STUDY OF KICK CONTROL THROUGH COMPUTATIONAL MODELING CONSIDERING THE EXPANSIBILITY OF THE WELL WALL AND OF THE FLUID COMPRESSIBILITY

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Abstract. *In this work, kick control in an oilwell is simulated considering that the fluids are compressible and the well walls are expandable. This paper aims to add these effects and solve the flow by means of the conservation of mass and momentum transient equations. The development of the governing equations leads to a pair of partial differential equations, which are solved by the method of characteristics and finite difference. The model incorporates the elastic characteristics of the system. A kick of oil is simulated for different properties of viscosity and compressibility, both for the oil and for the drilling fluid. The compressibility is beneficial in reducing the pressur surgese, but not significantly. The viscosity has a more significant impact on the dynamic pressure. There is a residual pressure, which remains even after stopping the movement of the kick, because of the compressibility of the fluid and of the expansibility of the well walls. Oscillating phenomena arise, depending on the viscosity of drilling fluid. Thus the pressures and speeds during the movement of a kick can always be calculated considering the transient effects that exist in these situations.*

Keywords*: kick, compressibility, expansibility, well*

1. INTRODUCTION

When a well is being drilled and the bottom hole pressure inside the wellbore happens to be lower than the formation pore pressure in a given moment, an influx of formation fluid may enter the well. The influx constitutes a problem and must be circulated out of the well before continuing the operations. This influx is called a kick (Santos, 2005). When a kick is detected, precautions should be taken so that the volume of the formation fluid entering the wellbore is the lowest possible, in order not to diminish excessively the bottom hole hydrostatic pressure and not to favor losing control, causing a blowout. Thus, once detected, the BOP shall be closed immediately and the fluid circulated out of the well by fluid circulating and displacement in a reduced flow rate. A good study of kick control should consider a model of two-phase flow, which should define what flow pattern is under simulation. There are basically four flow patterns of upward vertical flow (Taitel, 1980), namely, bubble flow, slug flow, churn flow and annular flow. The kick may be considered as a large isolated bubble or mixed up with the drilling fluid.

2. DESCRIPTION OF THE PROBLEM

The objective here is to develop models for the calculation of pressure and velocity in the displacement of an oil kick, rather than a gas kick, considering the compressibilities of the fluids and the expansibilities of the walls of the drill pipe, of the annulus between the open well and the drill pipe and between the casing and the drill pipe, through which the kick is circulated. It is not considered the temperature change in developing the model, so that a hypothesis of isothermal flow is adopted. Figure (1) shows the well schematic adopted for the development of the equations. To calculate the expansibilities in different parts of the well, the model should adopt a schematic that is closer to the geometry of the problem and a set of tube passages that fits the description of the well, as shown in Fig. (1a). Some basic concepts of the theory of elasticity allow the forecast of the behavior of the annular and the drill pipe cross-sectional areas under pressure. An axi-symmetrical distribution of tension is considered.

The direction to the longitudinal axis of the well will be x. The flow is parallel along the tube axis. The dependent variables are the pressure $p(x, t)$ and average speed $v(x, t)$ in a cross section. The independent variables are the distance, measured along the pipe, x, and the time, t.

3. MATHEMATICAL MODEL

3.1. Conservation of the Moment

To implement Newton's second law of motion, the control volume of Fig.(1b) of length δx is considered. According to Fig.(1b), it follows:

$$(pA) - (pA) - \left(\frac{\partial (pA)}{\partial x}\delta x\right) + \left(p\frac{\partial A}{\partial x}\delta x\right) + \rho A g \delta x - \left(\frac{dF_{fric}}{dx}\delta x\right) = \delta m a \qquad (1)$$

Developing the derivatives and the frictional force:

$$\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{dv}{dt}-g+\frac{1}{\rho}\frac{dP_{fric}}{dx}=0 \tag{2}$$

