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Velocity and heat transfer parameters mapping: thermal quadrupoles and infrared image processing

Olivier Fudym
RAPSODEE UMR 2392 CNRS, Ecole des Mines d'Albi, 81013 Albi Cédex, France
fudym@enstimac.fr

Abstract. *Thermal images provided by infrared cameras associated with physical models of heat transfer enable to compute thermophysical properties and velocity mappings. Due to the very important flow of processed data, as well as the high number of estimated parameters, the corresponding inversion methods are generally based on local linear estimation processes that study indirectly the correlations between pixels. These kind of methods are not computationally intensive and have the advantage of yielding a confidence domain for the estimated field.The main problem arising when estimating heat transfer parameter mappings is that the measured superficial 2D field is correlated to the 3D structure. The challenge is then to find some convenient transfer function for the inversion process. The analytical thermal quadrupole method is suitable for the modeling of multidimensional transient heat diffusion in homogeneous media, especially when applied to multilayered media. Some semi-analytical extensions are proposed in order to discuss the separability of the 3D temperature field. We propose herein some examples and analyze the performance of these different methods in the case of thermal diffusivity mapping, solid plate velocity measurement, microchannels thermal characterization and granular media thermal contact resistance evaluation.*

Keywords**. **Infrared Thermography; microfluidics; thermal conductivity; full field estimation; inverse problem

1. Introduction

Infrared thermography is a powerful experimental tool for thermal imaging applications, such as local heat transfer parameters mapping. Recently, this technology gained a CCD-like revolution, with the Focal Plane Array sensors development. Simultaneously, the microelectronic and computer advances offer fast rate recording and processing capacities. Moreover, many fields of the continuum mechanics are concerned with the same kind of instrumentation revolution, yielding a wide family of imaging techniques designed for mapping velocity , stress, concentration, density, etc…

The main challenge is then to find the convenient inversion methods able to process the measured fields and estimate the corresponding parameter maps. These methods are defined by both a direct model and an estimation procedure. The main problem arising is that the measured superficial 2D field is correlated to the 3D structure and boundary conditions. Unfortunately, a complete 3D heat transfer model yielding to non linear estimation of thermal parameters is not computationally efficient, due to the very important flow of processed data, as well as the high number of estimated parameters. Thus, the usual inversion methods are generally based either on *(i)* local 1D heat transfer model, where the pixels are assumed to be spatially uncorrelated or *(ii)* local estimation with 2D model that takes into account the spatial correlations between pixels *(iii)* global 2D estimation obtained from a reduced model, for instance using orthogonal transforms, such as Fourier or SVD.

From a general point of view, estimating heat transfer parameters mapping from a 2D temperature field, without any assumption about the distribution of the properties, is an « ill-posed » problem. Linear estimation, such as Ordinary Least Squares, directly based on the minimization of the prediction error, is often preferred, because this kind of methods are not computationally intensive and have the advantage of yielding a confidence domain for the estimated field. The general problem has only been studied in stationary state (Jones et al, 1995), but such a situation would be quite difficult to be implement experimentally.

Thus it is of interest, for every type of methods (local 1D, local 2D or global 2D) to build some convenient transfer functions between the measured 2D field and the input data, such as the 2D heating flux or boundary conditions. The basic thermal quadupole formalism is a very efficient method for 3D linear heat conduction modeling and calculation, when involved in multilayered systems [Maillet, 2000]. For transient conduction in an homogeneous material, a linear intrinsic transfer matrix is relating the input and output temperature and heat flux after a Laplace transformation and integral space transforms (Fourier or Hankel transforms). The main advantage of this relationship is to make easy the representation of multilayered systems by multiplying the corresponding quadrupole matrices. Recently, a semi-analytical extension was proposed [Fudym, 2002] for heterogeneous stratified media, where the input/output linear relationship are given in a vectorial form between the corresponding fields. This kind of representation can be helpful to analyze the homogenization / constriction heat conduction effects in heterogeneous media and thus deduce simplified models [Fudym, 2004].

