A NON LINEAR NUMERICAL PROCEDURE FOR NON PLANAR LIFTING SURFACES

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Abstract. *This paper describes a fast method for the calculation of aerodynamic characteristics of a complete aircraft where a simple three-dimensional numeric model is corrected based on the characteristics of two-dimensional flow around the aerodynamic profile experimentally obtained, and including a free wake model and the measurement of induced drag through the Momentum variation. This procedure results in a non linear method capable of predicting lift, induced drag, parasite drag, aerodynamic moments both in linear regions as well as stall regions in the lift curve of a complex non-planar set of lifting surfaces, including effects due to yaw, pitch and roll, and the influence between surfaces. This paper presents some results obtained with this procedure and compares them to experimental results.*

Keywords*: vortex lattice, wing, stall, free-wake, Treftz-Plane*

1. INTRODUCTION

In initial phases of design activities, the detailed knowledge of all the flow around the aircraft is not relevant, the designer is interested only in the resulting aerodynamic forces (drag, lift and moments), which motivate the development of diverse specific techniques for determination of these aerodynamic forces, without the need for solution of the entire flow fiels (Boundary Element Method), highlighting the methods: Lifting Line (Prandtl, 1921), Vortex-Lattice (Lamar, 1976) and Panels (Hess and Smith, 1966).

Since the objective of this paper regards design activities for complete aircraft and related work, an analysis procedure is considered efficient when, aside from supplying coherent results in comparison with experimental measurements, it is also robust and fast, as well as applicable to a great variety of cases. The condition of necessity of applicability to multiple lifting surfaces of complex geometries makes it impossible to use the Classic Lifting Line method, while the condition of processing speed makes it impossible to use the panel method, which makes the Vortex-Lattice method the natural choice.

2. THE VORTEX LATTICE METHOD

The Vortex-Lattice method is based on the solution of the Laplace equation through the distribution of singularities (horseshoe vortex) along the body, which responds to the condition of impermeability (flow cannot pass throughout a non-porous surface). In its classic formulation, the vortex-lattice method has singularity distributions both along the span as well as in the camber line of the airfoil. However, in the proposed method, since two-dimensional information on the aerodynamic airfoil would be used, the distribution along the chord becomes unnecessary. This method is also known as Wessinger (Wessinger, 1947), or modern lifting line (Phillips and Snyder, 2000). The distribution of bounded vortex is at every ¼ of chord, and the control points are at ¾ of chord, satisfying the Kutta's condition in the trailing edge (Figure 1).

Instead of using the classic flat horseshoe vortex, a more interesting form is used (Figure 1) in which the horseshoe vortex is composed by discrete vortex segments which, follows the surface until the trailing edge and than align to the free stream (Miranda, Elliott, Baker,1977) . This method can be modified to include a free wake and unsteady aerodynamic models.

The solution of the Vortex-Lattice model is based on a development of the Bio-Savart theorem (Katz and Plotkin, 1991). The speed in point P induced by a straight vortex line segment that goes from point A to point B, as shown in Figure 1, can be calculated through the equation (1)

$$\vec{V} = \frac{\Gamma}{4\pi}\frac{\vec{r}_1 \times \vec{r}_2}{\left|\vec{r}_1 \times \vec{r}_2\right|^2}\vec{r}_0 \cdot \left(\frac{\vec{r}_1}{\left|\vec{r}_1\right|} - \frac{\vec{r}_2}{\left|\vec{r}_2\right|}\right) \tag{1}$$

Where $\vec{V}$ denotes the speed induced by a vortex segment, Γ denotes the intensity of the vortex and $\vec{r}_0$ to $\vec{r}_2$ are the distances indicated in Figure 1.