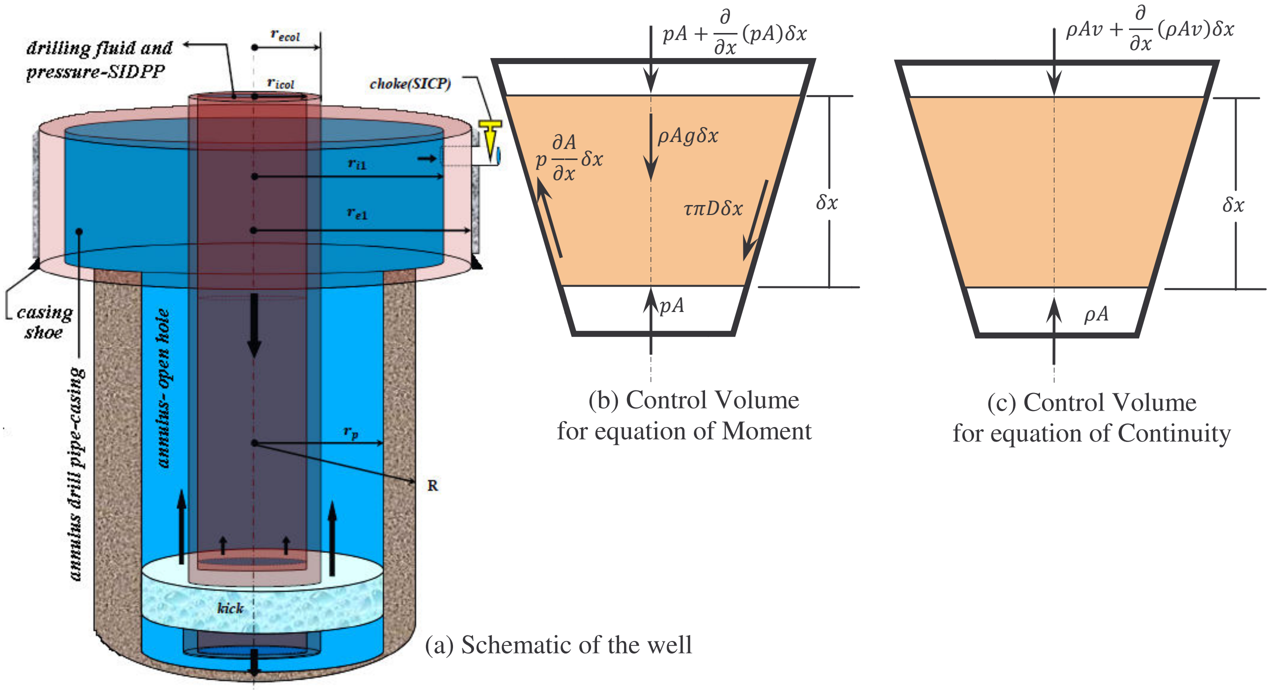


Figure 1. Schematic of the well and control volume for equations of motion and continuity.

Using a relationship for the total derivative, represented by Eq. (3), the Eq. (4) is obtained. This is the governing equation of the fluid motion.

$$\frac{dv}{dt}=v\frac{\partial v}{\partial x}+\frac{\partial v}{\partial t} \tag{3}$$

$$\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}-g+\frac{1}{\rho}\frac{dP_{fric}}{dx}=0 \tag{4}$$

3.2. Continuity Equation

The equation of continuity for a transient regime is applied to the control volume of Fig.(1c) .

$$(\rho Av\delta t)_x-(\rho Av\delta t)_{x+\delta x}=(\rho A\delta x)_{t+\delta t}-(\rho A\delta x)_t \tag{5}$$

Dividing Eq.(5) by $\delta x\delta t$, taking the limit $\delta t_{\to 0}$ and using the relationships of Eq.(6), the Eq.(7) of continuity is arrived at:

$$\frac{1}{\rho}\frac{d\rho}{dt}=\frac{1}{\rho}\frac{\partial\rho}{\partial t}+\frac{v}{\rho}\frac{\partial\rho}{\partial x}\ ;\ \frac{1}{A}\frac{dA}{dt}=\frac{1}{A}\frac{\partial A}{\partial t}+\frac{v}{A}\frac{\partial A}{\partial x} \tag{6}$$

$$\frac{1}{A}\frac{dA}{dt}+\frac{1}{\rho}\frac{d\rho}{dt}+\frac{\partial v}{\partial x}=0 \tag{7}$$

3.3. Compressibility of Fluid and Expansibility of the Well Walls

The introduction of the concept of fluid compressibility is one of the steps that, together with the expansibility of the well walls, will allow calculating the speed of the sound, making Eq. (7) to depend on the pressure. From the definition of compressibility, *C*, and dividing by *dt*, it follows:

$$C=\frac{1}{\rho}\frac{d\rho}{dp}\Rightarrow\frac{d\rho}{dt}=C\rho\frac{dp}{dt} \tag{8}$$

Eq. (8) represents the change of the density of fluid as a function of the change of pressure. In the case of the wall expansibility (α) it is necessary to take into account the characteristics of the formation, drilling pipe and coating. The expansibility is defined as:

$$\alpha = \frac{1}{A}\frac{dA}{dp} \Rightarrow \frac{1}{A}\frac{dA}{dt} = \alpha\frac{dp}{dt} \quad (9)$$

It is important to notice that each section of the well have a different expansibility (α). It is necessary to extend the concept of expansibility to the geometry of annulus space and drill pipe, taking into account the mechanical properties of the material. Thus, for the drill pipe:

$$\frac{1}{A_{ecol}}\frac{dA_{ecol}}{dt} = \alpha_{col}\frac{dp}{dt} \quad (10)$$

where A_{ecol} is the outside area of the drill pipe. For the annulus in the open hole the reasoning is similar to the above discussion,

$$\frac{1}{A_p}\frac{dA_p}{dt} = \alpha_p\frac{dp}{dt} \quad (11)$$

where A_p is the open hole area and α_p, its expansibility. Thus, for this annulus between drill pipe and open hole, where $A_{an} = A_p - A_{ecol}$, it is obtained:

$$\frac{1}{A_{an}}\frac{dA_{an}}{dt} = \alpha_{an}\frac{dp}{dt} \quad (12)$$

Combining adequately the expansibilities of the drill pipe and of the open hole, it is possible to define the expansibility of the annular space (α_{an}). Working out Eq. (12), the expansibility will depend on the area of the open hole and the external area of the drill pipe.

