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Numerical modeling of the heat transfer in the turbulent boundary layer

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Abstract. *The main goal of this work is the numerical modeling of the forced convective turbulent heat flux , for the turbulent incompressible flow of air over a flat plate, with unheated starting lengths, for a variety of four step positions at Reynolds number up to 5.6 x* 10^6*. The numerical algorithm applied a consolidate Reynolds and Favre averaging process for the turbulent variables. The turbulence model is the classical* $\kappa - \epsilon$*. The turbulent inner layer is modeled by velocity wall laws and temperature wall laws. The remaining non-linearities, due to laws of wall, are treated by a minimal residual method. The numerical results for heat transfer, velocity, temperature, hydrodynamic and thermal boundary layers are compared with experimental data, in addition the numerical results are also compared with empirical correlation results. The Reynolds number range of the modeled flows, based on the length of flat plate is placed between* $10^5 < Re < 10^7$*. Two different physical cases are analyzed, in the first one the velocity and the temperature field are uncoupled. In the second case the velocity and the temperature field are strongly coupled.*

keywords: *Convection, Flat plate, turbulence, wall laws, turbulence models, finite elements.*

1. Introduction

The objective of this work is to analyze the efficiency of the methodology used by the solver Turbo2D in the modeling of the forced convection heat transfer in turbulent boundary layers, with and without the coupling between the velocity and the temperature fields.

To make these analysis two distinct physical situations, in the point of view of the relation between the velocity and temperature fields, were chosen. In the flows studied experimentally by Reynolds et al. (1958) and by Taylor et al. (1990) the thermal and hydrodynamic boundary layers are clearly uncoupled. In the Ng (1981) test case a strong dependency between both fields is observed.

The extension and the complexity of the coupling existing between the turbulent fields of momentum and energy depends on the intensity of the velocity, pressure and temperature gradients involved, on the geometry of the physical solid boundary of the flow and also, on the thermodynamic behavior of the fluids involved in the process. The degree of difficulty and the computational cost of the numerical modeling of the problem are directly related to existing coupling degree between the turbulent fields of momentum and energy.

The test cases of Reynolds et al. (1958) and of Taylor et al. (1990) represents forced, turbulent and incompressible flows of air over horizontal, smooth, flat plates. The temperature fields are specified so that the beginning of the thermal and the hydrodynamic boundary layers are separated by an unheated starting length ξ.

There is an adiabatic length in the beginning of each plate, created by the equality of the temperatures of the wall and the flow. The thermal boundary layer begins in the point were both temperatures are differentiated, originating a heat flux between the wall and the flow. In all the studied situations the thermal boundary layer is developed over isothermal walls.

The turbulent air flows studied by Reynolds et al. (1958) and by Taylor et al. (199), occurs with such velocities, that so in the faster situation, creates Mach numbers under 0,2. The geometry in the solid boundary of the flow is incapable to provide the detachment of the hydrodynamic boundary layer or changing the curvatures of the fluid movement. Under these conditions, the existing pressure gradients are of low intensity. The higher differences of temperature imposed are of 18 K, this means that the heated plate can not change in a significant way the specific mass, the thermal conductivity and the dynamic viscosity of the air.

In the absence of significant pressure and temperature gradients, it is possible the adoption of the hypothesis that the thermodynamic proprieties of the fluid are constants, this condition is capable to break the coupling existing between the equations of conservation of momentum and energy and to make linear the conservation equation of energy.

The system of governing equations, simplified by the non variance of the thermodynamic proprieties, admits analytical solution and, in the computational point of view, allows the use of less expensive computational algorithms of numerical resolution.

Even with the restriction represented by the hypothesis of constant values for the fluid thermodynamic proprieties, many problems of technological interest can be modeled with this formulation, being distinguished because of its recent importance, the problem created by the necessity of cooling plates used in electronic circuits used in the computational area.

In the Ng (1981) test case, a turbulent flow of air at 293 K, totally developed in a wind tunnel is conduced to an horizontal smooth flat plate, heated up to 1250 K. The great difference between the temperature of the wall and of the flow, produces important variations in the specific mass, in the dynamic viscosity, in the specific heats and in the thermal conductivity of the fluid. The variation of those proprieties with temperature is responsible by the coupling between the equations of conservation of momentum and energy and by the non linear behavior of the energy conservation equation.

The solver Turbo2D is a program of research, that is been developed in the Group of Complex Fluid Dynamics - Vortex, of the Mechanical Engineering Department of the University of Brasília. This solver is based on the adoption of the finite elements technique, under the formulation of pondered residuals proposed by Galerkin, adopting in the spatial discretization of the calculus dominium the triangular elements of the type P1-isoP2, as proposed by Brison, Buffat, Jeandel and Serres (1985).

Considering the uncertainties normally existing in the initial conditions of the problems that are numerically simulated, it is adopted the temporal integration of the governing equations system. In the temporal integration process the initial state corresponds the beginning of the flow, when velocity and pressure fields are considered nulls. The end of the process occurs when the temporal variations of the velocity, pressure, temperature and other turbulent variables stop. The temporal discretization of the system of the governing equations, implemented by the algorithm of Brun (1988), uses sequential semi-implicit finite differences, with truncation error of order $0(\Delta t)$ and allows to make linear the equation system at each step of time.

