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The effects of the number Mach and vorticity dynamic in asymmetric wake

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Abstract. *In an aircraft, high-lift devices operating at a high angle of attack promote the formation of large wakes. Such wakes influence the aerodynamic performance. The slat wake, for example, affects the transition point of the wing is main element and is the most important source of noise in the airframe. Owing to the generation of lift these wakes are asymmetric. There are aspects of such flows that require investigation. A simplified model has been proposed to investigate the influence of the Mach number and vorticity dynamics on a two-dimensional asymmetric wake profile under temporal development. The asymmetric wake profile was obtained by a combination between a Gaussian wake profile and a hyperbolic tangent mixing layer profile. Direct numerical simulation of compressible Navier-Stokes equations in characteristic formulation and non-conservative were performed and compared with the two-dimensional linear stability theory analysis of the profile. The results showed that the increase of Mach number reduced the maximum rate amplification and the unstable band. Moreover, the results indicate a deformation in the vortex wake with increasing asymmetry.*

Keywords: *asymmetric wake, Mach number, temporal amplification rate, vorticity dynamic*

1. INTRODUCTION

The study of bluff body wake flows has been a subject of interest to engineers and scientists for many years as they have direct engineering applications. In flow past a bluff body there is a critical value to free stream velocity. When it is exceeded, the vortex shedding occurs due to the flow instability in the near wake. The alternate shedding of vortices may cause, among other things, structural vibrations and acoustic noise. Therefore, this study presents many difficulties. Bluff body wakes are complex, as they involve the interaction of various shear layers in the same problem, namely, a boundary layer, a separating mixing layer and a wake, (Williamson, 1996). Because of this complexity, a flow past circular cylinder has been investigated. The cylinder geometry has less complexity in relation to other bluff body and is representative of the phenomenon. Also, a cylindrical structures are found in several engineering applications such as risers, transmission cabled and landing gears and etc, which shows be a subject of interest to several researchers.

Several researchers investigated the stability of a two-dimensional wake behind a circular cylinder, "Sato and Kuriti, 1961", "Betchov and Criminale, 1966", "Hultgren and Aggarwal, 1987", etc. They studied the instability of the symmetric wake flow using a velocity profiles, a simplified model to two-dimensional wake. In these studies has been done a linear analysis of the cylinder wake.

In an aircraft, high-lift devices operating at a high angle of attack promote the formation of relatively large wakes. Such wakes influence the aerodynamic performance. The slat wake, for example, affects the transition point of the wing main element and is the most important source of noise in the airframe (Gennaro and Medeiros, 2007). Owing to the generation of lift these wakes are asymmetric. Besides, bodies immersed in a shear flow also produce asymmetric wakes, with important applications to the oil industry.

Quite a few studies have been performed on uniform shear flow over a cylinder, along with the adopted flow conditions in Table 1. Theys have mainly investigated the effects of Reynolds number and shear rate on the vortex-shedding frequency, the magnitude of the mean drag, and so on in the uniform shear flow. Despite many achievements to date, some controversial issues have to be further resolved for improved undertanding of the bluff body flows (Kang, 2006). A few disrepancies among studies on the uniform shear flow remain unresolved yet (Lei et al., 2000 and Sumner and Akosile, 2003). It is implied that more investigations on the uniform shear flow are necessary, which motivates the present study. Representative controversial issues among previous studies are enumerated in the Table 1.

"Kiya et al., 1980, Kwon et al. 1992, Tamura et al. 1980 and Yoshino and Hayashi, 1984" claimed that the Strouhal number $St = fD/U_c$ increased with increasing shear rate. On the other hand, "Vitola, 2006, Kang, 2006, Sumner, and Akosile, 2003 and Lei et al. 2000" claimed that the Strouhal number decreased with increasing shear rate. In this present study, this shear rate represents the asymmetry of the wake.

"Recently, Gennaro and Medeiros (2008)" investigated the asymmetry effects in a wake profile. About this controversy, the results also shed some light into the apparent discrepancy, in particular regarding the variation of the Strouhal number. The discrepant results were to be corrected for the most appropriate reference scales, they showed a clear tendency for reduction of the Strouhal number for increasing asymmetry. Those results may strongly suggest that the controversy can be given as resolved (Gennaro, 2008).