$$\frac{dA_{an}}{dt} = \frac{dA_p}{dt} - \frac{dA_{ecol}}{dt} \quad (13)$$

While α_{an} is constant and after combining Eq. (10), Eq. (11) and Eq. (13), the expansibility of the annular space can be written as a function of expansibilities of the open hole and of the drill pipe. For the annular space between the drill pipe and the first coating (α_{an1}) the procedures are the same, only changing the area of the annulus.

$$\alpha_{an} = \frac{A_p\,\alpha_p - A_{ecol}\,\alpha_{col}}{A_p - A_{ecol}} \quad (14)$$

$$\alpha_{an1} = \frac{A_{i1}\,\alpha_{r1} - A_{ecol}\,\alpha_{col}}{A_{i1} - A_{ecol}} \quad (15)$$

Where A_{i1} is the inner area of the first coating and α_{r1} its expansibility. It is necessary to determine the expansibilities of each element, namely, of the formation and of the coating α_{col}, α_p e α_{r1} respectively, as a function of the elastic parameters of the drill pipe, α_{col}, α_p e α_{r1} respectivelyso that the terms of the annulus space expansibilities are completed. For this, it is important to know the radial displacement of a hollow cylinder, when it is subjected to pressure, and how the stress around it, that represents a well, will behave. The equation for purely radial displacement of a hollow cylinder and the stress associated with these strains are well known in the literature (Villaça, 2000). The expansibilities can be seen in Table 1 (Campos, 1986). Thus the calculations of the annulus expansibilities are completed.

Table1.Different Expansibilities

Between the column and open hole	Column	Between the column and the first coating
$\alpha_p = \frac{1}{G}$	$\alpha_{col} = -\frac{(1-\nu)r_{ecol}^2 + (1+\nu)r_{icol}^2}{(0{,}5)E\left(r_{ecol}^2 - r_{icol}^2\right)}$	$\alpha_{r1} = \frac{(1-\nu)r_{i1}^2 + (1+\nu)r_{e1}^2}{(0{,}5)E\left(r_{e1}^2 - r_{i1}^2\right)}$

4. METHOD OF CHARACTERISTICS

Using the equations of compressibility and expansibility, it is possible to express Eq. (7) in terms of pressure. Replacing the Eq. (9) and Eq. (11) in Eq. (7) and expanding the term of the total derivative of the pressure:

$$(\alpha + c)\left[\frac{\partial p}{\partial x}v + \frac{\partial p}{\partial t}\right] + \frac{\partial v}{\partial x} = 0 \qquad (16)$$

Performing a variable substitution, it is conveniently to rewrite the constants α e c as a function of the speed of the soundv_s. Dividing Eq. (16) byρv_s^2, the equation of continuity in terms of pressure will be:

$$(\alpha + c) = \frac{1}{\rho v_s^2} \Rightarrow v_s = \sqrt{\frac{1}{\rho(\alpha + c)}} \Rightarrow \frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + \rho v_s^2 \frac{\partial v}{\partial x} = 0 \qquad (17)$$

The speed in Eq. (17) depends on the density and compressibility of the fluid and on the expansibility of the region through which the fluid is moving. Solving the system formed by Eq. (4) and Eq. (17), another system of ordinary differential equation of the first order will result, represented by Eq. (18-I) and Eq. (18-II). It can be solved numerically by the method of characteristics, along their respective characteristic directions C^+ C^-. Each interval of the well will have a characteristic straight line, because the speed of the sound is different in each interval, due to the different expansibilities and compressibilities, as calculated in this work.

$$(I)\,\frac{dv}{dt} + \frac{1}{\rho v_s}\frac{dp}{dt} - g + \frac{1}{\rho}\frac{dP_{fric}}{dx} = 0,\ para\ \ C^+: \frac{dx}{dt} = +v_s\,;\,(II)\,\frac{dv}{dt} - \frac{1}{\rho v_s}\frac{dp}{dt} - g + \frac{1}{\rho}\frac{dP_{fric}}{dx} = 0,\ para\ \ C^-: \frac{dx}{dt} = -v_s \quad (18)$$

The system above formed by these equations represents the displacement of a compressible fluid within a tube of expandable walls. Since it is difficult to integrate the frictional pressure gradient in each equation, it is convenient to use the finite differences method and solve the system numerically.