The resolution of the coupled equations of continuity and momentum is done by a variant of Uzawa´s algorithm proposed by Buffat (1981). The statistical formulation, responsible for the obtaining of the system of average equations, is done with the simultaneous usage of the Reynolds (1895) and Favre (1965) decomposition. The Reynolds stress of turbulent tensions is calculated by the $\kappa - \varepsilon$ model, proposed by Jones and Launder (1972) with the modifications introduced by Launder and Spalding (1974). The turbulent heat flux is modeled algebraically using the turbulent Prandl number with a constant value of 0,9.

In the program Turbo2D, the boundary conditions of velocity and temperature can be calculated by four velocity and two temperature wall laws. The wall laws used in this work are of velocity are: Mellor wall law (1966), Nakayama and Koyama wall law (1984), the classic logarithm wall law and the velocity wall law of Cruz and Silva Freire (1998). The temperature wall laws available are: the Cheng and Ng (1982) and the Cruz and Silva Freire temperature wall law. In this work, as the pressure gradients are very low, we used only the logarithm wall law for velocity and the Cheng and Ng law for temperature. The numerical instability resulted of the explicit calculus of the boundary conditions of velocity, trough the evolutive temporal process, is controlled by the algorithm proposed by Fontoura Rodrigues (199). The numerical oscillations induced by the Galerkin formulation, resulting of the centered discretization applied to a parabolic phenomenon, that is the modeled flow, are cushioned by the technique of balanced dissipation, proposed by Huges and Brooks (1979) and Kelly, Nakazawa and Zienkiewicz (1976) with the numerical algorithm proposed by Brun (1988).

In order to quantify the wideness of range and the consistence of the numerical modeling done by the solver Turbo2D, the parietal heat fluxes obtained numerically are compared to the experimental data of Reynolds et al. (1958), Taylor et al. (1990) and Ng (1981), and also to empirical correlations.

2. Governing Equations

The system of non-dimensional governing equations, for a dilatable and one phase flow, without internal energy generation, and in a subsonic regime is:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial \underline{p}}{\partial x_i} + \frac{1}{Re}\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] - \frac{2}{3Re}\frac{\partial}{\partial x_j}\left(\mu\frac{\partial u_k}{\partial x_k}\delta_{ij}\right) + \frac{1}{Fr}\rho\frac{g_i}{\|g\|}, \tag{2}$$

$$\frac{\partial(\rho T)}{\partial t} + \frac{\partial(\rho u_i T)}{\partial x_i} = \frac{1}{RePr}\frac{\partial}{\partial x_i}\left(k\frac{\partial T}{\partial x_i}\right) \tag{3}$$

$$\rho(T+1) = 1. \tag{4}$$

In this system of equations ρ is the fluid specific mass, t is the time, x_i are the space cartesian coordinates in tensor notation, μ is the dynamic viscosity coefficient, δ_{ij} is the Kronecker delta operator, g_i is the acceleration due to gravity in tensor notation, T is the temperature, τ_{ij} is the viscous stress tensor in tensor notation, γ is the ratio of specific heats, k is the thermal conductivity, Re is the Reynolds number, Fr is the Froud number, Pr is the Prandtl number, M_o is the Mach number, and the non dimensional components of pressure are:

$$\underline{p} = \frac{p - p_m}{\rho_o U_o^2} \qquad \text{and} \qquad \widehat{p} = \frac{p}{p_o} \tag{5}$$

More details about the dimensionless process are given by Brun (1988). In order to simplify the notation adopted, the variables in their dimensionless form have the same representation as the dimensional variables. The Reynolds, Prandtl, Froude and Mach numbers are defined with the reference values adopted in this process.

2.1. The Turbulence Model

In this work all the dependent variables of the fluid are treated as an average value plus a fluctuation of this variable in a determinate point of space. In order to account variations of specific mass, the model used applies the well known Reynolds (1985) decomposition to pressure and fluid density and the Favre (1965) decomposition to velocity and temperature. In the Favre (1965) decomposition a randomize generic variable φ is defined as:

$$\varphi(\vec{x},t) = \widetilde{\varphi}(\vec{x}) + \varphi''(\vec{x},t) \qquad \text{with} \qquad \widetilde{\varphi} = \frac{\overline{\rho\varphi}}{\bar{\rho}}\ . \tag{6}$$

Applying the Reynolds (1895) and Favre (1965) decompositions, to the governing equations, and taking the average value of those equations, we obtain the mean Reynolds equations:

$$\frac{\partial\overline{\rho}}{\partial t} + \frac{\partial}{\partial x_i}(\overline{\rho}\widetilde{u}_i) = 0, \tag{7}$$

$$\frac{\partial}{\partial t}(\overline{\rho}\widetilde{u}_i) + \frac{\partial}{\partial x_j}(\overline{\rho}\widetilde{u}_j\widetilde{u}_i) = -\frac{\partial\overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\overline{\tau_{ij}} - \overline{\rho u_j'' u_i''}\right] + \overline{\rho}g_i, \tag{8}$$

where

$$\overline{\tau_{ij}} = \mu\left[\left(\frac{\partial\widetilde{u}_i}{\partial x_j} + \frac{\partial\widetilde{u}_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial\widetilde{u}_l}{\partial x_l}\delta_{ij}\right], \tag{9}$$

$$\frac{\partial(\overline{\rho}\widetilde{T})}{\partial t} + \frac{\partial(\widetilde{u}_i\widetilde{T})}{\partial x_i} = \frac{\partial}{\partial x_i}\left(\alpha\frac{\partial\widetilde{T}}{\partial x_i} - \overline{\rho}\overline{u_i'' T''}\right) \tag{10}$$

$$\overline{p} = \overline{\rho}R\widetilde{T} \tag{11}$$

In these equations α is the molecular thermal diffusivity and two news unknown quantities appear in the momentum (8) and in the energy equation (10), defined by the correlations between the velocity fluctuations, the so-called Reynolds Stress, given by the tensor $-\overline{\rho}\overline{u_i'' u_j''}$ and by the fluctuations of temperature and velocity, the so-called turbulent heat flux, defined by the vector $-\overline{\rho}\overline{u_i'' T''}$.