The aim of the present work is to investigate the influence Mach number and dynamics of vorticity on a two-dimensional asymmetric wake profile under temporal development.

Table 1. Flow conditions used in previous studies.

References	**Re**	**K**	**St**
"Vitola, 2006"	1800	$0 - 0,2$	decrease
"Kang, 2006"	$50 - 160$	$0 - 0,2$	decrease
"Sumner and Akosile, 2003"	$4.0 \times 10^4 - 9.0 \times 10^4$	$0,02 - 0,07$	decrease
"Lei, Cheng, and Kavanagh, 2000"	$80 - 1000$	$0 - 0,25$	decrease
"Kwon, Sung, and Hyun, 1992"	$600 - 1600$	$0 - 0,25$	increase
"Tamura, Kiya and Arie, 1980"	40 and 80	$0 - 0,4$	increase
"Kiya, Tamura and Arie, 1980"	$35 - 1500$	$0 - 0,25$	increase
"Yoshino and Hayashi, 1984"	80	$0 - 0,4$	increase

2. STATEMENT OF PROBLEM

A numerical simulation to the presence of the body leads to a large computational cost, since this cost is related to the size of the computational dominium. The initial condition generally produces a lot of noise could even cause numerical instability, this contradicts the temporal amplification rate of infinitesimal disturbance.

Linear stability properties of a flow can be studied by investigating its impulse response, generally, finding the time and space evolution of an impulse disturbance. A disturbance ideally contains modes of all frequency and wavenumbers from which the flow amplifies the modes or band of modes, that are unstable. However, the aim this work is to investigate the linear regime considering the effects of the Mach number Ma, thus, the Linear Stability Theory consists of investigating the amplification rate of the first unstable mode. The stability analysis provides a dispersion relation that determines the sign of ω_i, namely, an imaginary part of frequency ω, for a given wavenumber α. For $\omega_i < 0$ the wave grows exponentially in time and is termed a temporally unstable disturbance.

In order to investigate the effects of asymmetry in the wake, a simplified model has been proposed, namely, a parallel and periodic far wake, details can be found in "(Gennaro, 2008)". Thus, it has been to possible to perform a temporal analysis considering the periodic flow in streamwise direction. Besides, the flow is uniform in $y \to \pm\infty$, where y is the normal direction of the flow, unlike the real case where the flow is shear in this region. The asymmetric wake profile was obtained by a combination between a Gaussian wake profile and a hyperbolic tangent mixing layer profile.

In numerical simulations performed this work, the hypothesis of parallel flow was obtained through the cancellation, for the base flow, of the viscous effects in normal direction of flow. For the base flow has been used the Gaussian velocity profile given by

$$U(y) = 1 - \exp(-y^2 ln2) + K \tanh(2y), \tag{1}$$

where K is the asymmetry parameter nondimensional that controls the amount of asymmetry of the wake.

The local half-widths of the wake b have been used as reference scale. This reference length scale of a wake profile is the local half-width b and is defined implicitly

$$U(b) = (U_{max} + U_{min})/2. \tag{2}$$

where U_{max} and U_{min} are maximum and minimum velocity.

For the current asymmetric case, the length scale was the average b, between both sides of the profile. The reference velocity was the maximum velocity. Considering only the velocity profile is possible reproduce numerically the parallel hypotheses by canceling the viscous diffusion at the base flow in the y-direction.

3. METHODOLOGY

As usual, the normal mode assumption. It is assumed that the physical perturbation velocity is given by

$$\tilde{u}(x, y) = \hat{u}(y) \exp[i(\alpha x - \omega t)], \tag{3}$$

with an analogous assumption for spanwise velocity and the perturbation pressure, where $\hat{u}$ is autofunction, α and ω are wavenumber and frequency, respectively. Substitution of (3) into the Navier-Stokes equation and continuity, linearization and application of the parallel-flow assumption then leads to the Orr-Sommerfeld equation for the y-direction velocity component

$$\alpha(U - c)(\hat{v}'' - \alpha^2 \hat{v}) - U'' \alpha \hat{v} = \frac{-i}{Re} \left(\hat{v}'''' - 2\alpha^2 \hat{v}'' + \alpha^4 \hat{v} \right) \tag{4}$$

where $c = \omega/\alpha$, $\hat{v}$ is an autofunction of y-direction and the primes denote the derivates with respect to y.