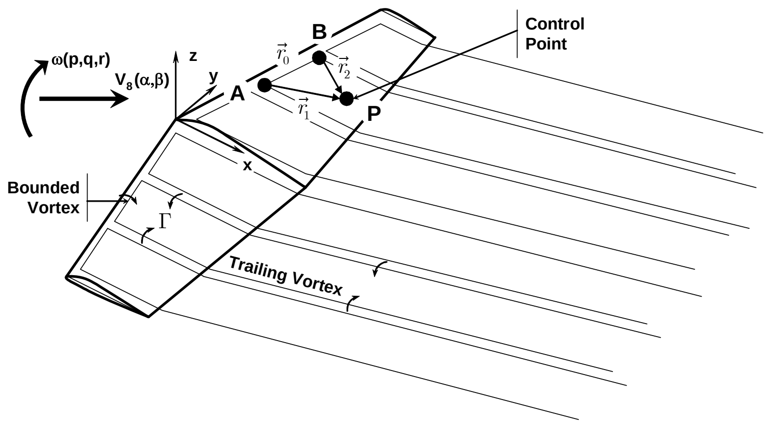


Figure 1 – Vortex Lattice Arrangement adopted in the proposed aerodynamic procedure.

The solution of the potential flow problem will be the determination of the intensities Γ of each horseshoe vortex through a system of linear equations as shown in equation (2), with the boundary condition being the condition of body impermeability.

$$\begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \\ w_{21} & w_{22} & \cdots & w_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ w_{m1} & w_{m2} & \cdots & w_{mn} \end{bmatrix} \times \begin{Bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_n \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{Bmatrix} \tag{2}$$

Where w denotes the geometric influence on normal induced speed on the panel m by the horseshoe vortex n, Γ_n denotes the intensity of the horseshoe vortex n and B denotes the speed of normal free flow at the panel surface in the control points, including the components due to maneuver (rolling, pitch and yaw).

Once the intensity of each horseshoe vortex is determined, the aerodynamic forces can be calculated according to the Kutta-Joukowsky theorem as show in Eq. (3) (Katz and Plotkin, 1991) resulting in the distribution of force, in each panel, as shown in Figure 4.

$$\vec{F} = \rho \vec{V} \times \vec{\Gamma} \tag{3}$$

Where $\vec{F}$ denotes the resulting force at ¼ of the chord, ρ the fluid density, $\vec{V}$ the resulting vector for the speed at ¼ of chord and $\vec{\Gamma}$ the bounded vortex.

3. NONLINEARITY APPROACH

Since the distribution of horseshoe vortex occurs only along the span, with only one control point along the chord disregarding the line of camber of the airfoil, the results obtained with the traditional method refer to a wing that uses a symmetric aerodynamic profile, which, according to the linear theory is equivalent to a flat plate, with a lift slope, constant equal to 2π and the null lift angle constant and equal to zero. In order to avoid this restriction and correct the three-dimensional results of flow obtained with the flat plate as a function of aerodynamic characteristics of the real airfoil, an iterative process based on the method proposed by Mukherjee and Gopalarathnam (Mukherjee and Gopalarathnam, 2003) will be used.

This iterative algorithm is capable of computing the influence of two-dimensional flow characteristics on three-dimensional flow and can be described through the following steps:

(1) Initial values of δ and ΔC_L are assumed for each section of the wing.

(2) The aerodynamic characteristics are calculated using the traditional Vortex-Lattice algorithm

(3) The effective attack angles for each section ($\alpha_{e\_\sec}$) are calculated using the local lift value obtained in step (2) using the equation (4)

$$\alpha_{e\_\sec} = \frac{CL_{\sec}}{2\pi} - \delta \qquad (4)$$

(4) The value of $\Delta C_L = C_{Lvisc} - C_{L\sec}$, is calculated, where $C_{L\sec}$ is the value obtained with the conventional Vortex-Lattice method, and C_{Lvisc} is the value obtained using the information of the airfoil and effective attack angles for each section of the span (two-dimensional polar curves).

(5) The new value of δ is calculated using equation. (5) and the new angle of attack of witch sections became the initial angle of attack plus the new value of δ , as show in equation (6) .

$$\delta = \delta + \frac{\Delta C_L}{2\pi} \qquad (5)$$

$$\alpha_{e\_\sec} = \alpha_{initial} + \delta \qquad (6)$$

(6) Return to (2), where $\alpha_{e\_\sec}$ is the new value for the attack angle of each section to be used for the calculation with the traditional method. This process is repeated until $CL_{\sec}$ converges.