In this paper, it is shown how the thermal quadrupole formalism can be used jointly with thermal images processing for the thermal charaterization of heterogeneous media. First, the basic quadrupole formalism is briefly recalled. Some important aspects related to the case of in-plane diffusion and local heat transfer parameters estimation will be further considered. It is shown how some separability considerations derived from the quadrupole approach allow in some particular cases to define a new variable corresponding to a simple 2D estimation, such as vertical cracks detection in thick medium or the solid plate velocity measurement. Finally, a macroscopic two-temperature analytical model is used for microchannels thermal characterization.

2. Local estimation with the assumption of spatially uncorrelated pixels

2.1 One dimensional in-depth heat transfer

In one dimensional experiments each pixel of the IR image is assumed to be spatially uncorrelated, that is independent of the neighbors. Usually in-plane diffusion is avoided by experimental care, such as spatially uniform excitation, thin sample... The transient temperature images can be processed pixel by pixel. Detecting delaminations in thin plates of composite materials by IR thermography has been studied since 1980 (see for example Balageas et al, 1991). Most of the methods associated to delamination detection consists in considering only a 1D heat transfer following the thickness direction, even if 2D or 3D heat transfer corrections have been studied (see Bendada et al, 1998). Two families of excitation methods can then be considered : flash methods or periodic methods. Periodic methods [Wu, 1996] consist in making a periodic thermal excitation at a surface of a sample. The excitation is generally realised with laser or lamps systems. The recording of the images is often synchronised with the transient excitation. A convenient estimation methods of the thermal diffusivity can be implemented by applying a Fourier transform on time and then considering the amplitude variation of the signal, or the phase lag. It can be noticed that the phase lag is non depending on the absolute temperature level and can then be obtained without a calibration of the detectors. When the characteristic frequency of the plate or the thermal diffusivity is unknown, even with thin samples, the flash method can be more advantageous than periodic methods, since the power spectral density of heat source is then spreaded all over the frequencies. Other orthogonal transforms can be considered such as Principal Component Analysis [Marinetti 2004].

The characteristic time of the heating source is related to the characteristic penetration depth of the signal. Therefore, the recording time step of the thermographic device at the pixel scale must be adapted to such penetration depth., and that could be critical for microscales applications.

2.2 In-plane diffusion

At the opposite, some methods based on in-plane diffusion in transient state have been implemented with very restrictive assumptions, for homogeneous samples, or assuming that the sample is locally homogeneous, as for in-depth transfer. They allow to estimate the macroscopic thermal diffusivity of anisotropic samples (Philippi et al, 1995). One of the main difficulties is then to take into account the spatial correlation of the pixels induced by 2D heat transfer. The so-called « flying spot » method (see Gruss and Balageas, 1992, Lepoutre et al, 1993) consists in moving a laser hot spot on the front face of the sample and analysing the transient temperature field around the spot. Such a method is efficient but experimentally difficult (scanning of the domain, implementation of optics and laser techniques…).

Hadisaroyo (Hadisaroyo, 1992) proposed the thermal diffusivity estimation applying the Laplace transform on time and defining the averaged temperature of two chosen areas (A_0 at position x_0 *and* A_1 at position x_1) on a semi-infinite fin, where the heat flux excitation is applied on the edge of the fin, such as shown in Fig. (1).

$$\ln\left(\frac{\tau_{A_0}\left(x_0,s\right)}{\tau_{A_1}\left(x_1,s\right)}\right)^2 = \left[\frac{s}{a}+\frac{2\cdot h}{k\cdot e}\right]\cdot\left(x_1-x_0\right)^2 \qquad (1)$$

where τ_{A_i} is the Laplace tranform of the temperature at position x_i averaged on the corresponding area A_i, k and a are the thermal conductivity and diffusivity respectively, and h is the convective heat losses coefficient.