5. METHOD OF FINITE DIFFERENCES

To apply the finite differences in Eq. (18-I) and Eq. (18-II), the well has been divided into a grid where the number of grid points will be determined by the characteristics straight line of each interval. Thus, in the finite form:

$$\frac{\delta x_i}{\delta t} = v_s^i \qquad (19)$$

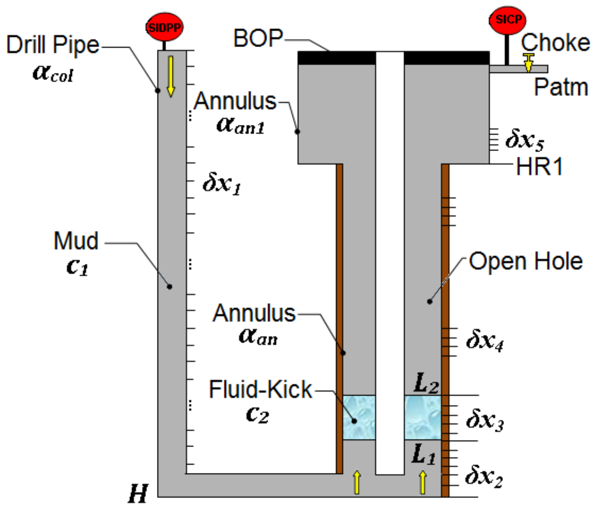


Figure 2. Schematic for finite differences

Let's consider two types of fluid and three different intervals through which these fluids will pass. The schematic can be seen in Fig.(2). According to this figure, L_1 is the boundary between the drilling fluid and the basis for kick and L_2 the top of the kick. In the contact interface between the two fluids, the density may be considered an average between their densities. Replacing in Eq. (18-I) and Eq. (18-II), their derivates by their finite differences expressions and using the schematic of Fig.(2) and Fig.(3b) as a basis, the numerical difference equations for the drill pipe are:

$$\frac{V_2(i) - V_1(i-1)}{dt} + \frac{1}{\rho V_s}\frac{P_2(i) - P_1(i-1)}{dt} - \delta_1 g + \frac{1}{\rho}\frac{dP_{fric}(i-1)}{dx} = 0\,;\,\frac{V_2(i) - V_1(i+1)}{dt} - \frac{1}{\rho V_s}\frac{P_2(i) - P_1(i+1)}{dt} - \delta_2 g + \frac{1}{\rho}\frac{dP_{fric}(i+1)}{dx} = 0 \quad (20)$$

The knowledge of the speeds and pressures in an instant earlier, in this case, are represented by: $V_1(i-1), V_1(i+1), P_1(i-1), P_1(i+1)$. The variables to be calculated in an instant later are: speed $V_2(i)$ and pressure$P_2(i)$. While δ_1 and δ_2 positive or negative *(+1 or -1),* depending on the sense of the flow direction. If it is flowing down through drill pipe *(+1)* or flowing up *(-1)*. Solving this system for the speed and pressure:

$$V_2(i) = \frac{V_1(i-1)+V_1(i+1)}{2} + \frac{P_1(i-1)-P_1(i+1)}{2\rho V_S} + \frac{(\delta_1+\delta_2)gdt}{2} - \frac{\mathrm{dt}}{2\rho}\left[\frac{dP_{fric}(i-1)}{dx} + \frac{dP_{fric}(i+1)}{dx}\right] \tag{21}$$

$$P_2(i) = \frac{\rho V_S[V_1(i-1)-V_1(i+1)]}{2} + \frac{P_1(i-1)+P_1(i+1)}{2} - \frac{(\delta_2-\delta_1)gdt\rho V_S}{2} - \frac{V_S dt}{2}\left[\frac{dP_{fric}(i-1)}{dx} - \frac{dP_{fric}(i+1)}{dx}\right] \tag{22}$$

Special procedures must be adopted in some boundaries, namely, entry of drill pipe, at the bottom of the well (passage of the drill pipe to the annulus between drill pipe and open hole), passage to open hole to first casing shoe and to choke.