The Reynolds stress of turbulent tensions is calculated by the $\kappa - \varepsilon$ model, proposed by Jones and Launder (1972) with the modifications introduced by Launder and Spalding (1974), where

$$-\overline{\rho}\overline{u_i'' u_j''} = \mu_t\left(\frac{\partial\widetilde{u}_i}{\partial x_j} + \frac{\partial\widetilde{u}_j}{\partial x_i}\right) - \frac{2}{3}\left(\overline{\rho}\kappa + \mu_t\frac{\partial\widetilde{u}_l}{\partial x_l}\right)\delta_{ij}, \tag{12}$$

with

$$\kappa = \frac{1}{2}\overline{u_i'' u_i''}. \tag{13}$$

and

$$\mu_t = C_\mu \bar{\rho} \frac{\kappa^2}{\varepsilon} = \frac{1}{Re_t} \quad . \tag{14}$$

The turbulent heat flux is modeled algebraically using the turbulent Prandl number Pr_t equal to a constant value of 0,9 by the relation

$$-\bar{\rho}\overline{u_i'' T''} = \frac{\mu_t}{Pr_t}\frac{\partial \widetilde{T}}{\partial x_i}. \tag{15}$$

In the equation (14) C_μ is a constant of calibration of the model, that values $0,09$, κ represents the turbulent kinetic energy and ε is the rate of dissipation of the turbulent kinetic energy. Once that κ and ε are additional variables, we need to know there transport equations. The transport equations of κ and ε were deduced by Jones and Launder (1972), and the closed system of equations to the $\kappa - \varepsilon$ model is given by:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho}\widetilde{u}_i}{\partial x_i} = 0 \ , \tag{16}$$

$$\frac{\partial\left(\bar{\rho}\widetilde{u}_i\right)}{\partial t} + \widetilde{u}_j\frac{\partial\left(\bar{\rho}\widetilde{u}_i\right)}{\partial x_j} = -\frac{\partial \bar{p^*}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\left(\frac{1}{Re}+\frac{1}{Re_t}\right)\left(\frac{\partial \widetilde{u}_i}{\partial x_j}+\frac{\partial \widetilde{u}_j}{\partial x_i}\right)\right] + \frac{1}{Fr}\bar{\rho}g_i \ , \tag{17}$$

$$\frac{\partial\left(\bar{\rho}\widetilde{T}\right)}{\partial t} + \widetilde{u}_j\frac{\partial\left(\bar{\rho}\widetilde{T}\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{1}{Re\,Pr}+\frac{1}{Re_t\,Pr_t}\right)\frac{\partial \widetilde{T}}{\partial x_j}\right] \ , \tag{18}$$

$$\frac{\partial\left(\bar{\rho}\kappa\right)}{\partial t} + \widetilde{u}_i\frac{\partial\left(\bar{\rho}\kappa\right)}{\partial x_j} = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{Re}+\frac{1}{Re_t\,\sigma_\kappa}\right)\frac{\partial \kappa}{\partial x_i}\right] + \Pi - \bar{\rho}\varepsilon + \frac{\bar{\rho}\beta g_i}{Re_t\ Pr_t}\frac{\partial \widetilde{T}}{\partial x_i} \ , \tag{19}$$

$$\begin{aligned}\frac{\partial\left(\bar{\rho}\varepsilon\right)}{\partial t} + \widetilde{u}_i\frac{\partial\left(\bar{\rho}\varepsilon\right)}{\partial x_j} &= \frac{\partial}{\partial x_i}\left[\left(\frac{1}{Re}+\frac{1}{Re_t\,\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right] \\ &+\frac{\varepsilon}{\kappa}\left(C_{\varepsilon1}\Pi - C_{\varepsilon2}\bar{\rho}\varepsilon + C_{\varepsilon3}\frac{\bar{\rho}\beta g_i}{Re_t\ Pr_t}\frac{\partial \widetilde{T}}{\partial x_i}\right) \ ,\end{aligned} \tag{20}$$

$$\bar{\rho}\left(1+\widetilde{T}\right) = 1 \ , \tag{21}$$

where:

$$\frac{1}{Re_t} = C_\mu \bar{\rho}\frac{\kappa^2}{\varepsilon} \ , \tag{22}$$

$$\Pi = \left[\left(\frac{1}{Re_t}\right)\left(\frac{\partial \widetilde{u}_i}{\partial x_j}+\frac{\partial \widetilde{u}_j}{\partial x_i}\right) - \frac{2}{3}\left(\bar{\rho}\kappa + \frac{1}{Re_t}\frac{\partial \widetilde{u}_l}{\partial x_l}\right)\delta_{ij}\right]\frac{\partial \widetilde{u}_i}{\partial x_j} \ , \tag{23}$$

$$p^* = \bar{p} + \frac{2}{3}\left[\left(\frac{1}{Re}+\frac{1}{Re_t}\right)\frac{\partial \widetilde{u}_l}{\partial x_l} + \bar{\rho}\kappa\right] \ , \tag{24}$$

with the model constants given by:

$$C_\mu = 0,09 \ , \ C_{\varepsilon1} = 1,44 \ , \ C_{\varepsilon2} = 1,92 \ , \ C_{\varepsilon3} = 0,288 \ , \ \sigma_\kappa = 1 \ , \ \sigma_\varepsilon = 1,3 \ , \ Pr_t = 0,9 \ .$$