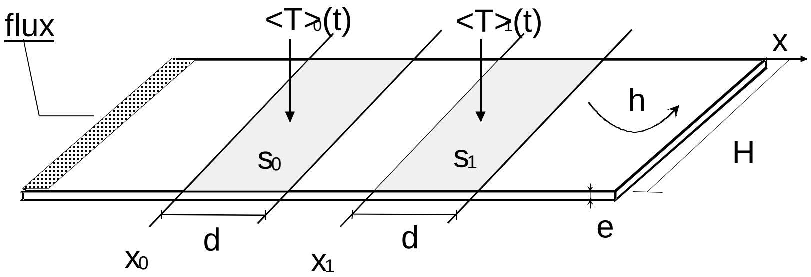


Figure 1. Macroscopic thermal diffusivity estimation of a thin plate from infrared image processing

The main advantage of this method is that the knowledge of the transient heat excitation is not necessary. Moreover, the influence of the measurement noise is statistically reduced due to this kinf of pre-processing by surface averaging and Laplace transform

3. Two dimensional local estimation with the assumption of correlated pixels

Two dimensional velocity and diffusivity mappings from the processing of infrared image sequences with a front flash heating can be performed following an approach similar to the optical flow method used in computer vision [Jähne,1998]. The practical implementation consists in creating a spatially random initial temperature distribution by putting a mask in front of the "Flash". As shown in this section, the sensitivity matrices are built directly with spatial and time correlations between the pixels, such as discrete derivatives or Laplacian approximations. Spatially random heating is used to optimize in-plane diffusion, in order to avoid as much as possible that these sensitivity coefficients tend to zero, making the sensitivity matrices rank deficient and ill-conditioned. Moreover, a necessary condition for the Ordinary Least Square approach to be the minimum unbiased estimator is that the sensitivity matrix is noise free. The effect of getting noisy experimental data as sensitivity coefficients yields a bias, especially where the sensitivity coefficient tend to zero, that is why random heating is preferred to periodic heating [Batsale, 2004].

We consider now the following 2D governing equation:

$$\rho c v_x \frac{\partial T}{\partial x} + \rho c v_y \frac{\partial T}{\partial y} + \rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right). + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + g - h\left(T - T_\infty\right) \quad (2)$$

Clearly Eq. (2) applies at every pixel position on the measurement surface, and it is apparent that the velocity, thermal diffusivity, thermal conductivity gradient, heat source term as well as the convective heat losses coefficient can be estimated locally from the computation of the spatial and time first derivatives and Laplacian operator applied to the temperature signal. Obviously, for an heterogeneous medium, the thermal conductivity derivative and velocity are corraleted, thus velocity can be estimated only where the thermal conductivity is constant.

3.1 Thin medium: two dimensional heat transfer

The thickness of the sample is assumed to be small compared to the in-depth diffusion time, so that the temperature is assumed to be uniform in the direction, and the sample is seen as a fin. The governing equations are given as

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right). + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) - \frac{2h}{e}.\left(T - T_\infty\right) \quad (3)$$

The experiment is shown in Fig. (2). The discretization of Eq. (3), yields:

$$\mathbf{T^{t+\Delta t}} - \mathbf{T^t} = \mathbf{A}.*\mathbf{\Delta T^t} + \delta_x \mathbf{A}.*\delta_x \mathbf{T^t} + \delta_y \mathbf{A}.*\delta_y \mathbf{T^t} - H(\mathbf{T^t} - T_\infty) \quad (4)$$

where the discretized temperatures are arranged in a vectorial form, and **A** is the thermal diffusivity matrix.

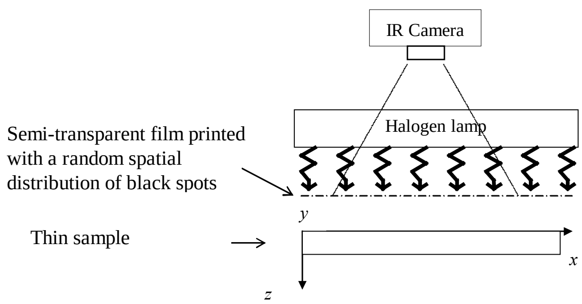


Figure 2. Spatially random Flash heating

The estimation of the vector of parameters corresponding to the thermal diffusivity field is obtained from Eq. (4) by a linear least square approach, by considering the minimization of the least square functional *S* such as:

$$S = \left(\left(\hat{\mathbf{T}}^{\mathbf{t}+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}} \right) - \hat{\mathbf{X}}\boldsymbol{\beta} \right)^t \left(\left(\hat{\mathbf{T}}^{\mathbf{t}+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}} \right) - \hat{\mathbf{X}}\boldsymbol{\beta} \right) \quad (5)$$