When the effective attack angle for each station and the profile polar is known, beyond getting the lift coefficient, it is possible to obtain the drag parasite and aerodynamic moment coefficients for each wing section.

Mukherjee and Gopalarathnam, tested the algorithm only in individual wings in simple flight conditions without roll, pitch and yaw speeds. However, in order to use the algorithm in a complete aircraft with multiple wings and tails with complex geometries and in flight conditions with movements such as roll, pitch and yaw, it was necessary to use a damping and dissipation coefficient, which are not included in the original algorithm suggested by Mukherjee and Gopalarathnam, which brings more stability to the numeric method, transforming the equation (5) into the form shown in equation (7) and (8).

$$\delta_i = \delta_i + \frac{1}{K+1}\frac{\Delta C_L}{2\pi} \qquad (7)$$

$$\delta_i = \frac{\left(\delta_i + \Pi\frac{(\delta_{i-1} + \delta_{i+1})}{2}\right)}{1+\Pi} \qquad (8)$$

Where i denotes the section throughout the span, K is the damping factor and Π is the dissipation factor.

It is important to note that the increase in damping factor not affect the final result, increasing Just the total number of iterations need to convergence. However, the increase in dissipation factor can change the final result been important its correct choice Table 1 and Table 2, and Figure 2 shows examples of the influence of these factor in final result. These results correspond to a rectangular wing at 22° of attack angle, with aspect ratio equal to 5 using NACA-0012 airfoils.

Table 1- Damping factor influence

Damping Factor	Lift Coefficient	Iterations
0	0.98185	58
.5	0.98185	86
2	0.98185	162

Table 2- Dissipation factor influence

Dissipation factor	Lift Coefficient	Iterations
0	0.98185	58
0.01	0.98474	55
0.1	1.0034	36
0.5	1.0311	26

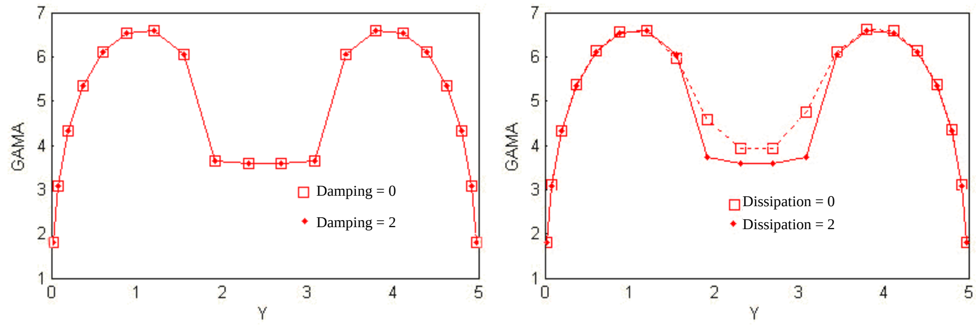


Figure 2 – Influence of damping factor (a) and dissipation factor (b) on lift distribution.

4. CALCULATION OF INDUCED DRAG

Because the difficulties associated with the calculation of the induced drag present in a lifting surface, there is many ways to estimate it, such the backward component of force calculated with the Kutta-Joukowsky theorem, a modified Lifting Line method proposed by Eppler which computes the downwash in the trailing edge, and pressure integration (Mortara, Straussfogel, Maughmer, 1992).

A more accurate way to estimate induced drag given by Kutta-Joukowsky theorem is measuring the variation of kinetic energy (*Momentum*) in a plane behind the aircraft (Giles, M. B. and Cummings R. M., 1999), because its valid for any kind of wing geometry and multiple surfaces even for a complete aircraft including the fuselage (Jie, Fengwei and Qin, 2003). The variation of *Momentum* in the perpendicular direction to free stream in the gray area of Figure 3 is the drag caused by the aircraft and can be computed with the equation (9). This technique is known as Treffts-Plane.