2.2. Near Wall Treatment

The $\kappa - \epsilon$ model is incapable of properly representing the fluid behavior in the laminar sub-layer, in the buffer sub-layer and in the beginning of the logarithmic regions of the turbulent boundary layer. To solve this inconvenience, the standard solution is the use of wall laws, capable of properly representing the flow in the inner region of the turbulent boundary layer. There are four velocity and two temperature laws of the wall implemented on Turbo2D code. In this work, considering that no significative pressure gradients are involved, are used the logarithm law for velocity. The logarithm law of the wall for velocity is already well known, and further explanations are unnecessary.

For the near wall temperature, Cheng and Ng (1982) derived an expression similar to loga-rithmic law of the wall for velocity. For the numerical calculation purposes, the intersection point between laminar and logarithmic sub-layers are defined at $y^* = 15,96$, with $y^* = u_f\delta/\nu$, where u_f is the friction velocity calculated by the relation

$$u_f = \left[\left(\frac{1}{Re}+\frac{1}{Re_t}\right)\frac{\partial \widetilde{u}}{\partial x_j}\right]_\delta , \tag{25}$$

ν is the kinetic viscosity and δ is the distance until the wall. The temperature wall laws for laminar and logarithmic sub-layers are respectively

$$\frac{(T_0 - T)_y}{T_f} = y^* \; Pr \quad \text{and} \quad \frac{(T_0 - T)_y}{T_f} = \frac{1}{K_{Ng}} ln \; y^* + C_{Ng} \;\; , \tag{26}$$

where T_0 is the environmental temperature and T_f is the friction temperature, defined by the relation

$$T_f u_f = \left[\left(\frac{1}{Re \, Pr} + \frac{1}{Re_t \, Pr_t} \right) \frac{\partial \widetilde{T}}{\partial x_j} \right]_\delta . \tag{27}$$

In the equation (26) the constants K_{Ng} and C_{Ng} are, respectively, 0,8 and 12,5. The turbulent Prandtl number Pr_t is assumed constant and equal to 0,9.

For the turbulent kinetic energy κ and for the rate of dissipation of the turbulent kinetic energy ε, the near wall values are taken by the following relations

$$\kappa = \left[\frac{{u_f}^2}{\sqrt{C_\mu}} \right]_\delta \quad \text{and} \quad \varepsilon = \left[\frac{{u_f}^3}{K \delta} \right]_\delta , \tag{28}$$

with $K = 0,419$.

2.3. The Stanton number

In this work the parietal heat flux is estimated in a non dimension form by using the local Stanton number, calculated numerically by two distinct manners. The first one is based on a classical way to turn the local parietal heat flux q_x in a dimensionless form:

$$St_x = \frac{q_x}{\rho c_p u_\infty (T_w - T_\infty)} \qquad \text{where, for a flat plate} \qquad q_x = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} . \tag{29}$$

In the equation above, the numerical method to compute the temperature gradient was by taking the temperature difference between the first node of our mesh and the temperature in the wall, and dividing it by the distance of the first node and the wall. This procedement was made, because the solver uses wall laws, and by this reason our calculus does not goes until the wall.

Another way to compute the local Stanton number is derived from a diversification of the Reynolds analogy made by Colburn (1933), for fluids with the Prandtl number equal or larger than $0,5$. The Colburn (1933) empirical correlation establish a relationship between the local Stanton number St_x, the local friction coefficient C_{fx} and the Prandtl number Pr:

$$St_x = \frac{C_{fx}}{2 Pr^{\frac{2}{3}}} . \tag{30}$$

The second form to calculate the local Stanton number used in this work, proposed by Kays and Crawford (1993), is an improvement of the equation (30) by the use of the $\frac{1}{7}$ power law for temperature and velocity profiles, where δ_u and δ_T are, respectively, the thickness of the velocity and temperature boundary layers,

$$St_x = \frac{C_{fx}}{2 Pr^{\frac{2}{5}}} \left(\frac{\delta_u}{\delta_T} \right)^{\frac{1}{7}} . \tag{31}$$

In the equation above, the numerical values for δ_u and δ_T were obtained by taking the heights of 300 points in the direction of the flow, with velocities equal to $0,99 u_\infty$ for the hydrodynamic boundary layer, and temperatures equal to $T_w - 0,99(T_w - T_\infty)$ for the thermal boundary layer.

