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HYBRID SOLUTION FOR HYDRODYNAMICALLY DEVELOPING FLOW IN CIRCULAR TUBES

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Abstract. *The Generalized Integral Transform Technique (GITT) is employed in the solution of the continuity and momentum equations in hydrodynamically developing laminar flow inside circular ducts. The streamfunction formulation is adopted to automatically satisfy the continuity equation and to eliminate the pressure field, thus providing improved computational performance. A novel eigenvalue problem in this eigenfunction expansion approach is considered, thus eliminating the difficulties associated with the singularity at the channel centerline. Results for the velocity field and for the product of the friction factor-Reynolds number are computed along the entrance region for different governing parameters, which are tabulated for reference purposes and graphically presented as functions of the dimensionless axial coordinate. Critical comparisons with previous results in the literature are also performed, in order to validate the numerical code here developed and to provide a set of benchmark results.*

Keywords. *Developing flow, circular tubes, integral transforms, friction factor, laminar flow.*

1. Introduction

Laminar flow within pipes is found in several engineering applications, and has therefore been studied in different geometric configurations and operational conditions. The most usual geometry utilized in pipelines is the circular tube, which has been extensively analyzed in laminar flow conditions by many researchers (Shah and London, 1978). In such studies, the aim has been the determination of the velocity field, and as a consequence parameters of practical interest such as pressure drop and friction factors, both in the entrance and fully developed regions. These computations are generally difficult to accomplish by analytical approaches in light of the nonlinearity of the momentum equations, either in the full Navier-Stokes equations formulation or as the simpler boundary layer equations. In this context, purely numerical schemes have been employed to solve the equations for the classical problem in developing laminar flow in circular tubes (Hornbeck, 1964; Liu, 1974).

In the search for a solution with automatically controlled accuracy, the Generalized Integral Transform Technique (GITT) (Cotta, 1993; Cotta, 1994; Cotta, 1998; Santos et al., 2001; Cotta and Mikhailov, 2006) with its automatic global error control capability is a reliable path for obtaining benchmark results, allowing for a more definitive critical evaluation of previously published numerical results of this classical problem. The GITT has already been utilized to find a hybrid analytical-numerical solution for laminar flow development inside parallel-plates channels (Carvalho et al., 1993; Machado and Cotta, 1995; Pérez Guerrero and Cotta, 1995; Figueira da Silva et al., 1996; Figueira da Silva et al., 1999), by using both the primitive variables and streamfunction-only formulations, in either the Navier-Stokes or boundary layer formulations. The application of the GITT under the streamfunction formulation, leads to the solution of an eigenvalue problem of fourth order, inherent to the analytical treatment of the biharmonic equation, which is readily solved in analytic form (Cotta, 1993, Perez Guerrero and Cotta, 1995). The same solution methodology has been employed in the solution of developing flow within annular channels (Pereira et al, 1998; Pereira et al., 2000), by handling the associated biharmonic-type eigenvalue problem in the cylindrical coordinates system.

Until now, however, the use of this hybrid approach in conjunction with the streamfunction formulation for the flow development in circular tubes was not yet possible, due to the need of finding an appropriate fourth order eigenvalue problem in the cylindrical coordinates system, which would alleviate the singularities of the Bessel functions at the duct centerline. This difficulty is here circumvented by adopting a recently introduced eigenvalue problem (Fedele et al., 2005) which accounts for the singularities at the central radial position. The present work is thus aimed at utilizing the ideas in the generalized integral transform technique to construct a hybrid analytical-numerical solution for the flow development in circular tubes, and to accurately compute the velocity field along the entrance and fully developed regions, as well as the product of friction factor-Reynolds number. Critical comparisons with other previously published works are also performed in order to assess the consistency of the available numerical results.

2. Analysis

Laminar flow of a Newtonian fluid inside a circular tube is considered as shown in Fig. (1). The continuity and incompressible steady two-dimensional Navier-Stokes equations in cylindrical coordinates are used to model the flow inside this circular geometry. The longitudinal velocity component v_z is assumed to be known at the channel entrance, and the inlet flow is assumed to be parallel ($v_r = 0$). Fully developed flow conditions are attained at a sufficiently large channel length, recovering the parabolic flow structure. Therefore, the governing equations, in the region $0 < r < 1$ and $z > 0$, are written in dimensionless form as:

$$\frac{1}{r}\frac{\partial (r v_r)}{\partial r}+\frac{\partial v_z}{\partial z}=0 \tag{1}$$

$$v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}=-\frac{\partial p}{\partial r}+\frac{2}{Re}\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r\,v_r)\right)+\frac{\partial^2 v_r}{\partial z^2}\right] \tag{2}$$

$$v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}=-\frac{\partial p}{\partial z}+\frac{2}{Re}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{\partial^2 v_z}{\partial z^2}\right] \tag{3}$$