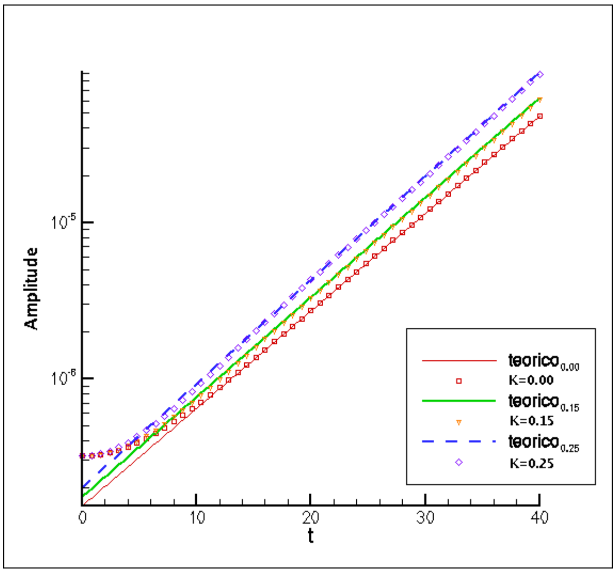

Figure 1. Comparison between numerical and theoretical results. Temporal evolution of the fundamental mode amplitude to several values of the asymmetry parameter K, $Re = 1000$ and $\alpha = 0,60$.

The eigenvalues of the problem defined by "Eq. 4" were determined numerically by using a shooting method. The starting values for the numerical procedure were obtained from the asymptotic solutions of "Eq. 4". More details these numerical procedure can be seen in "Gennaro 2008". The fourth order Runge-Kutta method was the used to numerically integrate the Orr-Sommerfeld equation. Orthogonalization of the solutions used Gram-Schmidt process was performed whenever deemed necessary to preserve their linear independence.

The study also was carried out by solving the compressible two-dimensional Navier-Stokes equations in nonconservative form, written in a characteristic formulation (Sesterhenn, 2001). The code used 6th order compact finite difference scheme for the spatial derivatives and a 4th order Runge-Kutta scheme for the time integration. A filtering scheme was also employed to keep the simulation without aliasing problems (Germanos and Medeiros, 2005). In normal-to-the-flow y-direction a free-slip non-reflecting boundary conditions was used. For both the theoretical and numerical analysis the flow was considered periodic in streamwise x-direction. This numerical code was subject to tests according with the linear stability theory from results of the "Eq. 4".

The "Fig. 1" shows the temporal development of the fundamental mode for several values of the asymmetric parameter K. There is a good agreement of the results obtained from the solution of the Orr-Sommerfeld equation with the results of direct numerical simulations of the Navier-Stokes equations.

4. RESULTS

The results about the influence of the asymmetry that promotes in the wake showed a reduction of both the amplification rates and unstable range of wavenumbers for the asymmetry parameter small. Therefore, the asymmetric wake was shown to be less unstable. Besides, the asymmetry parameter promotes a reduction of the frequency real part ω_r, consequently, a reduction in the Strouhal number, which is defined by $St = fD/U$. As the Strouhal number is proportional to the frequency, the reduction ω_r implies in the reduction of Strouhal number, results that can be seen in "Gennaro (2008)".

After verifying the numerical method, we conducted numerical simulations by varying the asymmetric parameter of $0 \leq K \leq 0,25$ and Mach number $0,1 \leq Ma \leq 0,6$ with Reynolds number $Re = 1000$.

4.1 Mach

The effect of the Mach number in asymmetric wake was investigated only numerically because the theory developed in this work was for incompressible flow.

As noted in other flows, the Mach number has a stabilizing effect. The expectation for an asymmetric wake was that the number also cause a stabilizing effect on this flow, this is confirmed by noting the "Fig. 2". In figures, the increased of the Mach number promotes a reduction in ω_i, the temporal amplification rate, indicative of the stabilizing effect of the Mach number. It is clear that the benchmarks used were the symmetric case in "Fig. 2" by the solid line.