To evaluate the accuracy of the numerical results is possible to determine the local Stanton number by using the empirical correlation for the unheated starting length problem over flat plate:

$$St_x = \frac{1}{2} \frac{C_{fx}}{Pr^{\frac{2}{5}}} \left[1 - \left(\frac{\xi}{x} \right)^{0,9} \right]^{-\frac{1}{9}} . \tag{32}$$

To calculate the local friction coefficient C_{fx} used in equation (31), it was used the numerical value of the local friction velocity calculated by the Turbo2D

$$\frac{C_{fx}}{2} = \frac{\tau_w}{\rho u_\infty^2} \qquad \text{with} \qquad \tau_w = \rho u_f^2 \qquad \text{so} \qquad \frac{C_{fx}}{2} = \frac{u_f^2}{u_\infty^2} . \tag{33}$$

To calculate the local friction coefficient C_{fx} used in the empirical correlation, equation (32), is employed the empirical value for turbulent boundary layer

$$\frac{C_{fx}}{2} = 0.0287 Re_x^{-\frac{1}{5}}. \tag{34}$$

3. Numerical methodology

The numerical solution of a dilatable turbulent flow, has as main difficulties: the coupling between the pressure, velocity and temperature fields; the non-linear behavior of the momentum and energy equations; the explicit calculations of boundary conditions in the solid boundary; the methodology of use the continuity equation as a manner to link the coupling fields of velocity and pressure.

The solution proposed in the present work suggests a temporal discretization of the system of governing equations with a sequential semi-implicit finite difference algorithm proposed by Brun (1988) and a spatial discretization using finite elements of the type P1-isoP2. The temporal and spatial discretization implemented in Turbo 2D is presented in Fontoura Rodrigues (1990).

4. Numerical Results

All the studied experimental test cases consisted in an horizontal heated flat plate, witch receives air in a parallel direction. The difference between the cases studied are in the value of properties such as the free stream velocity of the flow, the size and temperature of the plate, and the size of the unheated starting length. The physical parameter compared from the numerical simulation to experimental data is basically the local Stanton number. The numerical value of the local Stanton number was calculated of two different ways, by using the equations (29) called "numerical 1", and using equation (31) called "numerical 2".

The Reynolds et al. (1958) test case is characterized by free stream velocities of $19,5m/s$ and $21,9m/s$. In this work the variation range of the local Reynolds number is placed between $10^5 < Re_x < 3,5x10^6$. There is a difference of $11K$ between the temperature of the plate and of the free stream flow. The test section has $1,53m$ long.

The test cases of Taylor et al. (1990) are characterized by free stream velocities of $28m/s$ and $67m/s$. In this work the variation range of the local Reynolds number is placed between $10^5 < Re_x < 10^7$. There is a difference of $18K$ between the temperature of the plate and of the free stream flow for the flow of $28m/s$ and a difference of $12K$ for the flow of $67m/s$.The test section has $2,4m$ long.

In the test cases of Reynolds et al. (1958) and Taylor et al. (1990), the values of the unheated starting length ξ, change as the free stream velocity changes. In this work, the following profiles were considered: for $u_\infty = 67m/s$ the values of ξ are $0,56m$, $0,86m$, $1,36m$; for $u_\infty = 28m/s$ the values of ξ are $0,36m$, $0,76m$, $1,36m$; for $u_\infty = 21,9m/s$ the value of ξ is $0,7243m$; for $u_\infty = 19,5m/s$ the value of ξ is $0,415m$. The simulations were made for all available results, in both works.

The Ng (1981) test case is characterized by a free stream velocity of $10,7m/s$. In this work the variation range of the local Reynolds number is placed between $5x10^5 < Re_x < 7,8x10^5$. There is a difference of $957K$ between the temperature of the plate and of the free stream flow. The test section has $0,25m$ long. In this case the plate has the same temperature in all the length of the test section.

The inlet conditions for Reynolds et al. (1958) and Taylor et al. (1990) test cases are uniform profiles for velocity, temperature, kinetic turbulent energy and its dissipation rate. In the Ng (1981) test case, a turbulent developed profile for velocity, temperature, turbulent kinetic energy and its dissipation rate. In the top part of both meshes was imposed conditions of atmospheric pressure, ambient temperature and null derivations for κ, ε, temperature and velocity. In the exit of both meshes was imposed condition of atmospheric pressure and null derivations of all other variables.

In the figure bellow, are presented ampliations of the wall region of the two meshes used to simulate the three test cases.

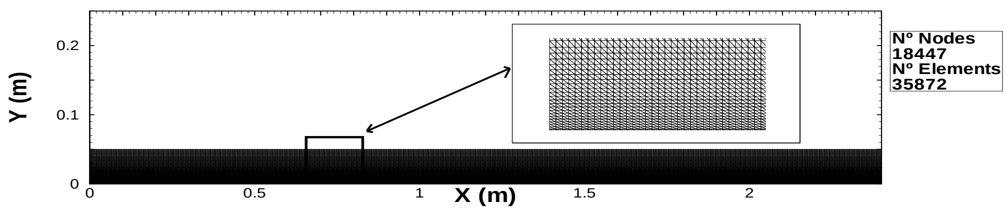

Figure 1: Velocity mesh used in Taylor et al. (1990) and Reynolds et al. (1958) test cases

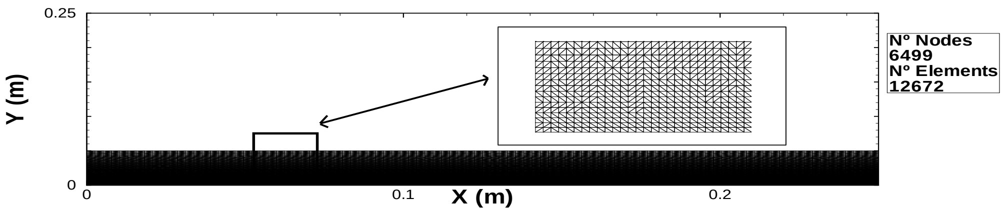

Figure 2: Velocity mesh used in Ng (1981) test case

The adoption of the same mesh in two different test cases was based in the fact that the plate used in the Reynolds et al. (1958) simulations is smaller than the plate used in the Taylor et al. (1990) simulations, besides, by the first analysis done for Taylor et al. (1990) test cases, was observed that the refinement level of the mesh was sufficient to describe the physical phenomenon.