Equations (1) to (3) are subjected to the following inlet and boundary conditions:

$$v_z\,(r,0)=1\,;\quad v_r\,(r,0)=0 \tag{4,5}$$

$$v_z\,(r,\infty)=4q\,(1-r^2)\,;\quad v_r\,(r,\infty)=0 \tag{6,7}$$

$$v_r\,(0,z)=0\,;\quad \frac{\partial v_z(0,z)}{\partial r}=0 \tag{8,9}$$

$$v_z\,(1,z)=0\,;\quad v_r\,(1,z)=0 \tag{10,11}$$

The dimensionless groups employed in the equations above are defined as

$$r = r^*/r_w;\quad z = z^*/r_w;\quad v_z = v_z^*/u_0;\quad v_r = v_r^*/u_0;\quad p = p^*/\rho u_0^2;\quad Re = 2u_0 r_w/\nu \tag{12}$$

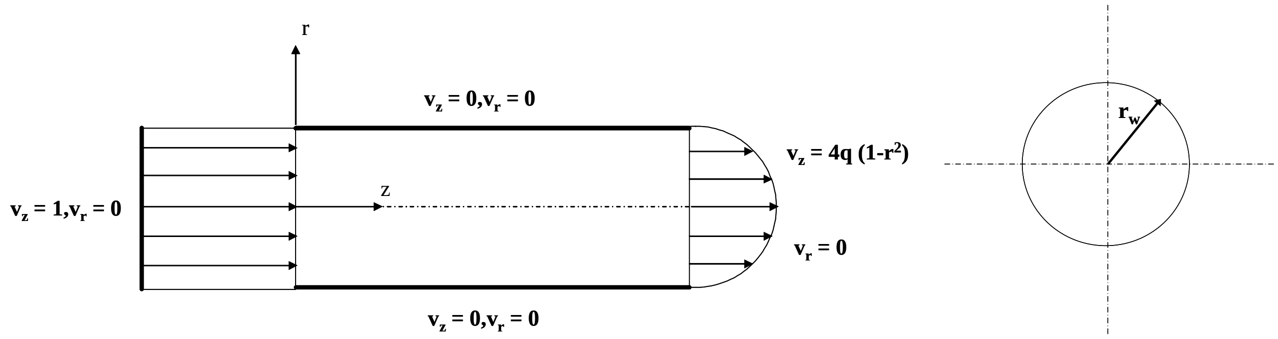

Figure 1. Geometry and coordinates system for developing flow in circular duct.

Now, expressing Eqs. (1) to (11) in the streamfunction-only formulation in order to automatically satisfy the continuity equation and to eliminate the pressure field, the flow modeling is represented by:

$$\frac{1}{r}\frac{\partial\psi}{\partial z}\left[\frac{\partial}{\partial r}(E^2\psi)-\frac{2}{r}(E^2\psi)\right]-\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z}(E^2\psi)=\frac{2}{Re}(E^4\psi) \tag{13}$$

where the operators E^2 and E^4 are defined as

$$E^2=\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\,;\quad E^4=E^2(E^2)=\frac{\partial^4}{\partial r^4}-\frac{2}{r}\frac{\partial^3}{\partial r^3}+\frac{3}{r^2}\frac{\partial^2}{\partial r^2}-\frac{3}{r^3}\frac{\partial}{\partial r}-\frac{2}{r}\frac{\partial^3}{\partial r\partial z^2}+2\frac{\partial^4}{\partial r^2\partial z^2}+\frac{\partial^4}{\partial z^4} \tag{14,15}$$

The streamfunction is defined in terms of the dimensionless velocity components in the longitudinal (r) and transversal (z) directions, respectively, as

$$v_r(r,z) = \frac{1}{r}\frac{\partial\psi(r,z)}{\partial z}\,; \quad v_z(r,z) = -\frac{1}{r}\frac{\partial\psi(r,z)}{\partial r} \tag{16,17}$$