There is still that the Mach number reduces the unstable band, especially the upper limit and the maximum rate of amplification, reducing the wavenumber the most amplifies. The simulations were performed for limited wavenumber, namely, for $\alpha = 0,10\ 0,30\ 0,60\ 0,90\ 1,20$ and $\alpha = 1,40$, in view of the high cost computational of the direct numerical simulations (DNS). Thus, we could not get exactly the wavenumber the most amplifies. Therefore, the simulations performed throw some light on their behavior, it decreased with increased of the Mach number. The increased of the Mach number also promoted a reduction in the unstable band and the maximum rate of amplification, therefore, the reduction in the band is small. Besides, the results presented in the "Fig. 3" show a more stabilizing effect to large wavenumbers.

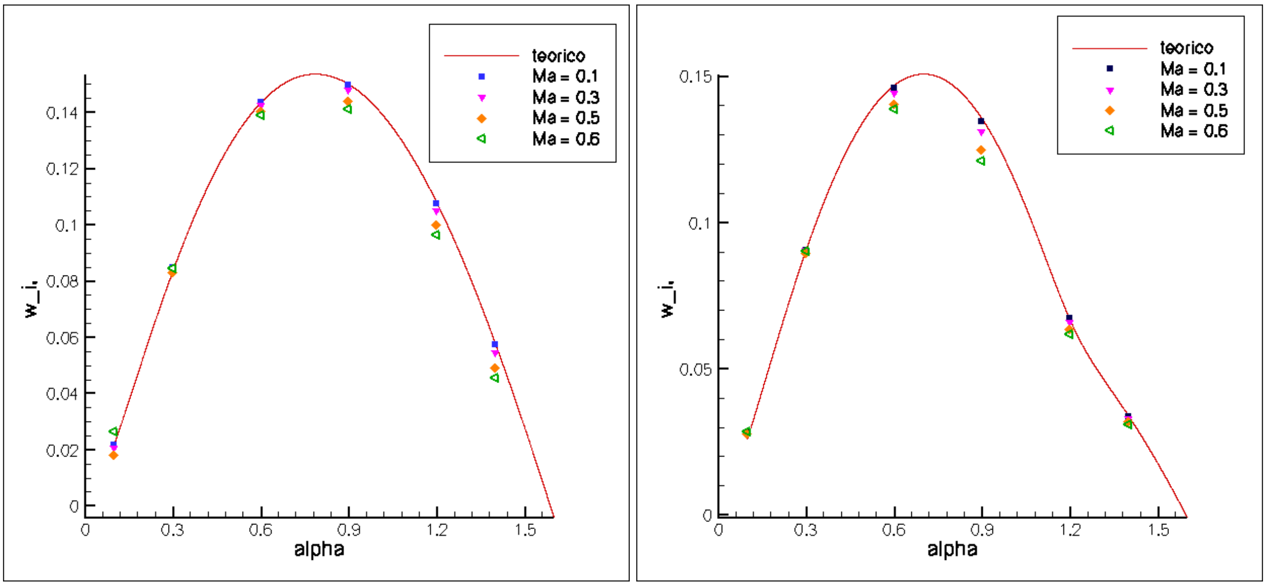

Figure 2. Effect of the Mach number (Ma) in temporal amplification rate for $Re = 1000$, $K = 0$ and $K = 0,15$, respectively. In the figure, the line refers to symmetric case that was obtained solving the equation of Orr-Sommerfeld.