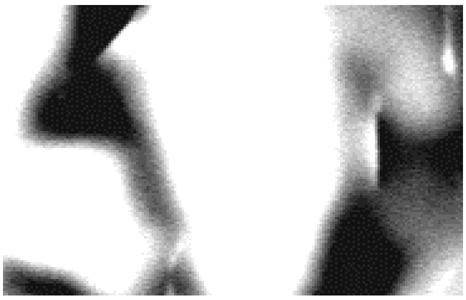
with the vector of unknown parameters $\boldsymbol{\beta} = \begin{bmatrix} \mathbf{A} & \delta_x \mathbf{A} & \delta_y \mathbf{A} & H \end{bmatrix}^t$ and $\hat{\mathbf{X}}$ the corresponding sensitivity matrix directly filled with the shift and difference operations applied on the images:

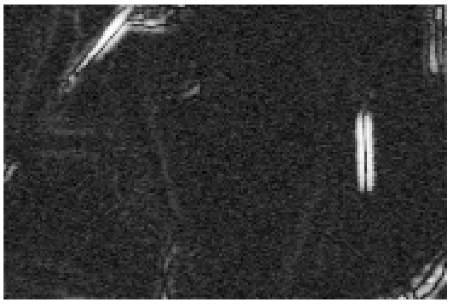
$$\hat{\mathbf{X}} = \begin{bmatrix} \Delta\hat{\mathbf{T}}^{\mathbf{t}_0} & \delta_x \hat{\mathbf{T}}^{\mathbf{t}_0} & \delta_y \hat{\mathbf{T}}^{\mathbf{t}_0} & \hat{\mathbf{T}}^{\mathbf{t}_0} - T_\infty \\ . & . & . & . \\ \Delta\hat{\mathbf{T}}^{\mathbf{t}} & \delta_x \hat{\mathbf{T}}^{\mathbf{t}} & \delta_y \hat{\mathbf{T}}^{\mathbf{t}} & \hat{\mathbf{T}}^{\mathbf{t}} - T_\infty \\ . & . & . & . \end{bmatrix} \quad (6)$$

The Ordinary Least Square estimator is:

$$\hat{\boldsymbol{\beta}}_{OLS} = \left(\hat{\mathbf{X}}' \hat{\mathbf{X}} \right)^{-1} \hat{\mathbf{X}}' \left(\hat{\mathbf{T}}^{\mathbf{t}+\Delta\mathbf{t}} - \hat{\mathbf{T}}^{\mathbf{t}} \right) \quad (7)$$

In Fig. (3) are shown both the initial temperature field and the thermal diffusivity mapping of an aluminium thin plate obtained from Eq. (7) for an experimental infrared image sequence by spatially random heating. The two cracks are clearly visible on the map.





Figue 3. Detection of cracks in a thin Aluminium plate: Initial temperature field and diffusivity map

3. 2 Three dimensional structure

The general problem of estimating the 2D superficial heat transfer parameters mapping from the superficial temperature field in a three dimensional heat transfer problem is a quite ill-posed problem, due to the dependance of the solution to the thickness coordinate variable. It is shown in this section how this kind of problem can be solved by the previous 2D approach when the separability of the general solution can be demonstrated. The separability of the temperature field resulting from some specific three dimensional heat transfer problems has been investigated with the quadrupole formalism approach for heterogeneous stratified media [Fudym, 2004].

