympanic membrane deformation is higher for lower values of the elastic constant. Besides, it can be seen that the lower tympanic membrane elastic constant implies in faster returns to the equilibrium condition while the Eustachian tube is opened.

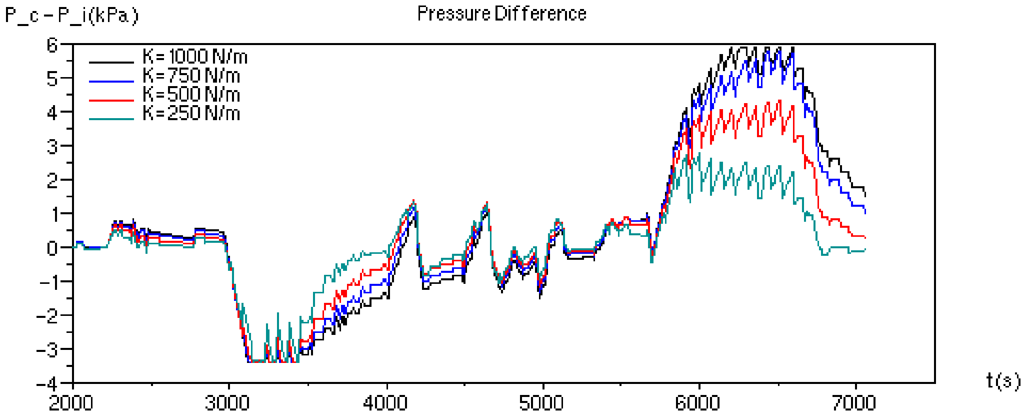

Figure 8: Tympanic membrane pressure difference for several values of the elastic constant, relative for the first cabin pressure curve.

Figure 8 shows that tympanic membrane with higher elastic constant supports higher pressure difference in spite of suffering lower deformation.

These parametric analyses allow to conclude that the Eustachian tube airflow resistance and the tympanic membrane elastic constant are very important parameters in the model to predict comfort and discomfort conditions during flight. Therefore, an effort should be put on the experimental determination of these parameters.

4. CONCLUSION

The development of the mathematical model was successfully conducted. Actual values of cabin pressure change were utilized in this model. The parametric analyses were conducted because two important parameters were not found with agreement in the literature. They showed that higher values of the Eustachian tube implies in higher deformation of the tympanic membrane. Also, parametric analyses showed that lower values of the elastic constant of the tympanic membrane implies in higher deformations but lower pressure differences between both sides of the tympanic membrane. Besides, it was shown that the pressure change rates have an important effect over the tympanic membrane deformation. Higher pressure rates implies in higher deformations.

5. REFERENCES

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