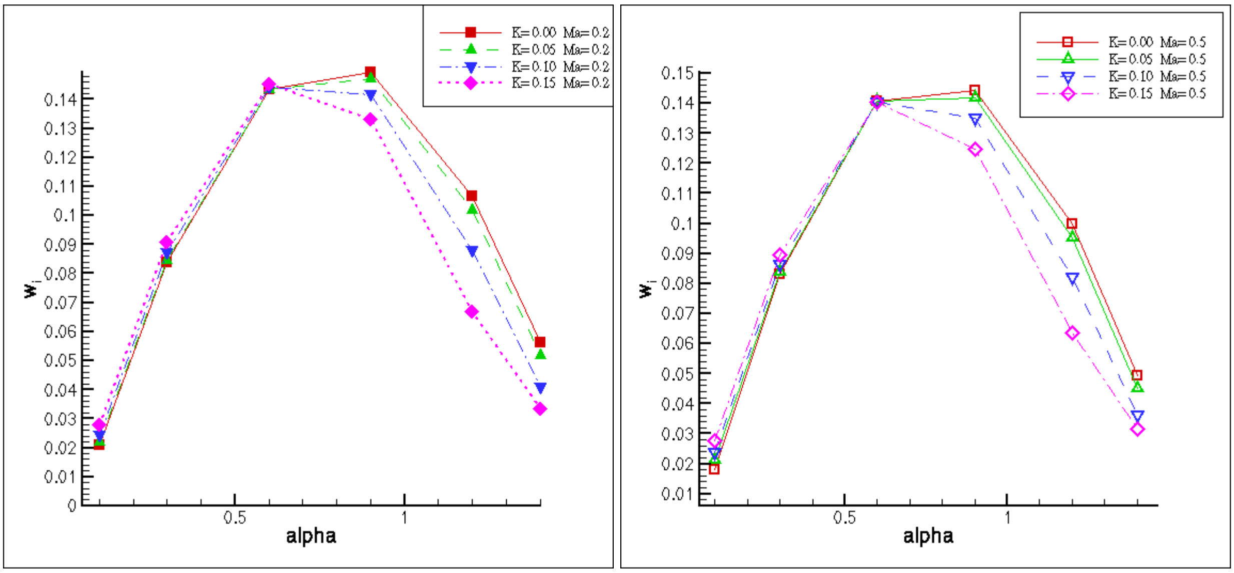

Figure 3. Temporal amplification rate versus wavenumber α for several values of K and $Re = 1000$. The first figure refers the $Ma = 0,20$ and the second the $Ma = 0,50$.

4.2 Vorticity dynamic

A study of the asymmetry effects in vorticity dynamic has been performed. The aspect of vortices dynamics is an important issue to search for aeroacoustics. In this subsection, we present some preliminary research results on this topic still in progress.

"Figures 4 to 6" show the vorticity field for $K = 0,0$, $0,10$ and $0,20$, $Re = 1000$ and wavenumber of maximum amplification. Their wavenumbers were obtained solving the Orr-Sommerfeld equation by a shooting method to incompressible flow. The vorticity field was obtained by direct numerical simulations of the compressible Navier-Stokes equations for Mach number $Ma = 0,1$.

The figures indicated a deformation in the wake with increased of the asymmetry parameter. There is still a stretch of vortices and a change in the position of vortices increasing asymmetry with increasing the asymmetry parameter arising in the vortex of deformation caused by the increase in smaller measure of asymmetry.

These changes in vortex dynamic of the wake may have implications for the generation of noise. This implications not were investigated.

5. FINAL REMARKS

In the present paper, results of numerical and theoretical calculations described which were performed for an asymmetric wake profile. The Mach number range $0 \sim 0,6$ was investigated for Reynolds number 1000 and the asymmetry parameter K was in the range of $0 \sim 0,20$.

The main results can be summarized as follows:

As expected, a von Kàrman vortex street similar to that behind a cylinder was observed even thought no cylinder was actually simulated. In fact, only a base flow of a wake profile was simulated.

The results showed the increased of the Mach number promotes a reduction in temporal amplification rate, besides, it reduces the unstable band.

Moreover, the results indicate a deformation in the vortex wake increasing asymmetry.