In some particular case, especially for semiinfinite media in the direction *z*, the separability of the temperature field can be demonstrated, and written as

$$T(x,y,z,t) = T_{x,y}(x,y,t) \cdot T_z(z,t) \quad (8)$$

Then a new pseudo-signal is defined such as

$$T_{x,y}(x,y,t) = \frac{T(x,y,z=0,t)}{T_z(0,t)} \quad (9)$$

This 2D fictive temperature $T_{x,y}(x,y,t)$ is governed by the corresponding 2D equation, such as Eq. (2), and the previous estimation method applies. The main difference is that the covariance matrix of the measurement error is modified when the temperature field is divided by $T_z(0,t)$. This approach has been implemented for the detection of cracks in a semiinfinite medium [Fudym, 2004]. The Maximum likelihood Estimator is used, since in that case the pseudo signal is obtained by dividing the temperature by the square root of time ($T_z(0,t) = C/\sqrt{t}$):

$$\hat{\boldsymbol{\beta}}_{ML} = \left(\hat{\mathbf{X}}'\mathbf{t}^{-1}\hat{\mathbf{X}}\right)^{-1}\hat{\mathbf{X}}'\mathbf{t}^{-1}\left(\hat{\mathbf{Y}}^{\mathbf{t+\Delta t}} - \hat{\mathbf{Y}}^{\mathbf{t}}\right) \qquad (10)$$

More recently, the same approach has been used for the estimation of both the thermal diffusivity and velocity mapping [Bamford, 2006] of a moving solid, as shown in Fig. (4). A diffusing pattern obtained by the flash heating pulse through a 2D mask is observed by the infrared imaging system. Velocity of the moving solid is responsible for the displacement of the grid, while thermal diffusion within the solid yields a diffuse behavior of the initial pattern. A Total Least Square (TLS) approach is implemented in order to take into account the effect of noise in the sensitivity matrix. The main difficulty of the TLS method is to determine a threshold for the dimension of the space where the solution has to be looked for. Interestingly, this threshold can be used for defining a confidence domain where the parameters can be reasonably identified.

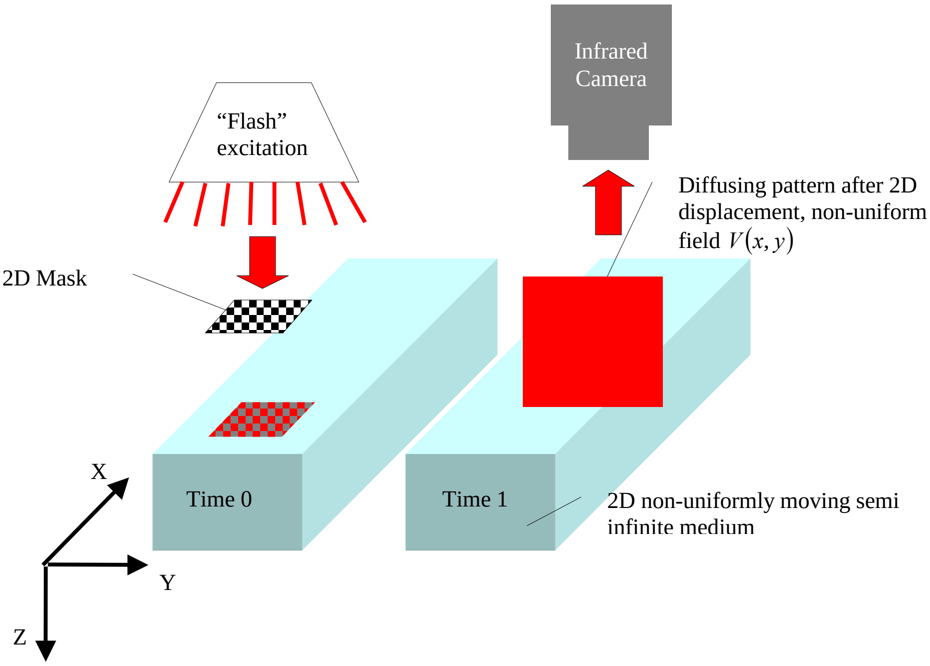


Figure 4. Velocity and thermal diffusivity map of a moving solid

Some experimental results obtained for a moving expansed polyvinyl chloride sample are shown in Figs. (5) and (6). In Fig. (5) are shown the initial and final images of the sequence obtained for a 25 Hz recording rate. Both displacement and diffusion of the initial pattern is clearly apparent on the second image. On Fig. (6) are plotted the thermal diffusivity and velocity maps