In the following graphics, figures 3, 4 and 5, done for Taylor et al. (1990) and Reynolds et al. (1958) simulations, the empirical correlation was obtained by the use of equation (32) with the local friction coefficient calculated by equation (34), and to the numerical solutions it was used equation (29), called "numerical 1" and equation (31), called "numerical 2". So we could compare, with the experimental data, the numerical and the empirical correlation adjust term of a step wall temperature distribution in the calculus of the local Stanton number.

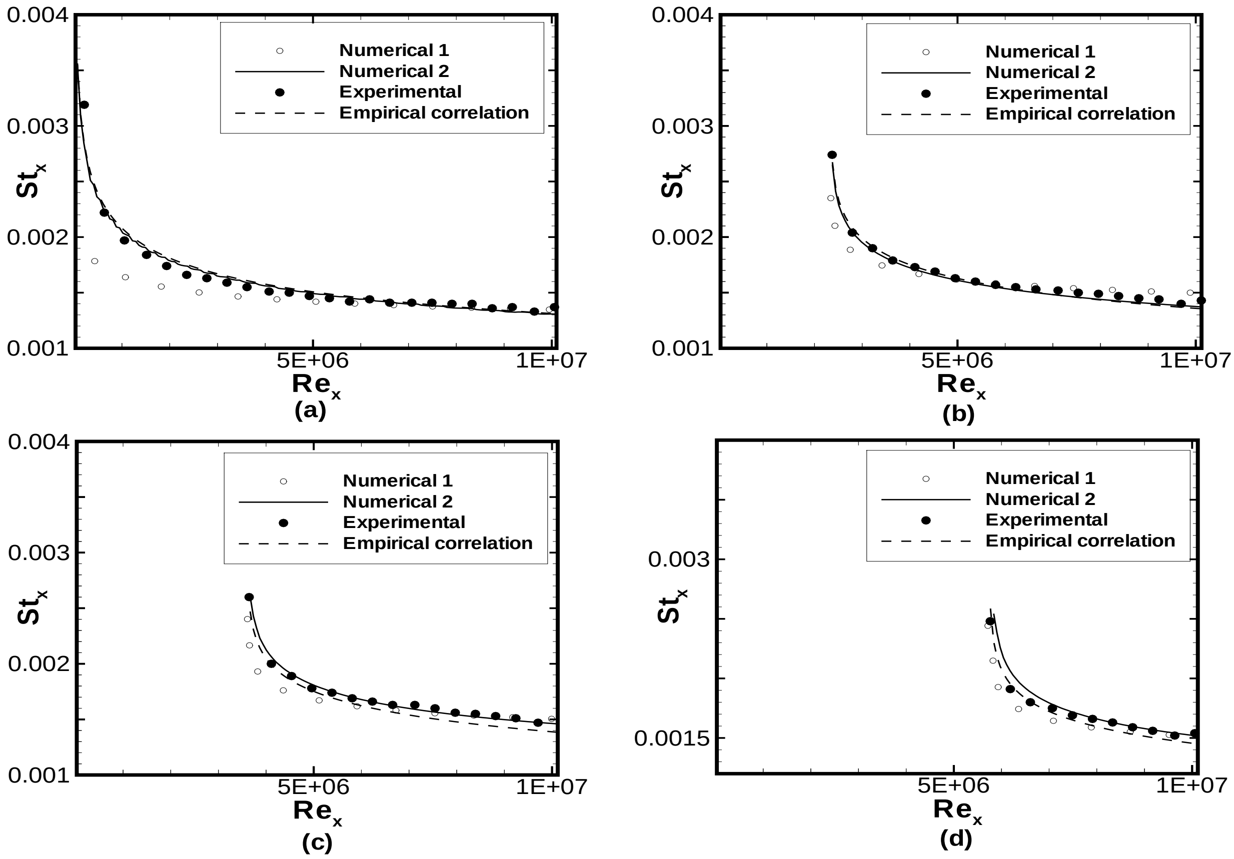

Figure 3: Local Stanton number for Taylor et al. (1990) test case. $U_\infty = 67$ - Isothermal plate (a), ξ=0,56 m (b), ξ=0,86 m (c) and ξ=1,36 m (d)