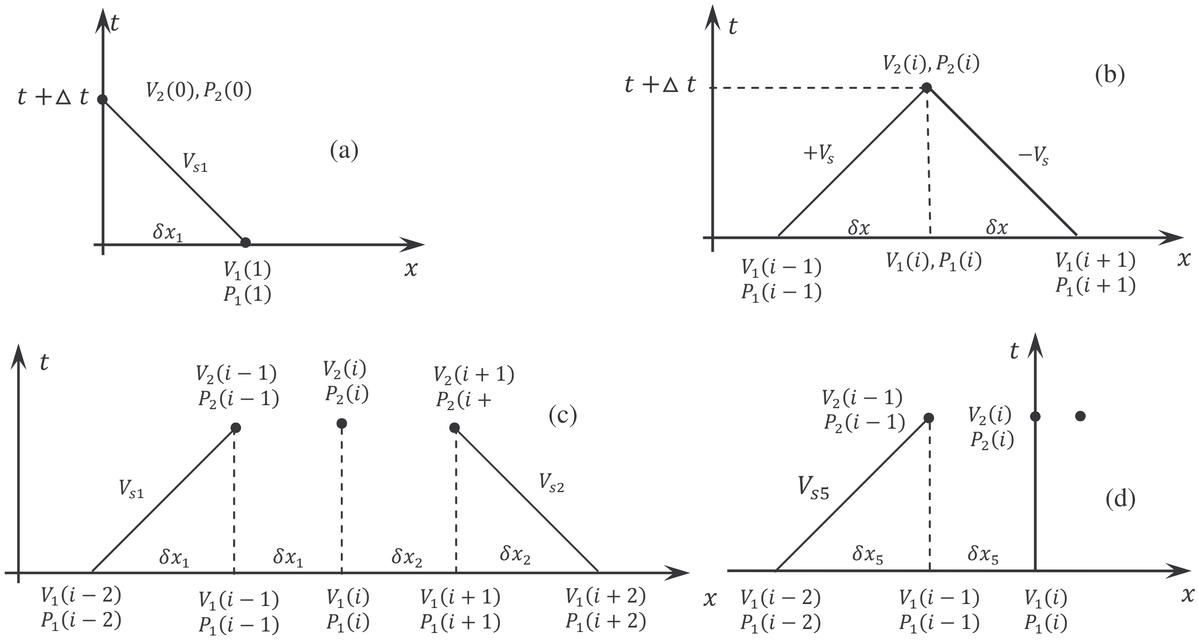


Figure 3. Schematic of characteristic equations to calculate the finite differences for different intervals of the well.

To calculate the point at the drill pipe entry the Eq. (21) and Eq. (22), index *i* must be ser to zero. The speed $V_2(0)$, will be provided by a plot that simulates the circulation reduced flow $Q(t)$, leaving only the task of calculating the boundary condition for the pressure that can be obtained by Eq. (22).

$$V_2(0) = \frac{Q(t)}{A_{icol}} \;\; ; \;\; P_2(0) = P_1(1) + \rho V_s[V_2(0) - V_1(1)] - \rho g V_s dt + V_s \mathrm{dt}\frac{dP_{fric}(1)}{dx} \tag{23}$$

There is a discontinuity of area at the well bottom, in the passageway of the drill pipe to the annulus between drill pipe and open hole. In points immediately before and after this discontinuity, the calculation of the pressure will obey the Bernoulli's Law. Even in this situation the time step must be the same for all the system. Thus the mesh will be:

$$\delta t = \frac{\delta x_1}{V_{S1}} \text{ and } \delta t = \frac{\delta x_2}{V_{S2}} \;\Rightarrow\; \frac{\delta x_1}{\delta x_2} = \frac{V_{S1}}{V_{S2}} \tag{24}$$

Since δx_1 is the division of the mesh inside the drill pipe and V_{s1} is its speed of the sound. Since δx_2 is the division of the mesh inside the annulus and V_{s2} its speed of the sound. In this point it is required to solve a system of four equations and four unknowns, described by Eq. (25), Eq. (26), Eq. (27) and Eq. (28).

$$\frac{V_2(i-1)-V_1(i-2)}{dt} + \frac{1}{\rho V_{S1}}\frac{P_2(i-1)-P_1(i-2)}{dt} - \delta_1 g + \frac{1}{\rho}\frac{dP_{fric}(i-2)}{dx} = 0 \tag{25}$$

$$\frac{V_2(i+1)-V_1(i+2)}{dt} - \frac{1}{\rho V_{S2}}\frac{P_2(i+1)-P_1(i+2)}{dt} - \delta_2 g + \frac{1}{\rho}\frac{dP_{fric}(i+2)}{dx} = 0 \tag{26}$$

$$\mathrm{A}_{\mathrm{icol}}V_2(i-1)=\mathrm{A}_{\mathrm{an}}V_2(i+1) \quad (27)$$

$$\frac{V_2^2(i-1)}{2}+\frac{P_2(i-1)}{\rho}=\frac{V_2^2(i+1)}{2}+\frac{P_2(i+1)}{\rho} \quad (28)$$

In this case δ_2 will be equal to *-1*, it shows that the fluid has already passed from the drill pipe to open well. Solving this system:

$$\begin{cases} V_2(i-1)=\dfrac{2\left(\frac{M-N}{\rho}\right)}{\left[V_{s1}+V_{s2}\frac{A_{icol}}{A_{an}}\right]^2-2\left[1-\left(\frac{A_{icol}}{A_{an}}\right)^2\right]\left(\frac{M-N}{\rho}\right)} \ ; \ P_2(i-1)=-\dfrac{2V_{s1}(M-N)}{\left[V_{s1}+V_{s2}\frac{A_{icol}}{A_{an}}\right]^2-2\left[1-\left(\frac{A_{icol}}{A_{an}}\right)^2\right]\left(\frac{M-N}{\rho}\right)}+M \\ V_2(i-1)=\dfrac{2\left(\frac{M-N}{\rho}\right)}{\left[V_{s1}+V_{s2}\frac{A_{icol}}{A_{an}}\right]^2-2\left[1-\left(\frac{A_{icol}}{A_{an}}\right)^2\right]\left(\frac{M-N}{\rho}\right)} \ ; \ P_2(i-1)=-\dfrac{2V_{s1}(M-N)}{\left[V_{s1}+V_{s2}\frac{A_{icol}}{A_{an}}\right]^2-2\left[1-\left(\frac{A_{icol}}{A_{an}}\right)^2\right]\left(\frac{M-N}{\rho}\right)}+M \\ M=P_1(i-2)+\delta_1\rho g V_{s1}dt-V_{s1}dt\dfrac{dP_{fric}(i-2)}{dx}+\rho V_{s1}V_1(i-2) \\ N=P_1(i+2)-\delta_2\rho g V_{s2}dt+V_{s2}dt\dfrac{dP_{fric}(i+2)}{dx}-\rho V_{s2}V_1(i+2) \end{cases} \quad (29)$$