6. ACKNOWLEDGMENTS

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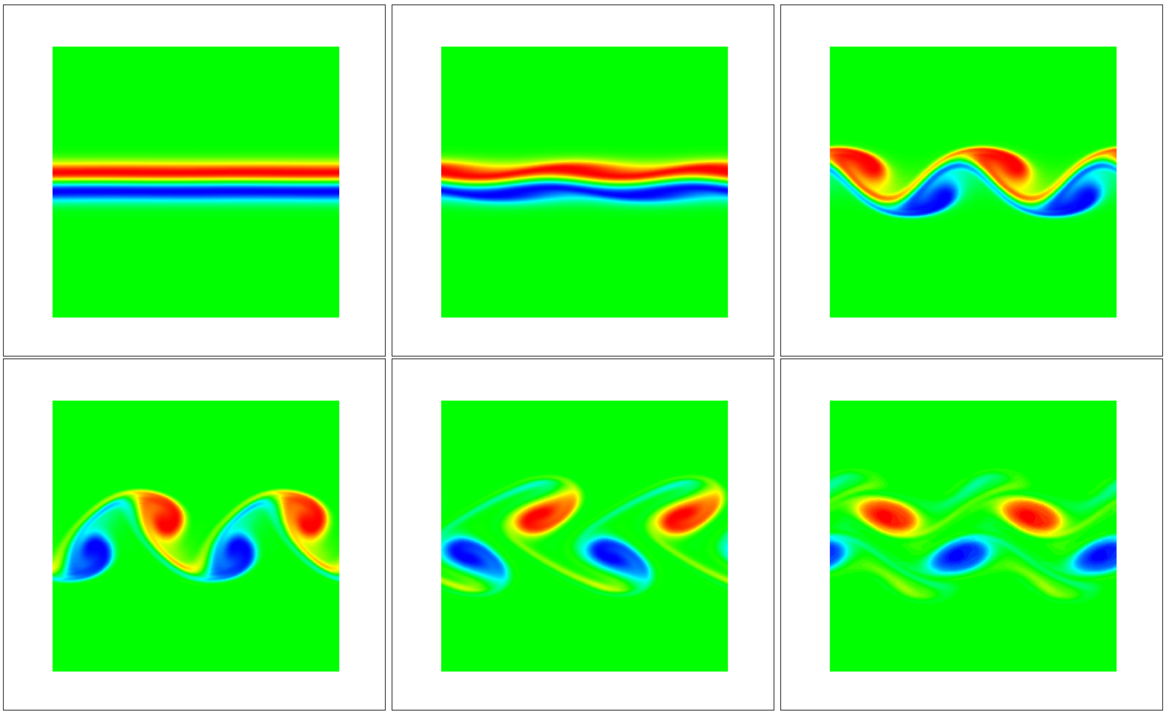

Figure 4. Vorticity contour for an asymmetric wake with $K = 0,00$, $Re = 1000$ and $\alpha = 0,7865$. The frames sequence correspond to adimensionais times $t = 90, 110, 125, 131, 141$ and 160.