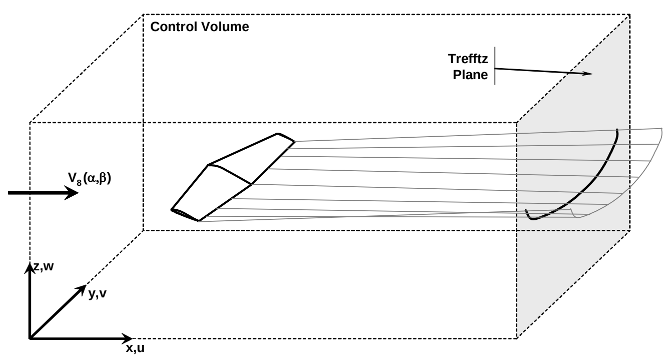


Figure 3 – Sketch of Trefftz Plane arrangement

$$D_{IND} = \frac{1}{2}\rho \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(v^2 + w^2\right) dydz \qquad (9)$$

However, a more convenient way of treating the equation (9) is obtained using the Green theorem (Katz and Plotkin, 1991) which converts an area integral into a line integral, and the induced drag equation assumes the form presented in equation (10).

$$D_{IND} = \frac{1}{2}\rho \int_{-b/2}^{b/2} \Delta\phi \cdot W_{tp} ds \tag{10}$$

Where $\Delta\phi$ denotes the difference of potential between the top and the bottom side of the wake, and W_{tp} is the normal flow speed in thought the wake.

As in the method of horseshoe vortex distribution, the difference in potential in the wake is the circulation ($\Delta\phi = \Gamma$), and the induced drag could be conveniently calculated through the equation (11).

$$D_{IND} = \frac{1}{2}\rho \sum_{i=1}^{n} \Gamma_i W_{tpi} s_i \tag{11}$$

Where D_{IND} denotes the induced drag, ρ denotes the density of the fluid, Γ the intensity of the horseshoe vortex, W_{tp} the normal induced velocity through the wake and s the width of the horseshoe vortex. All of these values is measured in points distant of the wing as shown in Figure 2. This formulation is also valid for any wake form, even for the rolled-up vortex sheet (Schlichting, H.; Truchenbrodt,1979).

5. FREE WAKE MODEL

The alignment of the horseshoe vortex with the free wake, as described previously (Figure 1), is a good approximation for the calculation of fixed wing aircraft in less severe flight conditions (no maneuvers), since the geometry of the wake has little influence in the aerodynamic results of interest (lift and drag coefficients) of the surface which generated it in steady flow.

However, when a more complex geometry of wing and maneuver conditions in which the wake passes very close to a lift surface, such as, for example, the tail in the trajectory of the wing wake, the precise calculation of the geometry of the wake can affect the result in some flight conditions and, unsteady flow (Katz and Maskew, 1987). In order to solve this problem, a free wake model can be added to the Vortex-Lattice model. This is a non linear and transient iterative process which demands greater processing time, since the wake is discretized in distinct elements and the horseshoe vortex trajectory is calculated.

The algorithm used for the calculation of the free wake is base on the ideas proposed by Katz and Maskew (Katz and Maskew,1987) and can be described as following :

(1) Entrance of the initial geometry of the horseshoe vortex where the trailing vortex extend on the surface of the body from the bonded vortex (¼ of chord) to the trailing edge only (instant t = 0 in Figure 4).

(2) The vortex-lattice method is used in order to calculate the intensity of the vortex and the velocity in the control points of the wake.

(3) With the velocities in the control points, the trajectory of a fluid particle located on these points is calculated (stream lines) using a numerical integrator, as shown in equations (12) and (13).

$$(x, y, z)_{t0+\Delta t} = (x, y, z)_{t0} + (\Delta x, \Delta y, \Delta z) \tag{12}$$

Where:

$$\left(\Delta x, \Delta y, \Delta z\right) = (u, v, w)\Delta t \tag{13}$$

(4) The trailing vortex is then extended over the calculated fluid particle trajectory (stream line).

(5) Return to (2), until the convergence criteria adopted or a pre-established number of iterations is obtained.

