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The Use of Genetic Algorithm for Transonic Airfoil Optimization

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Abstract. *The improvement in aircraft performance for fuel consumption and operational cost reductions have become important in the context of modern aviation. The aeronautical projects have as challenge a high level of optimization. In the present work a novel approach was used to optimize an airfoil for high speed airplanes. A compressible formulation (Euler equations) for subsonic and transonic flows was adopted. The numerical code for flow simulation was based in Jameson's method applied in a unstructured triangular mesh. The airfoil optimization was performed by a genetic algorithm. This algorithm creates a group of generations culminating in the adoption of the airfoil with best optimization. A family of parametric shapes (PARSEC) for super-critical airfoils was used to define the individuals used in the numerical simulations. The performance parameters used in the optimization were the values of: Cl, L/D, Cm, Cl^3/Cd^2 and the gradient of pressure. The results show that the optimization by using Genetic Algorithm requires a good fitness function, which has to consider all the parameters for optimization and their relationship.*

keywords: *Compressible flow, Finite Volume, Jameson Method, Genetic algorithm, Airfoil Optimization*

1. Introduction

For the development of modern aviation, researches are required to improve the aircraft performance. One of the most important researches is related to the flight cost reduction, minimizing the power and, consequently, the fuel consumption. One of the most prominent ways to commit these needs is by using the balance of forces over an aircraft. For this, one may consider the aircraft lift (L) equal to its weight (W) and its drag (D) equal to its trust (T). These parameters are used to calculate the power:

$$P = V \cdot T \tag{1}$$

where V is the speed necessary to a steady flight. Before adopting the Eq. (1), one may calculate the lift and drag under a certain flight condition. To calculate the lift – Eq. (2), the speed (V), the planform area (S) and the lift coefficient (CL) were considered. The drag is obtained by Eq. (3) where CD is the drag coefficient.

$$L = \frac{\rho \cdot V^2 \cdot S \cdot CL}{2} \tag{2}$$

$$D = \frac{\rho \cdot V^2 \cdot S \cdot CD}{2} \tag{3}$$

By using the previously presented equation, the power (P) can be written as:

$$\begin{aligned} P &= D \cdot V = 0.5 \cdot \rho \cdot V^3 \cdot S \cdot CD \\ &= 0.5 \cdot \rho \cdot S \cdot CD \cdot \left(\frac{2 \cdot W}{\rho \cdot S \cdot CL} \right) \\ &= \sqrt{\frac{2}{\rho \cdot S}} \cdot \sqrt{\frac{W^3}{\frac{CL^3}{CD^2}}} \end{aligned} \tag{4}$$

The best improvement of the airplane is related with the cost reduction too. According to Mair and Birdsall, 1992, to transonic airplanes, the specific range of an airplane is defined by Eq. 5.

$$r_a = \frac{V}{c\beta W} \tag{5}$$

where r_a is the specific range, W is the airplane weight. Considering the air temperature constant, V is proportional to Ma, where Ma is the Mach number. So to improve r_a in a fixed Ma, is necessary to maximize $\frac{L}{D}$.

For reducing the required power, we have to consider the weight reduction, the improvement of $\frac{Cl^3}{Cd^2}$, and the enlargement of the planform area. The maximization of range is taken maximizing $\frac{L}{D}$. The weight reduction is obtained by modifications in the aircraft structure. The planform area increases the structure weight and reduces aircraft speed. The aerodynamic performance is the result of forces produced by the airflow over the aircraft geometry.

The main aerodynamic structure for an aircraft, and the one which determine its performance, is the wing. The wing is responsible to generate the lift and for a great part of the drag. The most important geometric feature, which defines the aerodynamic, is the wing profile. Many works have been done in the development of new methods to optimize airfoil shapes and wing planforms (Ray and Tsai, 2004, Holst, 2004, Song *et al.*, 2003, Oyama *et al.*, 2000b, Oyama *et al.*, 2000a, Obayashi, 1995, Oyama *et al.*, 2001) with the objective of maximization of the aerodynamic efficiency.