Equation (l3) requires the specification of eight boundary conditions expressed in terms of the streamfunction. At the duct walls, no-slip and impermeability conditions are written as:

$$\frac{\psi(0,z)}{r} = C_1\,; \quad \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial\psi(0,z)}{\partial r}\right] = 0 \tag{18,19}$$

$$\psi(1,z) = C_2\,; \quad \frac{\partial\psi(1,z)}{\partial r} = 0 \tag{20,21}$$

$$\frac{\psi(r,0)}{r} = C_1 - \frac{r}{2}\,; \quad \frac{\partial\psi(r,0)}{\partial z} = 0 \tag{22,23}$$

$$\frac{\psi(r,\infty)}{r} = C_1 - 4q\left(\frac{r}{2} - \frac{r^3}{4}\right); \quad \frac{\partial\psi(r,\infty)}{\partial z} = 0 \tag{24,25}$$

C_1 and C_2 are constants that specify the streamfunction values at the channel centerline and wall. The constant q warrants the global mass conservation. Such constants are related by using the boundary conditions above, to yield

$$C_2 = C_1 - 1/2\,; \quad q = C_1 - C_2 \tag{26,27}$$

One may arbitrarily specify $C_1 = 0$, so that $C_2 = -1/2$ and $q = 1/2$.

Equation (13) and boundary conditions (18) to (25) complete the problem formulation in terms of the streamfunction only. Following the ideas in the generalized integral transform technique (Cotta, 1993; Cotta, 1994; Cotta, 1998; Santos et al., 2001; Cotta and Mikhailov, 2006), for improved computational performance, it is convenient to define a filter that reproduces the fully developed flow solution in order to homogenize the boundary conditions in the r direction, which later will be the coordinate chosen for the specification of the eigenvalue problem. Therefore, the filter is written as

$$\psi(r,z) = \psi_\infty(r) + \phi(r,z)\,; \quad \psi_\infty(r) = r^2\left(\frac{r^2}{2} - 1\right) \tag{28,29}$$

where the filter $\psi_\infty(r) \equiv \psi(\infty,r)$ represents the streamfunction in the fully developed region and $\phi(r,z)$ is now the filtered potential to be solved for. The resulting problem formulation is then given by:

$$\frac{1}{r}\frac{\partial\phi}{\partial z}\left[\frac{\partial}{\partial r}(E^2\phi) - \frac{2}{r}(E^2\phi)\right] - \frac{1}{r}\frac{d\psi_\infty}{dr}\frac{\partial}{\partial z}(E^2\phi) - \frac{1}{r}\frac{\partial\phi}{\partial r}\frac{\partial}{\partial z}(E^2\phi) = \frac{2}{Re}(E^4\phi) \tag{30}$$

$$\frac{\phi(0,z)}{r} = 0\,; \quad \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial\phi(0,z)}{\partial r}\right] = 0 \tag{31,32}$$

$$\phi(1,z) = 0\,; \quad \frac{\partial\phi(1,z)}{\partial r} = 0 \tag{33,34}$$

$$\frac{\phi(r,0)}{r} = \frac{r}{2}\left(1 - r^2\right); \quad \frac{\partial\phi(r,0)}{\partial z} = 0 \tag{35,36}$$

$$\frac{\phi(r,\infty)}{r} = 0\,; \quad \frac{\partial\phi(r,\infty)}{\partial z} = 0 \tag{37,38}$$

The present contribution is aimed at solving only the boundary layer equations. Therefore, within the range of validity of the boundary layer hypothesis, the momentum equation given by Eq. (30) and related boundary conditions become:

$$\frac{1}{r}\frac{\partial\phi}{\partial z}\left(\frac{\partial^3\phi}{\partial r^3} - \frac{3}{r}\frac{\partial^2\phi}{\partial r^2} + \frac{3}{r^2}\frac{\partial\phi}{\partial r}\right) - \frac{1}{r}\frac{\partial\phi}{\partial r}\left(\frac{\partial^3\phi}{\partial r^2\partial z} - \frac{1}{r}\frac{\partial^2\phi}{\partial r\partial z}\right) - \frac{1}{r}\frac{\partial\psi_\infty}{dr}\left(\frac{\partial^3\phi}{\partial r^2\partial z} - \frac{1}{r}\frac{\partial^2\phi}{\partial r\partial z}\right) = \frac{2}{Re}\left(\frac{\partial^4\psi}{\partial r^4} - \frac{2}{r}\frac{\partial^3\psi}{\partial r^3} + \frac{3}{r^2}\frac{\partial^2\psi}{\partial r^2} - \frac{3}{r^3}\frac{\partial\psi}{\partial r}\right) \tag{39}$$