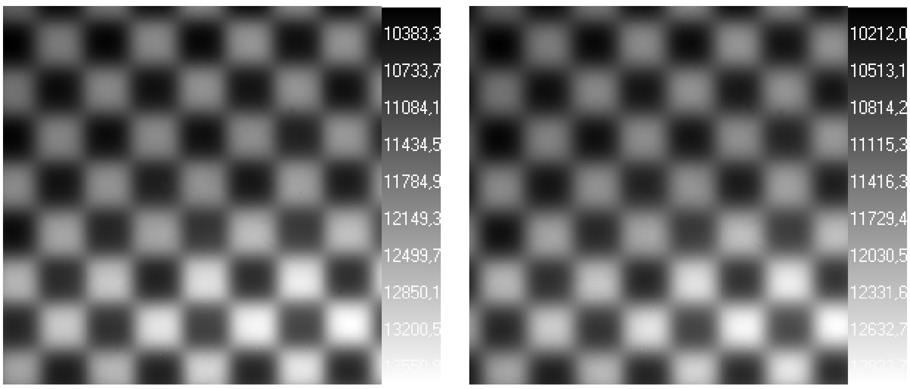


Figure 5. First and last images of an infrared sequence showing a moving and diffusing pattern

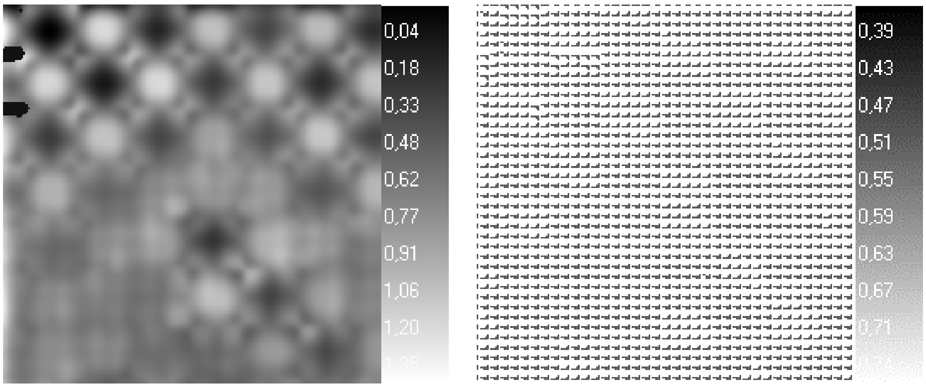


Figure 6. Diffusivity and velocity maps obtained from previous sequence

4. Macroscopic estimation obtained from averaged temperature fields

An other approach is presented in this section. When the investigated systems are small in relation with the spatial resolution of the infrared imaging system, a macroscopic approach is implemented. We propose herein an example of thermal characterization of a microchannel reactor, where the microchannel width has the same order of magnitude than the pixel size. The main idea is that the macroscopic gradients within the microreactor plate contain the pertinent information related to the heat source term within the microchannel. An analytical two-temperature model, derived from the thermal quadrupole approach is used in order to compute the average fields. The microsystem is shown in Fig. (7). The microchannel is filled with water. An electrical resistive heating film is used in order to simulate a step heating source distribution. The spatial resolution of the infrared camera is about 200 µm, while the microchannel width is 250 µm. The PDMS resin is assumed to be a perfect thermal insulator relatively to the glass cover. Hence heat transfer is mostly done within the microchannel and glass, and the glass cover is considered as a fin, due to the convective heat losses in the front wall.