To calculate the speed and pressure in the passageway at depth HR1 of coating, as shown in Fig.(2), the same developments of Eq.(29) must be considered, replacing their respective areas (A_{i1}) and making $\delta_1=-1$ and $\delta_2=-1$. To calculate the pressure at choke is needed to consider the formula for pressure nozzle, because the area of the choke is much smaller than the area of the annulus casing. So considering the Fig.(3d) is possible to generate the system formed by relationship of Eq. (30).

$$P_2(i)=PATM+\left(\frac{\rho A_{ir}}{2CdA_{choke}}\right)^2 V_2^2(i) \ ; \ \frac{V_2(i)-V_1(i-1)}{dt}+\frac{1}{\rho V_{s5}}\frac{P_2(i)-P_1(i-1)}{dt}+g+\frac{1}{\rho}\frac{dP_{fric}(i-1)}{dx}=0 \quad (30)$$

Since $C=-V_1(i-1)\rho V_{s5}-P_1(i-1)+gdt\rho V_{s5}+dtV_{s5}\frac{dP_{fric}(i-1)}{dx}$ $B=\left(\frac{\rho A_{ir}}{2CdA_{choke}}\right)^2$. Solving the system and performing some substitutions of variables it follows:

$$V_2(i)=\frac{-\rho V_{s5}+\sqrt{(\rho V_{s5})^2-4BC}}{2B} \quad (31)$$

5.1. Initial Conditions

To start up the calculation of speed and pressure, it is necessary to know what is the initial speed before the circulation and what is the pressure in the well at time *t = 0*. When the well is closed the pressures are read in the manometers at the drill string entrance and the casing exit, respectively called SIDPP (shut in drill pipe pressure) and SICP (shut-in casing pressure). The height of the kick is kwon by the gain of mud in the tank. Thus it is possible to calculate the distribution of pressure over the well when it is closed. The pressure in the entry column, that is, in the grid point *i=0* will be the SIDPP and will increase $\rho_i g\delta x_i$, reaching its maximum pressure at the bottom. Here ρ_i is the density of the fluid. For here, by ascending up through the annulus the pressure decreases by $-\rho_i g\delta x_i$ until it is reached the lowest pressure in the choke (SICP), that is, at the last grid point. The initial speed is zero for all points in the beginning.

$$V_1(i)=0, para\ i=0,..,N\ ; P_1(i)=\mathrm{SIDPP}+\rho_i g\delta x(i),+\ldots+,P_f,-\rho_i g\delta x(i),-\ldots-,SICP, para\ i=0,..,N \quad (32)$$

6. TUBE SIMULATION

Considering the developed theory, a case is simulated to analyze the parameters influence: drilling fluid viscosity and compressibility, oil kick viscosity, profile of kick circulation and the changing in choke area. A well of *2000 m* has been simulated. The drilling fluid and oil densities were *1294,13 kg/m³* and *872,43 kg/m³* respectively. The drill pipe area was *0,0015 m²*, the discharge coefficient (C_d) was 0,95 and the oil kick initial height was 100 *m*. The parameters that have been analyzed are described in Tab. 2. The simulation five was realized with a different manner of circulation.

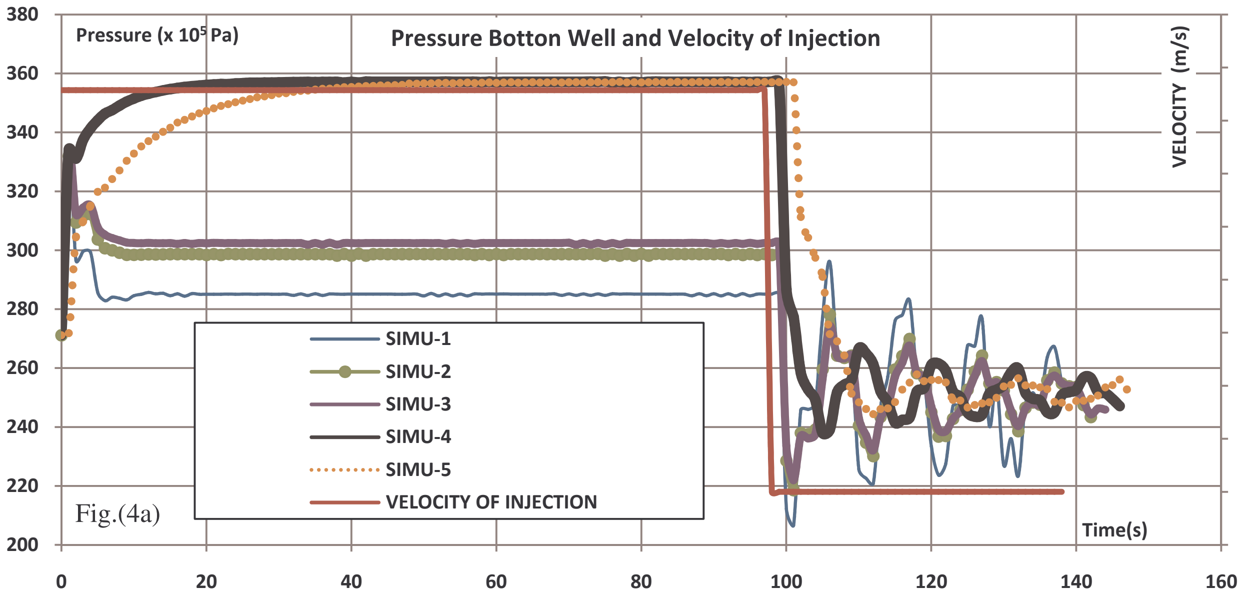
Table2.Parameters analyzed during simulations