$$\frac{\phi(0,z)}{r} = 0\;; \quad \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \phi(0,z)}{\partial r}\right] = 0 \tag{40,41}$$

$$\phi(1,z) = 0\;; \quad \frac{\partial \phi(1,z)}{\partial r} = 0 \tag{42,43}$$

$$\frac{\phi(r,0)}{r} = \frac{r}{2}\left(1-r^2\right) \tag{44}$$

2.1. Solution methodology

In applying the GITT approach in the solution of the PDE system given by Eqs. (39) to (44), due to the homogeneous characteristics of the boundary conditions in the r direction, it is more appropriate to choose this direction for the process of integral transformation. Therefore, the auxiliary eigenvalue problem is taken as (Fedele et al., 2005)

$$\mathcal{L}^2 X_i(r) = -\lambda_i^2\, \mathcal{L} X_i(r) \tag{45}$$

$$\frac{X_i(0)}{r} = 0\;; \quad \frac{d}{dr}\left[\frac{1}{r}\frac{dX_i(0)}{dr}\right] = 0 \tag{46,47}$$

$$X_i(1) = 0\;; \quad \frac{dX_i(1)}{dr} = 0 \tag{48,49}$$

where,

$$\mathcal{L} = r\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}\right) = \frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr}\;; \quad \mathcal{L}^2 = \mathcal{L}(\mathcal{L}) = \frac{d^4}{dr^4} - \frac{2}{r}\frac{d^3}{dr^3} + \frac{3}{r^2}\frac{d^2}{dr^2} - \frac{3}{r^3}\frac{d}{dr} \tag{50,51}$$

Problem (45) is analytically solved, to furnish:

$$X_i(r) = r^2 - \frac{r\,J_1(\lambda_i r)}{J_1(\lambda_i)} \tag{52}$$

where the eigenvalues λ_i's are computed from the following transcendental equation:

$$J_2(\lambda_i) = 0, \qquad i = 1, 2, 3, \ldots \tag{53}$$

The eigenfunctions satisfy the following orthogonality property:

$$\int_0^1 \frac{dX_i}{dr}\frac{dX_j}{dr}\frac{dr}{r} = -\int_0^1 X_i(\mathcal{L}X_j)\frac{dr}{r} = \begin{cases} 0, & i \neq j \\ N_i, & i = j \end{cases} \tag{54}$$

The normalization integral N_i is then computed from:

$$N_i = \int_0^1 \frac{1}{r}\left(\frac{dX_i}{dr}\right)^2 dr = \frac{\lambda_i^{\,2}}{2} \tag{55}$$

The eigenvalue problem defined by Eqs. (45) to (49) allow for the definition of the following integral transform pair:

$$\bar{\phi}_i(z) = -\frac{1}{N_i}\int_0^1 \phi(r,z)(\mathcal{L}X_i)\frac{dr}{r}, \qquad \text{transform} \tag{56}$$

$$\phi(r,z) = \sum_{i=1}^{\infty} X_i(r)\,\bar{\phi}_i(z)\,, \qquad \text{inverse} \tag{57}$$

The next step is thus to accomplish the integral transformation of the original partial differential system given by Eqs. (39) to (44). For this purpose, Eqs. (39) and the inlet condition (44) are multiplied by $[X_i(r)/r]$, and after that, integrated over the domain [0,1] in r, and the inverse formula given by Eq. (57) is employed, resulting in the following coupled ordinary differential system for the calculation of the transformed potentials $\bar{\phi}_i(z)$:

$$\sum_{j=1}^{\infty} D_{ij} \frac{d\bar{\phi}_j(z)}{dz} = \frac{2}{Re} \lambda_i^2 \bar{\phi}_i(z) \quad , \quad i=1,2,\ldots \tag{58}$$

$$\bar{\phi}_i(0) = \frac{1}{N_i} \int_0^1 (1-2r^2) \frac{dX_i}{dr} dr \tag{59}$$

where,

$$D_{ij} = \frac{1}{N_i} \left[B_{ij} + \sum_{k=1}^{\infty} A_{ijk} \bar{\phi}_k(z) \right] \tag{60}$$

$$A_{ijk} = \int_0^1 \frac{X_i}{r^2} \left(X_j X_k''' - \frac{3X_j X_k''}{r} + \frac{3X_j X_k'}{r^2} - X_j'' X_k' + \frac{X_j' X_k'}{r} \right) dr \tag{61}$$

$$B_{ij} = \int_0^1 \frac{X_i}{r^2} \left(\frac{X_j'}{r} - X_j'' \right) \frac{d\psi_\infty}{dr} dr \tag{62}$$