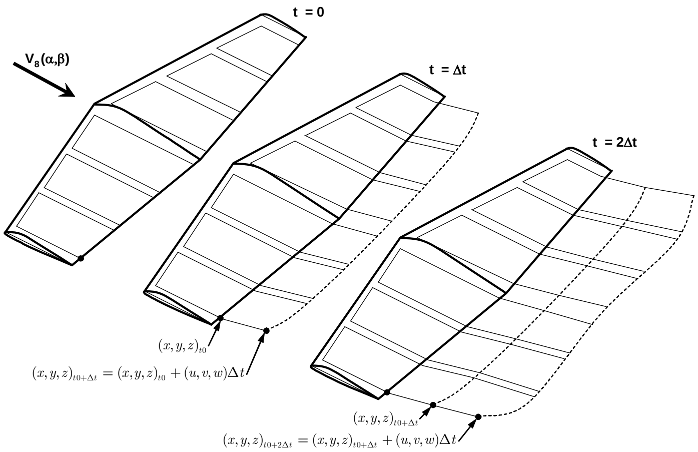


Figure 4 – Free wake integration schema

It is important to note that this is an elliptic problem, in other words, it is necessary to re-calculate the entire wake at each iteration, since the most distant vortex segment of the wing modifies the beginning of development of the wake, closest to the wing. It is also important to highlight that in step (2), the equations (12) and (13) refer to a first order integrator (Euler). It is advisable to use a numeric integrator of superior order, such as the Runge-Kutta of second or fourth order, which improves the precision of results.

6. COMPUTATIONAL IMPLEMENTATION

The presented procedure was implemented in a MATLAB environment using Windows Operational System. As a procedure based on linear equations solution its implementation in MATLAB takes the advantage on:

- Very efficient linear system solver available in MATLAB
- Very robust and easy implementation of arrays and structures under MATLAB

Another important aspect of this implementation is a graphical user interface (GUI) developed in order to access all functionalities implement. This GUI had been used in CEA-UFMG in the development of several different kind of projects with full success (Figure 5). A free version of this GUI and solver is available in www.demec.ufmg.br/cea/ for academic and non-commercial use. More information about this code, and authorization for commercial use, is available in contact with authors.

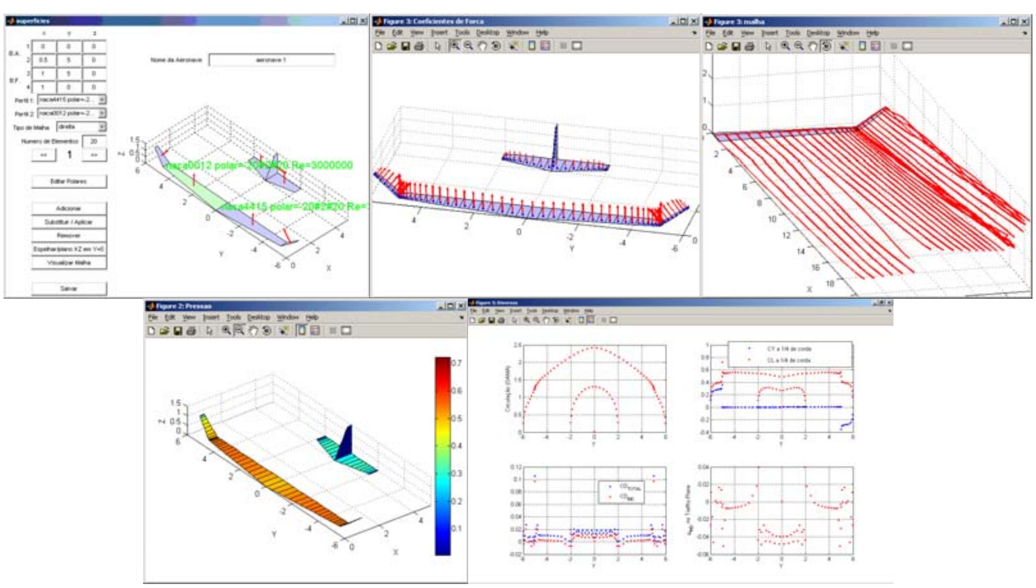
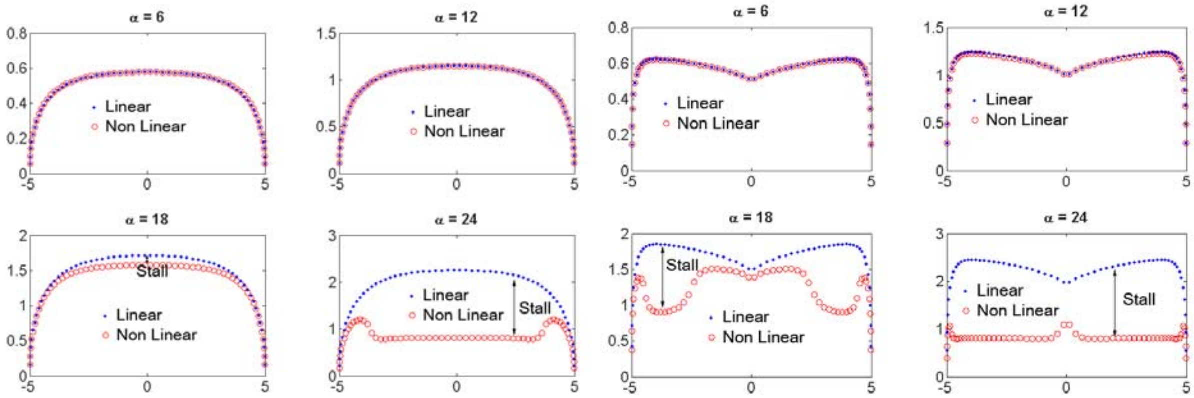