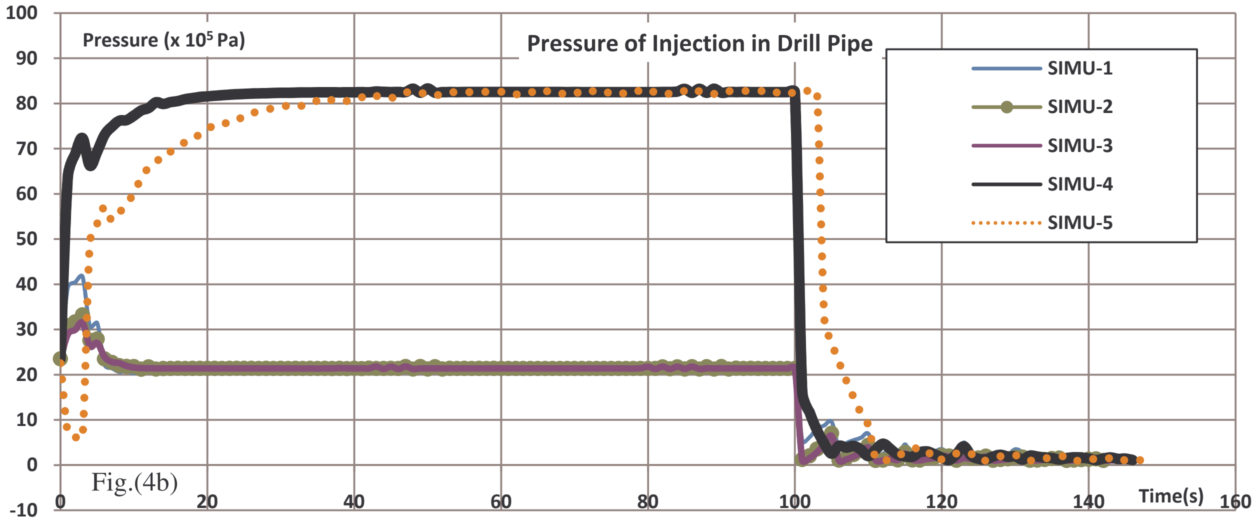
Parameters	Simulation-1	Simulation-2	Simulation-3	Simulation-4	Simulation-5
Viscosity- Fluid (Pa s)	0,002	0,02	0,05	0,02	0,02
Compressibility -Fluid (1/Pa)	$4{,}35 \times 10^{-10}$	$4{,}35 \times 10^{-10}$	$4{,}35 \times 10^{-10}$	$4{,}35 \times 10^{-10}$	$7{,}00 \times 10^{-10}$

Viscosity - Oil (Pa s)	0,003	0,03	0,06	0,03	0,03
Compressibility -Oil (1/Pa)	4,35 x 10^{-10}	4,35 x 10^{-10}	4,35 x 10^{-10}	4,35 x 10^{-10}	4,35 x 10^{-10}
Choke area (m^2)	0,0015	0,0015	0,0015	0,00075	0,00075

7. RESULTS AND CONCLUSIONS OF SIMULATIONS

With the input data, graphics of the reduced speed of circulation in the entry of the drill pipe were generated, which has the profile described in Fig.(4a). Plots of the pressure at the bottom of the well, at the choke and at the monitoring location of the top of the kick, were also generated. In the case of the fist simulation, Fig.(4c) has been generated for the pressure profile of the top kick during the circulation.





It can be noted that there are strong fluctuations in all graphs of Fig. (4) after the sudden stopping of the circulation. The increase in the viscosity causes the pressure to increase at the bottom of the well. On the other hand, increasing the viscosity results in the dampening of these oscillations. The reduction of the choke area had the expected effect of increasing pressure at all parts of the well. The increase of the compressibility has a beneficial effect, because the pressure takes longer to reach its maximum value, but after the cessation of circulation makes the well keep up pressure for longer. When changing the circulation profile as shown in Fig.(5), the fluctuations become smoother in the circulation ends. In a real situation, it is necessary to control the opening of the choke valve dynamically, so that the pressure is kept constant at the bottom.