In order to numerically handle the ODE system given by Eqs. (58) to (62), through the subroutine DIVPAG from the IMSL Library (1991), it is necessary to truncate the infinite series in a sufficiently high number of terms so as to guarantee the requested relative error in obtaining the original potentials. This subroutine solves initial value problems with stiff characteristics, and provides the important feature of automatically controlling the relative error in the solution of the ordinary differential equations system, allowing the user to establish error targets for the transformed potentials.

Once the transformed potentials are available, the velocity field is obtained from the definition of the streamfunction given by Eqs. (16) and (17), after introducing the inverse formula (57), to yield:

$$v_r(r,z) = \sum_{i=1}^{\infty} \frac{X_i(r)}{r} \frac{d\bar{\phi}_i(z)}{dz} \; ; \quad v_z(r,z) = 2(1-r^2) - \sum_{i=1}^{\infty} \frac{X_i'(r)}{r} \bar{\phi}_i(z) \tag{63,64}$$

In the realm of applications, one is concerned with quantities of practical interest such as the product of the Fanning friction factor-Reynolds number, fRe. The friction factor is defined as:

$$f = \tau_w \big/ (\rho u_0^2 / 2) \tag{65}$$

with the introduction of the inverse formula for the velocity field given by Eq. (64), it then results

$$f Re = 16 + 4 \sum_{i=1}^{\infty} \frac{d}{dr} \left[\frac{1}{r} \frac{dX_i(1)}{dr} \right] \bar{\phi}_i(z) \tag{66}$$

3. Results and discussion

Numerical results for the velocity profiles and the product of the Fanning friction factor-Reynolds number were produced along the entrance region of a circular tube. The computational code was developed in FORTRAN 90/95 programming language and implemented on a PENTIUM-IV 1.3 GHz computer. First, the numerical code was validated against those results presented by Hornbeck (1964), Liu (1974), Shah and London (1978), and Nascimento et al. (2006). The routine DIVPAG from the IMSL Library (1991) was used to numerically handle the truncated version of the system of ordinary differential equations (Eqs. (58) to (62)), with a relative error target of 10^{-8} prescribed by the user, for the transformed potentials. These results were produced with different truncation orders (N = 10, 20, 40, 60, 80 and 100) and for Re = 2000, but it should be noted that the dimensionless axial coordinate $X^+ = z/(2Re)$ makes the results independent of the Reynolds number for the boundary layer formulation.

Figure (2) and Table (1) show the convergence behavior of the longitudinal velocity component at the centerline of the circular tube, as well as its comparison with the results presented in Hornbeck (1964), Liu (1974), Shah and London (1978) and Nascimento et al. (2006), demonstrating a good agreement, which provides a direct validation of the numerical code here developed. From inspection of Figure (2), a monotonic convergence behavior for the longitudinal velocity component at the centerline of the circular tube is observed.

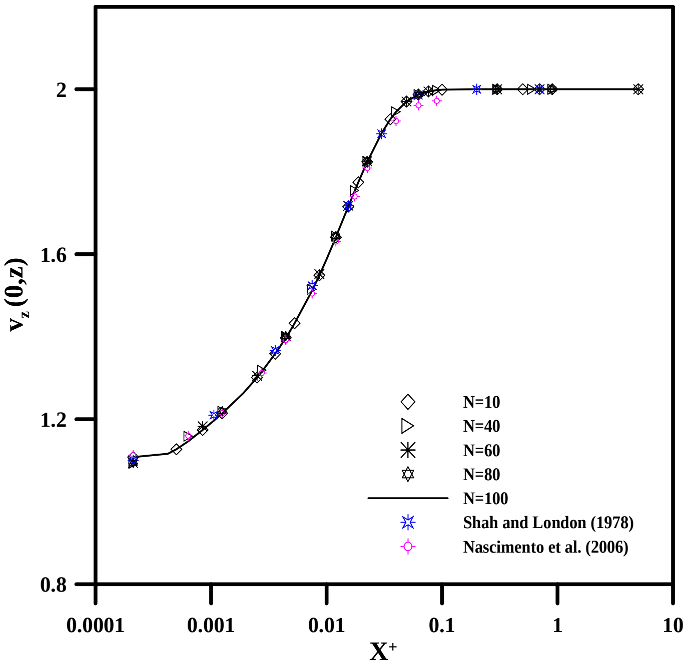

Figure 2. Development of the longitudinal velocity component v_z at the centerline of the duct.