In the commercial aviation, the high-speed aircraft with supercritical wings are predominant and the challenge is to minimize shock wave effects which increase the drag during cruiser flight. Studies have been addressed to determine good methods to optimize airfoils during design. Two different approaches are often employed in the aerodynamic design (Song and Keane, 2004): the inverse design and the direct numerical optimization (DNO).

The first method, named inverse design, tries to find out a geometry which produces a prescribed pressure distribution. The second, named the direct numerical optimization (DNO) method, considers a set of geometries and an aerodynamic analysis code, in an iterative process, to obtain an optimum design (Yamamoto and Inoue, 1995).

Other work considers the unconstrained single-objective airfoil design. Some of them, analyze optimization problems using evolutionary algorithms (EA), genetic algorithms (GA) with real number encoding, and hybrids comprised of GA and gradient-based methods. Constrained single-objective airfoil design problems have also considered solutions based on GA such as non dominated sorting genetic algorithm (NSGA), multiobjective GA, and NSGA coupled with artificial neural networks (Ray and Tsai, 2004). According to Ray, 2004, the GA's are good options because such problems involve highly nonlinear objectives and constraints often with functional and slope discontinuity that limits the effective use of gradient based optimization methods.

A relevant question in the aircraft design is how to consider the multiobjective optimization to improve performance measures such as the lift, drag and others. GA's and EA's are addressed to commit these needs what has been motivating many researchers. These algorithms have been also successfully applied to aerodynamic shape optimization problems such as airfoil shape design (Quagliarella and Cioppa, 1994; Yamamoto and Inoue, 1995), Multi-element airfoil shape design (Cao and Blom, 1996), subsonic wing shape design (Obayashi and Oyama, 1996) and supersonic wing shape design (Oyama *et al.*, 1999). Besides, these algorithms also aims at solving non-linear problems.

Motivated by the solutions provided by genetic algorithm, this paper proposes some multiobjective function to optimize the aerodynamic performance of airfoils. A Computational Fluid Dynamics (CFD) code was used to solve the Euler equation. The multiobjective function is applied over airfoil generations until reaching a profile which satisfies the performance needs. In the present work, the Parsec parametric airfoils (Sobieczky, 1998) were adopted.

This paper is divided as follows: section 2 shows the aerodynamic formulation used to evaluate the airfoil performance; section 3 explains the numerical methods used of this work; section 4 describes the geometry family that defines the airfoil shape; section 5 shows how the numerical code works; section 6 shows the code validation and the results obtained for three different fitness functions. The last section presents the main conclusions of the present work.

2. Formulation

The Navier-Stokes (NS) equations represent the mathematical model for any kind of flow. For transonic flow simulations over airfoils, one of the most important phenomena is related with the compressibility, ie. shock wave. When using the supercritical airfoil, the interaction between boundary layer and shock wave is important to represents the shock location and to analyze moderns wings profiles. These profiles are called *Natural Laminar Flow* (NLF), were the reduction of drag is caused by an increment of laminar flow percentage

($\geq$ 30% of chord) on the foil surface (Selig *et al.*, 1995). However, the viscous effects were neglected in the current work, simplifying the NS equations. This simplification leads do the called Euler equations. These equations in conservative form and in Cartesian coordinates becomes:

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0, \tag{6}$$

where

$$Q = [\rho, \rho u, \rho v, e]^t, \tag{7}$$

$$E = \left[\rho u, \rho u^2 + p, \rho uv, (e+p)u\right]^t, \tag{8}$$

$$F = \left[\rho u, \rho uv, \rho v^2 + p, (e+p)v\right]^t, \tag{9}$$

$$e = \rho(e_i + \frac{1}{2}(u^2 + v^2)), \tag{10}$$

$$p = \rho RT, \tag{11}$$

$$e_i = \frac{p}{(\gamma - 1)\rho}. \tag{12}$$

These equations were simulated in a numerical code. The details of the numerical code is presented in the next section.