Figure 5 – Examples of GUI implement with the presented procedure

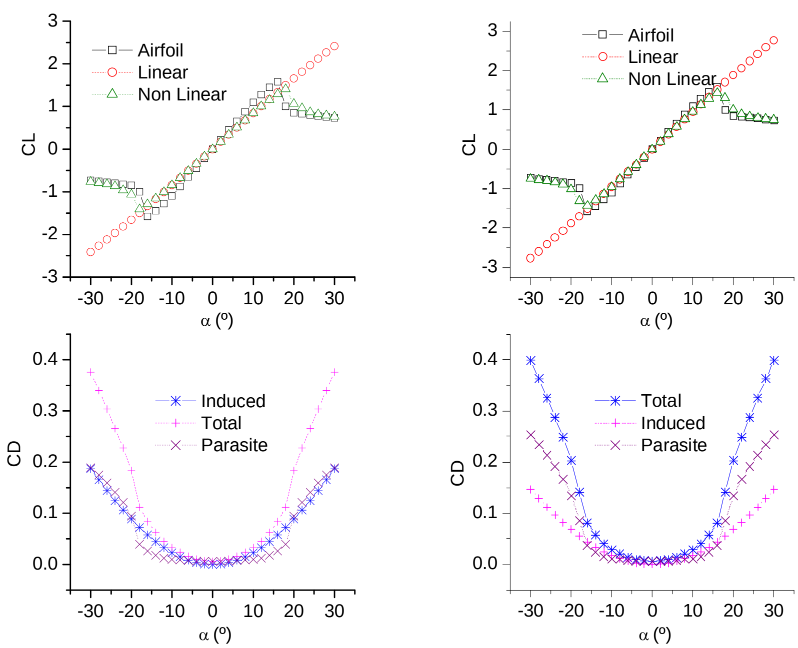
7. RESULTS

Two configurations frequently used in aircraft wings are rectangular and tapered plan forms, so these has being chosen to demonstrate the applicability of the proposed method , taking into consideration that both wings are constituted by the aerodynamic airfoil NACA 0012 (symmetrical). The results of the distribution of the lift coefficient for both wings in diverse attack angles can be seen in Figure 5, where one can note the occurrence of stall in elevated attack angles. Such a prediction is possible thanks to the use of the two-dimensional profile information, while, according to the traditional method, the wing would continue to afford lift. Figure 6 shows the integral aerodynamics characteristics of these wings, including lift, parasite drag and induced drag.