Results for the product of the Fanning friction factor-Reynolds number are shown in Fig. (3) as a function of the dimensionless axial positions, X^+. It can be observed that the product fRe diminishes until the fully developed region is reached, in which this parameter assumes a constant value. Also, it should be recalled that the product fRe presents higher values in the entrance region of the circular tube due to higher velocity gradients experimented by the fluid in this region, and for this reason convergence of the eigenfunction expansions is slower within the entrance. Also, a comparison with the results of Nascimento et al. (2006) is performed, showing an excellent agreement. This previous contribution with the GITT implemented a hybrid solution for the same boundary layer formulation, but under primitive variables mathematical description, which leads to different auxiliary eigenvalue problems, and thus different convergence behavior patterns.

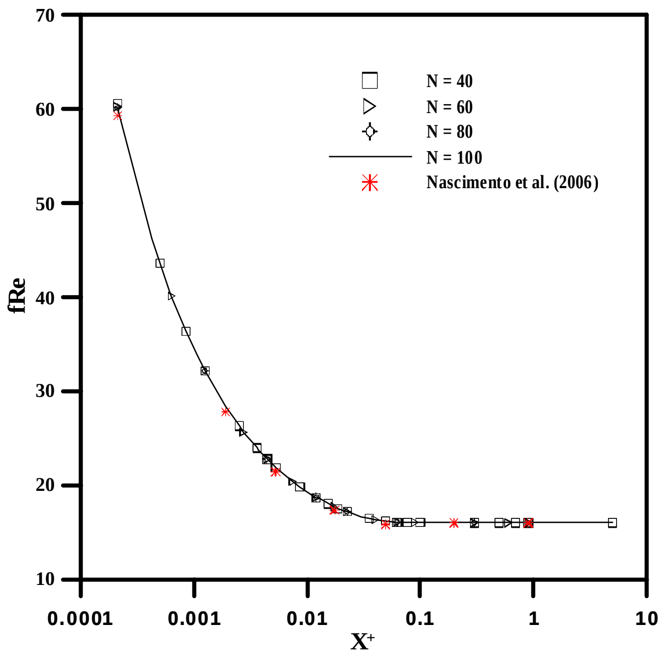

Figure 3. Product fRe along the entrance region of a circular tube.

Finally, Figure (4) illustrates the development of the longitudinal velocity component, as a function of the transversal coordinate R, at specific dimensionless axial positions X^+. From this figure it can be noticed that the velocity profile evolutes from flatter shapes in axial positions near the tube entrance, to parabolic forms in positions far from the entrance, where a fully developed region is attained.

Table 1. Convergence behavior and comparison of the longitudinal velocity component v_z at the duct centerline.