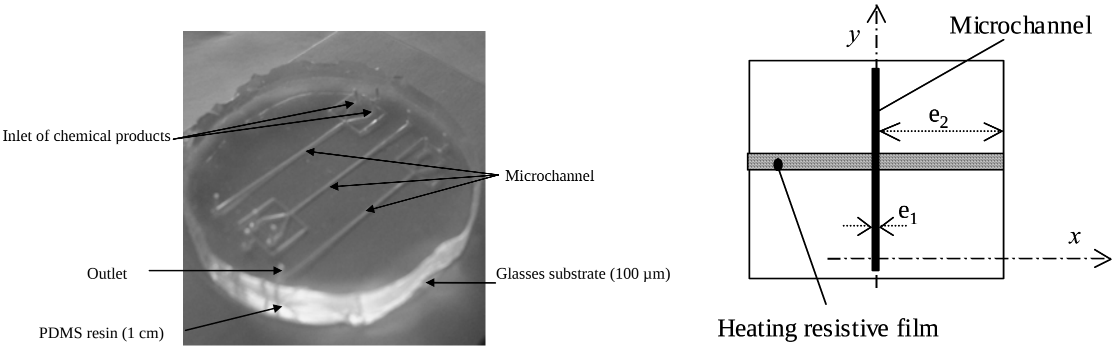


Figure 7. Picture of a microfluidic chip ; Top view of microchannel with heating resistive film.
The temperature is spatially averaged in the direction *x* along e_1 and e_2 respectively

The governing equations can be solved analytically by the quadrupole formalism, and the averaged temperatures, in the transformed space (here a Sinus Fourier transform applied in the *y* direction), are shown to be expressed by the Two temperature model such as

$$k_1\beta_1^2 e_1 <\theta_1> = G_1 e_1 - \frac{1}{Z}\left(<\theta_1> - <\theta_2>\right) \tag{11a}$$

$$k_2\beta_2^2 e_2 <\theta_2> = G_2 e_2 + \frac{1}{Z}\left(<\theta_1> - <\theta_2>\right) \tag{11b}$$

$$\beta_i^2 = \alpha_n^2 + H_i \tag{11c}$$

where β_i (i = 1,2) is a generalized frequency depending on the Fourier transform variable α_n and the convective heat transfer coefficient H_i ; $<\theta_i>$ is the Fourier transform of the temperature, spatially averaged on the width e_i, and only depending on *y* ; the thermal impedance *Z* is known analytically. G_i is the tranformed heat source term - here the heat source dissipated by the heating resistive film as shown in Fig. (7). Equations (11) can be represented as a two nodes problem by the analogical network depicted in Fig. (8).

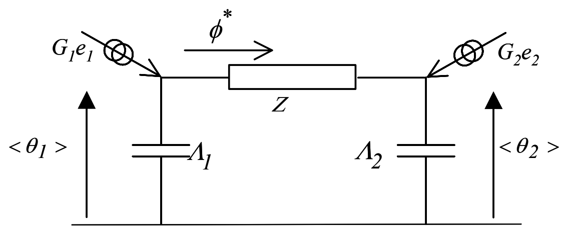


Figure 8. Analogical network relative to Eqs. (11), involving average temperatures

This model can be used to fit the experimental averaged temperatures obtained from the thermal images, in order to estimate the convective heat transfer coefficients H_i. and then retrieved the heat source term. Then, adding Eq. (11a) to Eq. (11b), and applying the inverse Fourier transform to the result yields in the real space the following equation, applied to the thermal field Y in order to recover the source term distribution :

$$-k_1e_1\frac{\partial^2 <Y_1>}{\partial y^2}-k_2e_2\frac{\partial^2 <Y_2>}{\partial y^2}+H_1k_1e_1<Y_1>+H_2k_2e_2<Y_2>=g_1e_1+g_2e_2 \qquad (12)$$

Equation (12) can be used directly on the thermal image in order to retrieve the heat source term distribution, as shown in Fig. (9), where the retrieved heat source is compared with the known distribution due to the heating resistive film.