(a) (b)

Figure 5 – Lift distributions for different attack angles for a rectangular wing (a) and a tapered wing (b)



(a) (b)

Figure 6 – Aerodynamics characteristics for a rectangular wing (a) and a tapered wing (b)

A comparison with experimental data can be seen in Figure 7, in which the results obtained numerically for rectangular wings with different aspect ratios are compared to experimental results presented by Mukherjee and Gopalarathnam (2006). As can be seen the present method presents good results, compatible with the experimental results even for the post-stall region of the lift curve.

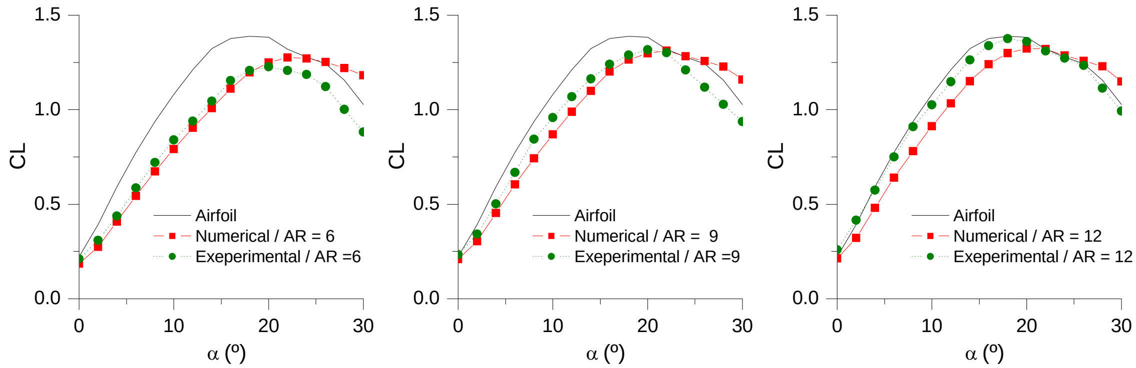


Figure 7 – Lift curve predicted with proposed method for different aspect ratios compared with experimental results (Mukherjee and Gopalarathnam, 2006).

Finally, a typical result using free wake and Treftz Plane induced drag is presented at Figure 8. In this case a rectangular wing with ailerons deflected is calculated. Is possible to see the effects of ailerons deflection at wake and lift (circulation) distribution. Must also be noted the distribution of induced drag calculated at Treftz Plane, which is non trivial due the complex lift distribution on aileron region.