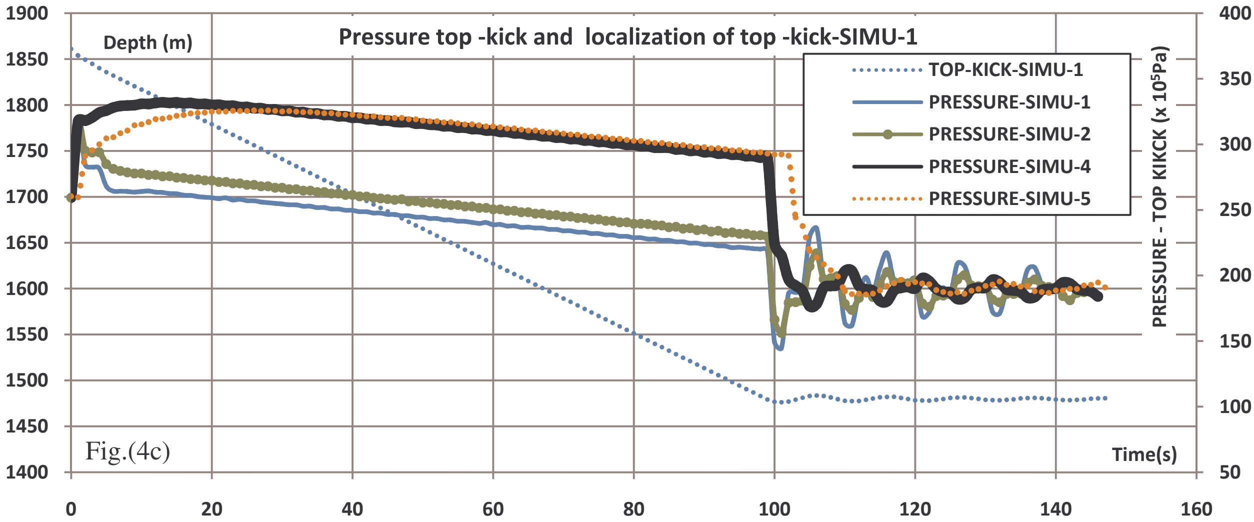


Figure 4. (a) Pressure at the bottom of the well; (b) Pressure at the choke; (c) Pressure at the top kick

The intent here is to test for the effects of the elastic parameters. A numerical simulation that represents all stages of a kick simulation needs to be done. It is possible to see in all cases that the dampening of the pressure oscillations is proportional to fluid viscosity. The closing of the choke has a strong effect on the appearance of the oscillations.

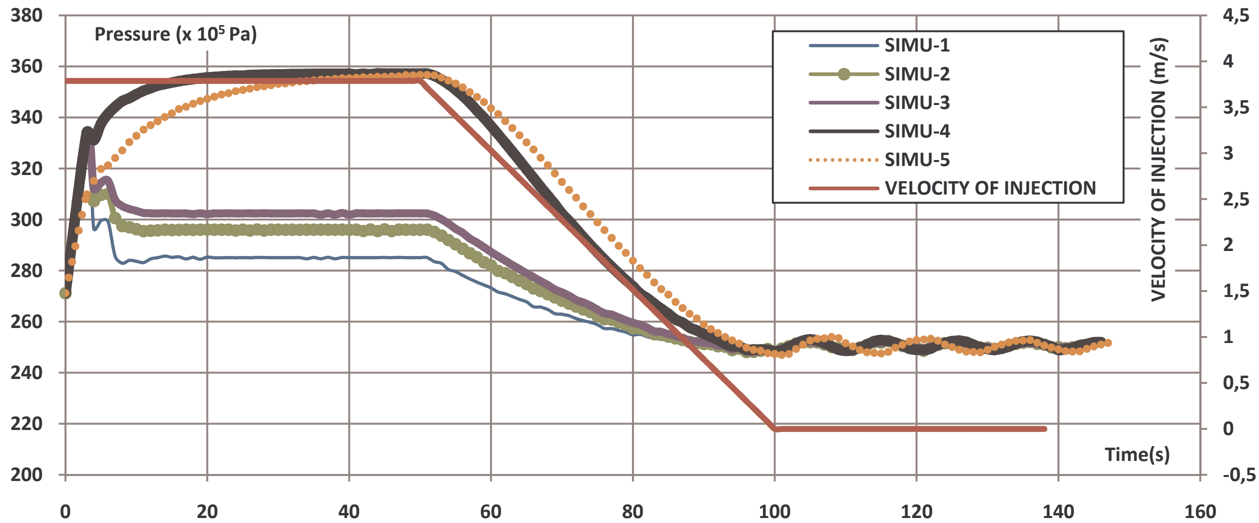


Figure 5.Pressure at the bottom of the well with another injection profile

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