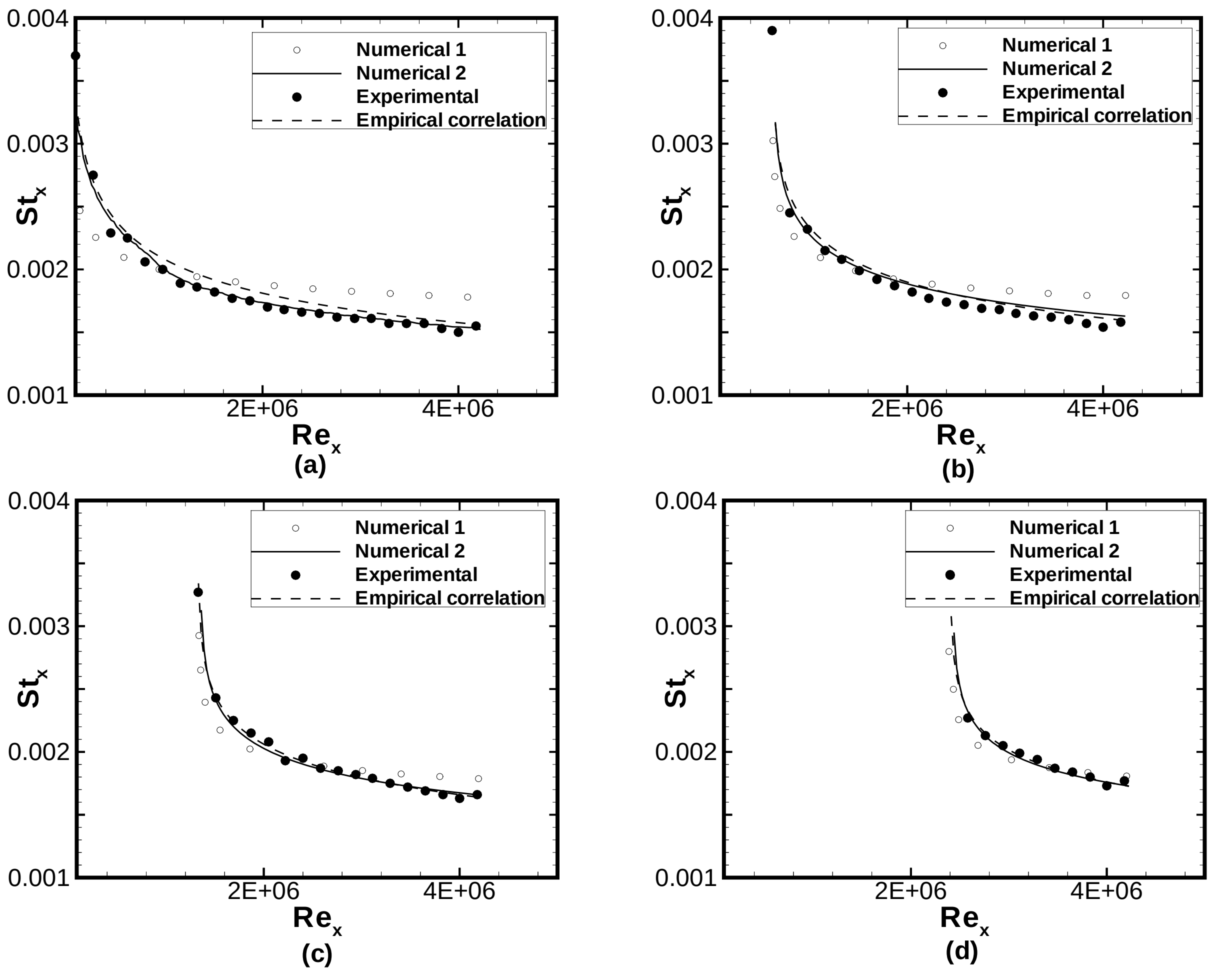

Figure 4: Local Stanton number for Taylor et al. (1990) test case. $U_\infty = 28$ - Isothermal plate (a), ξ=0,36 m (b), ξ=0,76 m (c) and ξ=1,36 m (d)

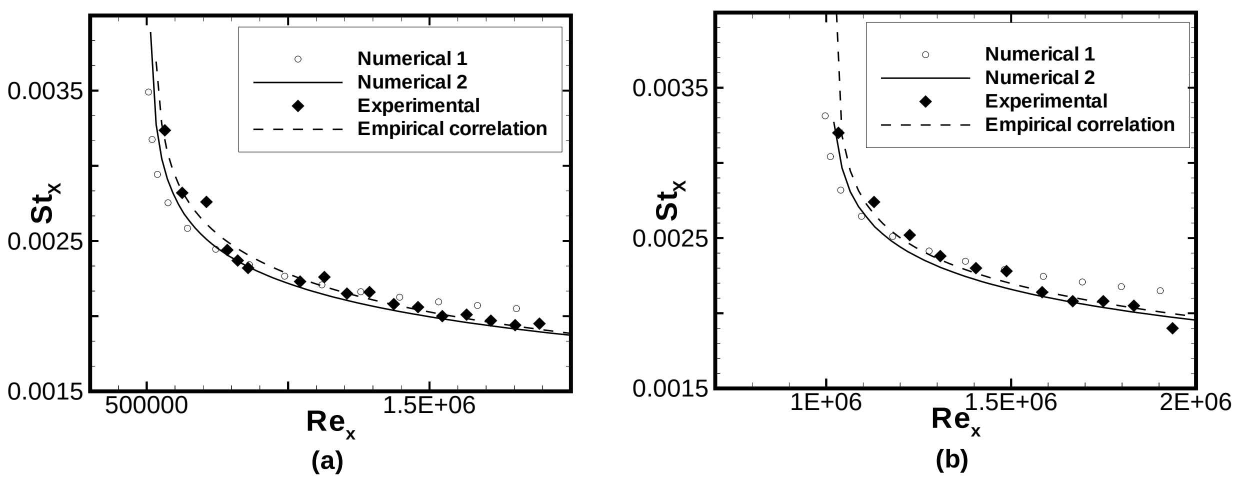

Figure 5: Local Stanton number for Reynolds et al. (1958) test case. $U_\infty = 19.5$ m/s with ξ= 0,415 m (a) - and 21.9 m/s with ξ=0,7243 m (b)

The following graphics, figures 6 and 7, done for the Ng (1981) test case, tries to show the precision of the code to calculate the thermal and hydrodynamic field, in cases were the velocity and the temperature field are strongly coupled.