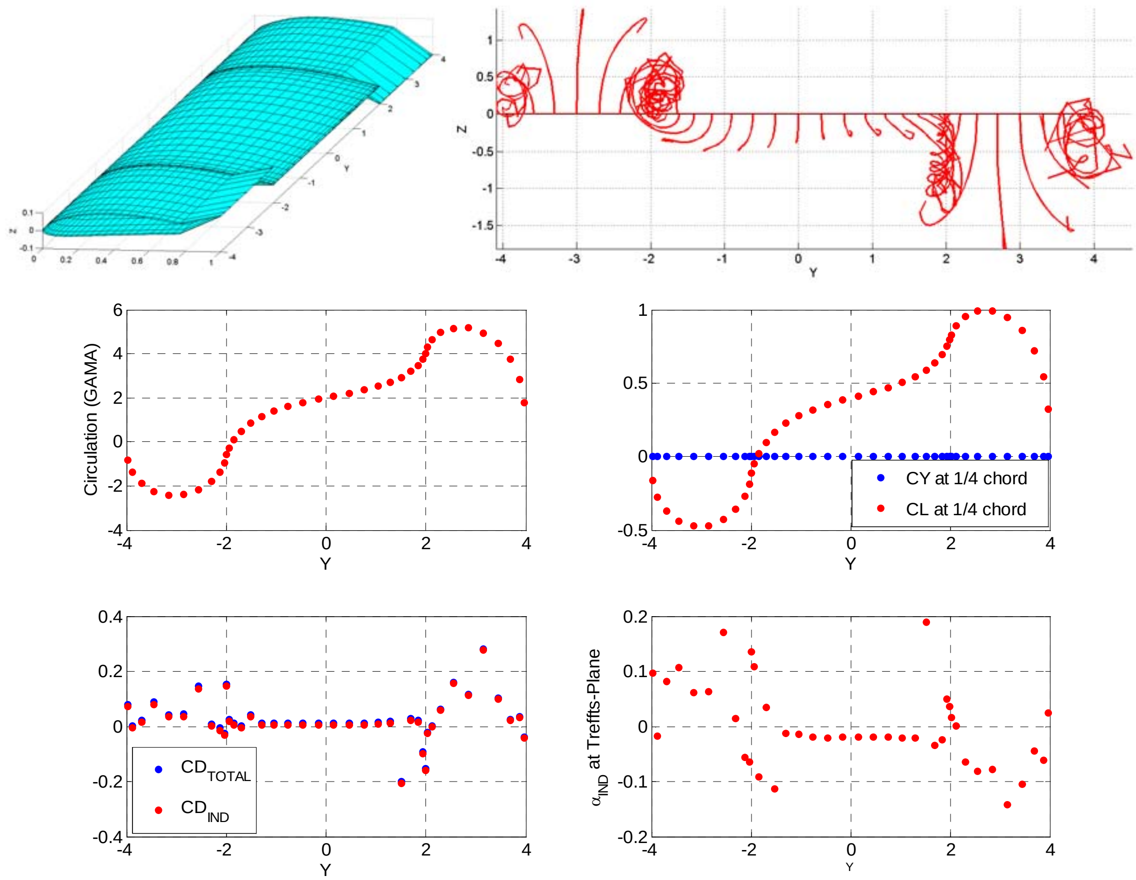


Figure 8 – Example of a free wake model of a rectangular wing using deflected ailerons

8. FINAL COMMENTS

Several case were studied at Centro de Estudos Aeronáuticos (CEA-UFMG) using this code. These studies include the design of new airplanes, optimization of lifting geometries, and stability derivates calculation for UAV's. In all cases the results obtained had been shown satisfactory and agreeing with available experimental results. Take in mind these positive experiences, the authors believe that the present method is reliable for the aeronautic design processing because of the follow benefic characteristics:

- Use of available two-dimensional information of the aerodynamic profile, obtained experimentally or calculated numerically, thus being able to calculate the non linear region of the lift curve (stall), include parasite drag effects and aerodynamic moment.
- Make it possible to use complex geometries with infinite planar and non-planar surfaces, including geometric torsion, dihedral and sweep.
- Allows aerodynamic torsion with different airfoil at root and tip of surfaces.
- Any kind of mesh distribution can be used, with four types available at this implementation.
- Calculation of forces, moments and their respective coefficients in three different coordinate systems (body axis, stability axis and wind axis).
- Aligned or not Aligned flat fix wake
- Additional model of free wake
- Calculation of induced drag using the Trefftz-Plane model
- Allows for diverse flight conditions including the incidence angle known as β , and maneuvers (roll, pitch and yaw).
- Great processing speed, making use of approximately 2 seconds for a complete aircraft analysis composed of 100 panels, including processing and graphic result (obtained in a PC, 1,4 GHz, 512 MB of RAM).
- Can incorporate a panel method for the fuselage

9. CONCLUSIONS

A computational aerodynamic procedure was presented in this paper. This procedure changes the modern lifting line procedure in order to account the non linear characteristics of airfoils that compose the wing. Additionally a free wake model and a Treftz Plane procedure to calculate induced drag were shown. Results of typical application of this procedure were shown including stall prediction, experimental results comparison and free wake and induce drag calculation of a complex geometry. Due to the successful application of this procedure in other design and analyses cases, the authors believe that the methodology described in this paper is of great value for the design of complete aircraft, meeting the needs for initial and less detailed phases, as well as phases which call for greater detail for the aircraft. It is also believed that this methodology can be extended to other areas of aerodynamics such as the design of blades and propellers due to the free wake model. It can also be used in conjunction with a flight simulator, since, due to its great processing speed, for a reduced number of panels (in flight simulators, precision can be smaller) the processing is almost immediate, substituting pre-established aerodynamic characteristics with values calculated in real time. For more real results for a complete aircraft a fuselage model with panel method must be done.

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