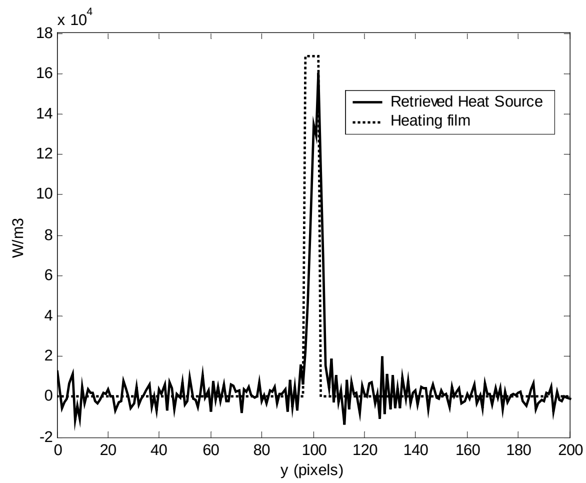


Figure 9. Retrieved heat source computed from Eq. (12) applied on the experimental field

A similar macroscopic approach can be used for the thermomechanical study of granular or dispersed media under mechanical stress. The problem is then to estimate the interfacial thermal contact resistances between grains within the granular medium, looking for the relationship with the applied mechanical stress. The temperature is averaged on each grain, and a characteristic frequency proportional to the thermal contact resistance is estimated. In Fig. (10) are shown some experimental results obtained with 1mm and diameter lead spheres. A periodic step heating is applied on the left sphere by two Constantan resistive wires inserted in the grain. The estimation is obtained from the following equation:

$$\frac{dT_i}{dt}=\sum_{j=1}^{j=3}\left[\frac{1}{\tau_{ij}}(T_j-T_i)\right]-\frac{1}{\tau_i}T_i \qquad (13)$$

where τ_{ij} are characteristic times corresponding to the interface of the grain *(i)* with the neighbor *(j)*.

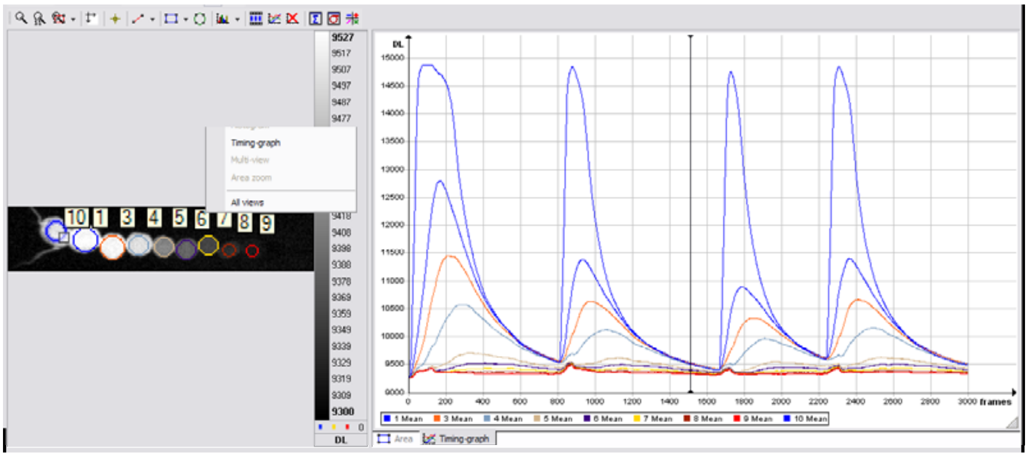


Figure 10. Periodic heating of stressed lead grains

5. Conclusion and perspectives

In this paper, a few recent developments for infrared images processing devoted to heat transfer parameters mapping have been presented. The widespread local 1D in-depth approach, usually implemented for Non Destructive Thermal Evaluation, can be adapted for quantitative thermal properties mapping, but with experimental limitations due to the necessary scanning pixel by pixel of the heating source. Local 2D estimations derived from the optical flow methods used in computer vision are quite efficient when coupled with the OLS or TLS linear estimators, but are suitable for in plane diffusion of 2D structures. In some particular cases, the 3D structures can be adressed by the same family of methods when the 3D temperature field is shown to be separable, by defining a new pseudo 2D signal to be processed. Estimating thermal mapping for this kind of structures in the general case is still an open problem, where a convenient relationship between the superficial field and the parameters has to be found. Reducing the problem by using orthogonal transforms such as SVD for a global estimation method could be an alternative. When the spatial resolution of the camera has the same order of magnitud as the studied sample, an different approach is to use a macroscopic approach coupled with analytical averaged models.

6. Acknowledgement

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