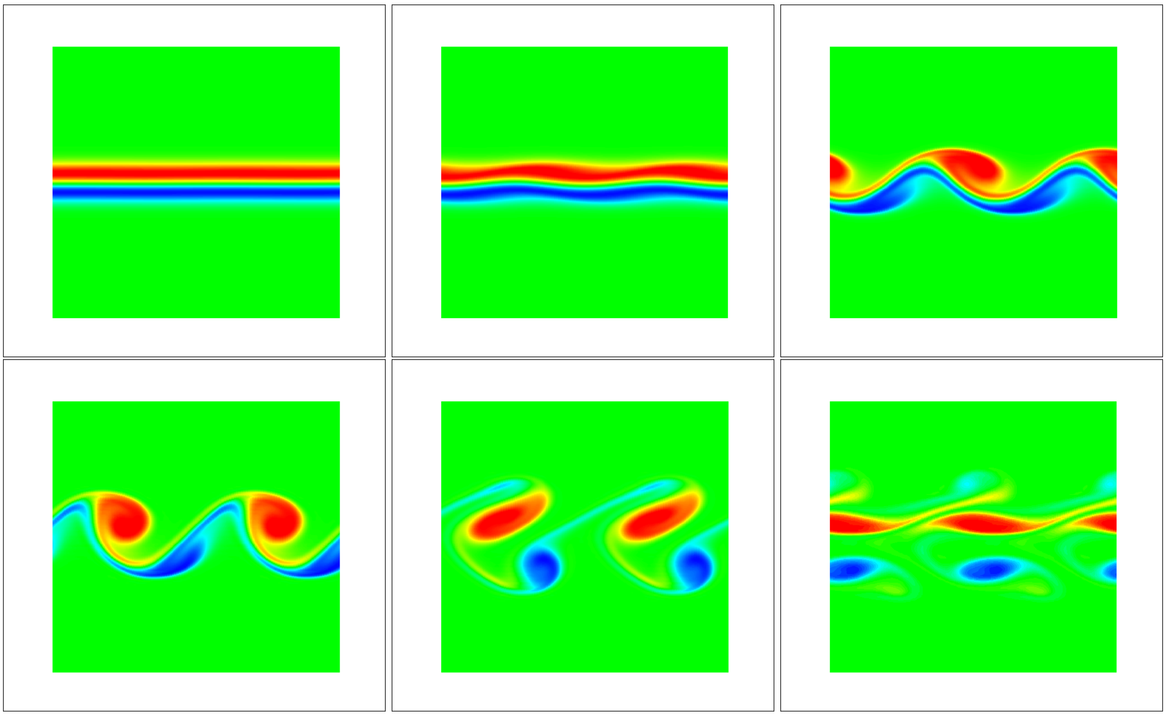

Figure 5. Vorticity contour for an asymmetric wake with $K = 0,10$, $Re = 1000$ and $\alpha = 0,7391$. The frames sequence correspond to adimensionais times $t = 90, 110, 125, 131, 141$ and 160.

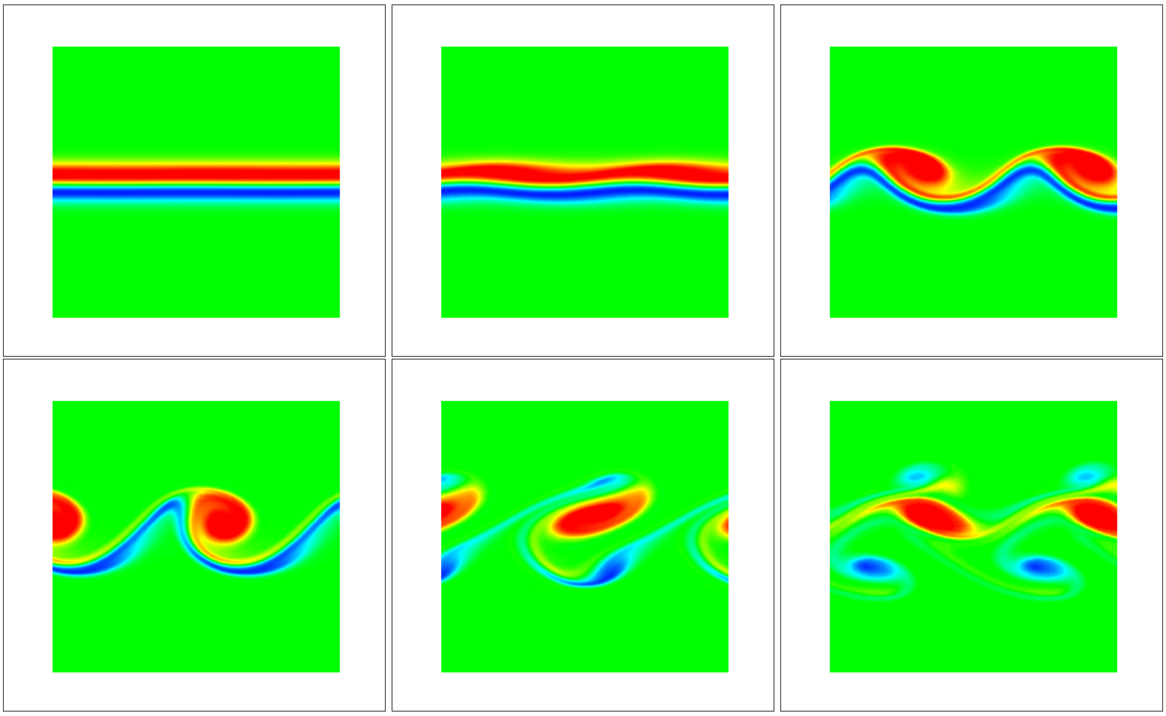

Figure 6. Vorticity contour for an asymmetric wake with $K = 0.20$, $Re = 1000$ and $\alpha = 0,6659$. The frames sequence correspond to adimensionais times $t = 90, 110, 125, 131, 141$ and 160.