X^+	N = 10	N = 20	N = 40	N = 60	N = 80	N = 100	Shah and London (1978)	Nascimento et al. (2006)
0.0002116	1.108	1.088	1.093	1.094	1.094	1.094	1.100	1.113
0.0004232	1.116	1.129	1.131	1.131	1.131	1.132	1.138	1.136
0.0005000	1.127	1.138	1.141	1.142	1.142	1.142	1.150	1.145
0.0006349	1.147	1.157	1.159	1.159	1.159	1.160	1.166	1.160
0.0008467	1.174	1.181	1.182	1.183	1.183	1.183	1.190	1.181
0.0010580	1.197	1.201	1.203	1.203	1.203	1.203	1.210	1.200
0.0012500	1.214	1.218	1.219	1.220	1.220	1.220	1.227	1.215
0.0019000	1.263	1.266	1.268	1.268	1.268	1.268	1.273	1.262
0.0025000	1.301	1.304	1.305	1.305	1.306	1.306	1.313	1.298
0.0027470	1.315	1.318	1.319	1.319	1.320	1.320	1.324	1.312
0.0035930	1.359	1.361	1.363	1.363	1.363	1.363	1.367	1.354
0.0037500	1.366	1.369	1.370	1.370	1.370	1.371	1.378	1.362
0.0044400	1.397	1.400	1.401	1.401	1.401	1.402	1.405	1.392
0.0050000	1.421	1.423	1.424	1.425	1.425	1.425	1.433	1.415
0.0052880	1.433	1.435	1.436	1.436	1.436	1.436	1.439	1.427
0.0075000	1.512	1.514	1.515	1.515	1.516	1.516	1.524	1.505
0.0086580	1.549	1.551	1.552	1.552	1.552	1.552	1.553	1.541
0.0100000	1.588	1.590	1.591	1.591	1.591	1.591	1.598	1.579
0.0120400	1.641	1.643	1.644	1.644	1.644	1.644	1.644	1.631
0.0125000	1.652	1.654	1.655	1.655	1.655	1.655	1.660	1.642
0.0154300	1.715	1.717	1.717	1.717	1.718	1.718	1.718	1.704
0.0175000	1.753	1.754	1.755	1.755	1.755	1.755	1.756	1.740
0.0188200	1.774	1.775	1.776	1.776	1.776	1.776	1.776	1.761
0.0222100	1.821	1.822	1.822	1.822	1.822	1.822	1.822	1.806
0.0225000	1.824	1.825	1.825	1.826	1.826	1.826	1.824	1.809
0.0300000	1.894	1.894	1.895	1.895	1.895	1.895	1.892	1.875
0.0356900	1.927	1.928	1.928	1.928	1.928	1.928	1.928	1.906
0.0400000	1.945	1.945	1.946	1.946	1.946	1.946	1.943	1.923
0.0492400	1.970	1.970	1.970	1.970	1.970	1.970	1.971	1.946
0.0500000	1.971	1.972	1.972	1.972	1.972	1.972	1.970	1.947
0.0625000	1.987	1.987	1.987	1.987	1.987	1.987	1.986	1.961
0.0628100	1.988	1.988	1.988	1.988	1.988	1.988	1.989	1.961
0.0763400	1.995	1.995	1.995	1.995	1.995	1.995	1.996	1.969
0.0899300	1.998	1.998	1.998	1.998	1.998	1.998	1.999	1.972
0.1000000	1.999	1.999	1.999	1.999	1.999	1.999	2.000	2.000
1.0000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000

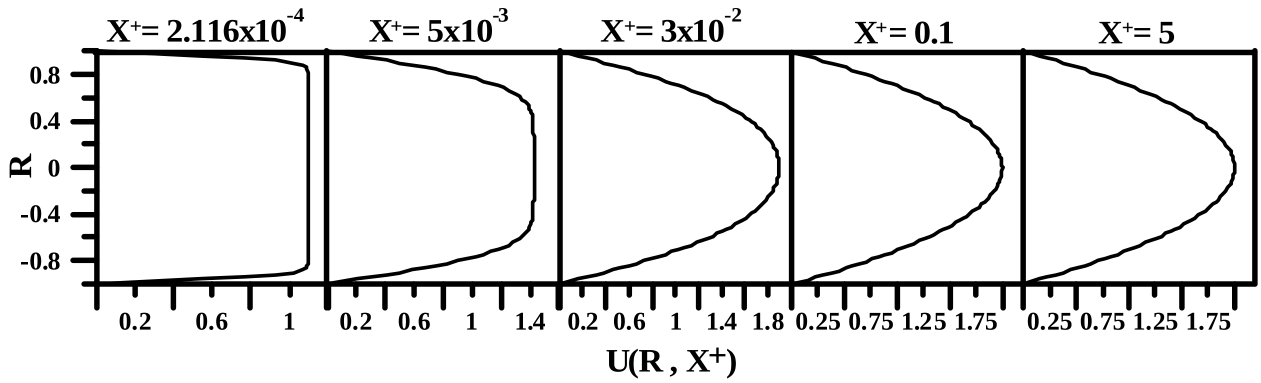

Figure 4. Development of the longitudinal velocity component profile along the tube length.

4. Conclusions

Numerical results for the velocity field and product Fanning friction factor-Reynolds number were produced by using the GITT approach in the solution of the continuity and momentum equations for the flow of Newtonian fluids within circular tubes. The adoption of a recently described eigenvalue problem in the cylindrical coordinates system made possible the application of a streamfunction formulation in such a geometry. The excellent agreement of the present results with previously reported ones demonstrates the consistency of this approach for benchmarking such class of problems. The approach presented, in addition, opens up new perspectives in the hybrid numerical-analytical solution of nonlinear heat and fluid flow problems in non-cartesian coordinates systems.