In the graphic of the figure 6 is shown the variation of the hydrodynamic and thermal boundary layer thickness as the air flows over the strongly heated plate of Ng (1981) test case.

In the figure 7 is shown the variation of the local Stanton number calculated of two numerical ways. The first numerical value was calculated though the derivation of the temperature in the first node of our mesh, Eq.(29), and the second one was obtained with the relation between the thermal and the hydrodynamic boundary layer thickness, Eq.(31), using the numerical value of C_{fx}.

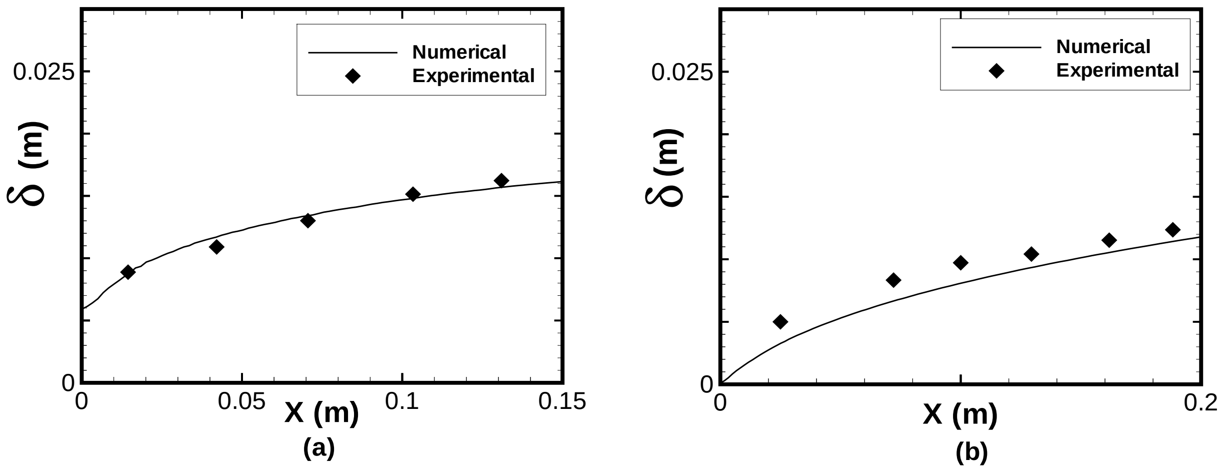

Figure 6: Hydrodynamic (a) and thermal (b) boundary layer thickness for Ng (1981) test case.

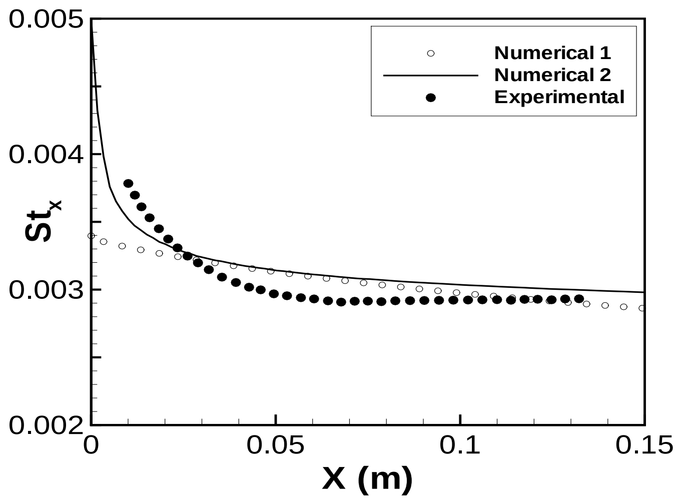

Figure 7: Stanton number - Ng (1981) test case.

5. Conclusions

We can conclude by this work, that the numerical simulation of a complex turbulent flow, done by the numerical methodology employed by the program Turbo2D, produces great results in the modeling of the forced convective turbulent heat flux, in problems were the velocity and temperature fields are coupled or uncoupled.

The results shown in figures 3, 4, 5, 6 and 7, shows that the numerical results calculated by Turbo2D solver produces very good agreement with the experimental and empirical correlation results. So, for a consolidate computational method that provides good values of the thermal and hydrodynamic boundary layer thickness along the wall, the numerical heat flux determined by using equation(31) is very accurate for turbulent flows over flat plate boundary layers.

All the results obtained numerically show that the determination of the turbulent parietal heat flux using equation (31), based on the thermal and hydrodynamical boundary layer thickness, produces better results than the ones obtained with equation (29), based on the temperature gradient in the normal direction to the wall. The results obtained with the equation (29) can be improved if more refined calculation meshes are employed.

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