5. Acknowledgements

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6. References

Carvalho, T.M.B., Cotta, R.M. and Mikhailov, M.D., 1993, “Flow Development in Entrance Region of Ducts”, Communications in Numerical Methods in Engineering, Vol. 9, pp. 503-509.

Cotta, R.M., 1993, “Integral Transforms in Computational Heat and Fluid Flow”, CRC Press, Boca Raton, USA.

Cotta, R.M., 1994, “Benchmark Results in Computational Heat and Fluid Flow: - The Integral Transform Method”, Int. J. Heat Mass Transfer (Invited Paper), Vol. 37 (Suppl. 1), pp. 381-393.

Cotta, R.M., 1998, “The Integral Transform Method in Thermal and Fluid Sciences and Engineering”, Begell House, New York.

Cotta, R.M., and Mikhailov, M.D., 2006, “Hybrid Methods and Symbolic Computations”, in: Handbook of Numerical Heat Transfer, 2nd edition, Chapter 16, Eds. W.J. Minkowycz, E.M. Sparrow, and J.Y. Murthy, John Wiley, New York, pp.493-522.

Fedele, F., Hitt, D.L. and Prabhu, R.D., 2005 “Revisiting the Stability of Pulsatile Pipe Flow”, European Journal of Mechanics B/Fluids, Vol. 24, pp. 237–254.

Figueira da Silva, E., Perez Guerrero, J.S., and Cotta, R.M., 1996, “A Comparison of Integral Transform Strategies for Solving the Boundary Layer Equations”, Proc. of the 6th National Thermal Sciences Meeting, ENCIT-96, Florianópolis, SC, Brasil.

Figueira da Silva, E., Perez-Guerrero, J.S., and Cotta, R.M., 1999, "Integral Transform Solution of Boundary Layer Equations in Streamfunction-only Formulation", Int. J. Non-Linear Mechanics, V.34, pp.51-61.

Hornbeck, R.W., 1964, “Laminar Flow in the Entrance Region of a Pipe”, Applied Scientific Research, Section A, Vol. 13, pp. 224-232.

IMSL Library, 1991, MATH/LIB, Houston, TX.

Liu, J., 1974, “Flow of a Bingham Fluid in the Entrance Region of an Annular Tube”, M. S. Thesis, University of Wisconsin – Milwaukee, USA.

Machado, H.A. and Cotta, R.M., 1995, "Integral Transform Method for Boundary Layer Equations in Simultaneous Heat and Fluid Flow Problems", Int. J. Num. Meth. Heat & Fluid Flow, V. 5, pp. 225-237.

Nascimento, S.C.C., Macêdo, E.N. and Quaresma, J.N.N., 2006, “Generalized Integral Transform Solution for Hydrodynamically Developing Non-Newtonian Flows in Circular Tubes”, Journal of the Brazilian Society of Mechanical Sciences and Engineering, Vol. 28, pp. 125-130.

Pereira, L.M., Perez Guerrero, J.S., and Cotta, R.M., 1998, "Integral Transformation of the Navier-Stokes Equations in Cylindrical Geometry", Computational Mechanics, V.21, no.1, pp.60-70.

Pereira, L.M., Cotta, R.M., and Perez Guerrero, J.S., 2000, "Analysis of Laminar Forced Convection in Annular Ducts Using Integral Transforms", Proc. of the 15th Brazilian Congress of Mechanical Engineering, COBEM 99, Águas de Lindóia, São Paulo, Brasil, December 1999 (CD-ROM); also, Hybrid Meth. Eng., V.2, no.2, pp.221-232.

Perez Guerrero, J.S. and Cotta, R.M., 1995, "Integral Transform Solution of Developing Laminar Duct Flow in Navier-Stokes Formulation", International Journal for Numerical Methods in Fluids, Vol. 20, pp. 1203-1213.

Santos, C.A.C., Quaresma, J.N.N., and Lima, J.A., eds., 2001, "Benchmark Results for Convective Heat Transfer in Ducts: - The Integral Transform Approach", ABCM Mechanical Sciences Series, Editora E-Papers, Rio de Janeiro.

Shah, R. K. and London, A. L., 1978, "Laminar Flow Forced Convection in Ducts", In Advances in Heat Transfer (Supplement 1), New York, Academic Press.

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