3. Numerical Methods

To accomplish the present work, it was necessary the implementation of some numerical methods to simulate and optimize the wing section leaving to computer the job to calculate and evaluate the performance of airfoil and decide which one is the best. This section is divided in two parts, the first one describes the numerical method used to solve the Euler equations and the second describes the Genetic Algorithm adopted.

3.1. Euler solver

To evaluate the performance of a wing section on a compressible flow, it is necessary to simulate the flow around the airfoil, using any method of solution for the Euler equations. There are a lot of these methods, that differ about the mesh, the discretization of equation and the accuracy.

In the current work, it was adopted a finite volume (FV) method proposed by Jameson (Jameson *et al.*, 1981; Jameson, 1982; Jameson and Mavriplis, 1986) on a unstructured triangular mesh. This mesh allows an easy definition of the geometry around the airfoil. The grid generation were made automatically by a scripted commands program. A detailed description of Jameson can be found in Hirsch, 1991.

3.2. Genetic Algorithms

Genetic Algorithms (GA) are being applied as search and optimization techniques in several domains. These algorithms are based on nature select mechanisms focusing at survival of the most capable individuals. GA does not always give the best possible solution, however provides good local solutions for NP-complete problems.

The problem solution using genetic algorithms involves two different aspects: solution encoding into the form of chromosomes, where each chromosome represents a possible solution, and a fitness function applied to find the best solution.

Different encoding techniques can be used for different kind of problems, such as binary strings, bitmaps, real numbers, and so on. The fitness function is responsible for the evaluation of possible solutions. This function receives a chromosome as parameter and returns a real number, informing the quality of the obtained solution, e.g., how adequate is the solution for the currently studied problem.

The most adequate chromosomes are identified and stored during the evolution process. The weakest ones, on the other side, are eliminated. Different techniques can be applied for the identification of the best chromosomes, such as the proportional selection, ranking selection and tournament-based selection (Back *et al.*, 1999b; Back *et al.*, 1999a).

In the proportional selection, individuals are transferred to the next generation according to their fitness value probability proportion. One of the possible implementations of this technique consists in the usage of a roulette, divided into N parts, N being the number of individuals (chromosomes) in the current population. The size of each part is proportional to the fitness value of each individual. The roulette is rotated N times afterward, and at each turn the appointed individual is selected and inserted into the new population.

Ranking-based selection can be subdivided into two steps. During the first one, the solutions are ordered according to their fitness function values. Once the list is ordered, each individual receives a new fitness function value equivalent to its position in the ranking. After that, a procedure that selects the individuals, according to their ranking position, is applied. Thus, the individuals with better ranking position have more chances to be selected.

Finally, a tournament-based selection does not automatically attribute probabilities to individuals. A tournament size (k) is defined, with $k \geq 2$ individuals. Then, k individuals are chosen randomly from the current population, and their fitness functions are compared. The individual with best fitness value is selected for reproduction. The k value is defined by the user, representing the selection pressure – e.g., the speed with the strongest individuals will dominate the population, generating the extermination of the weakest ones.

Once selected the individuals for the reproduction, it is necessary to modify their genetic characteristics using reproduction techniques known as genetic operators. The most common operators are crossover and mutation.

The crossover operator allows to exchange genetic material between two individuals, known as the parents, combining their information in a way that provides a significant chance of creation of new individuals with better characteristics than the original ones (Hinterding, 2000).

The single-point crossover operator is the most used one. In order to apply it, two individuals (parents) are selected and two new individuals are created from them (children). A single random splitting point is selected in parents chromosomes, and the new chromosomes are created from the combination of the parents, as shown in Tab. 1. In this table, label (a) shows the parent individuals and the splitting point marked by | symbol. The New individuals created from the combination of the parent chromosomes are shown in the same table with label (b), illustrating the crossover operator.

Table 1: Crossover operator

(a) Before the crossover	(b) After the crossover
$X_1X_2\|X_3X_4X_5X_6$	$X_1X_2\|Y_3Y_4Y_5Y_6$
$Y_1Y_2\|Y_3Y_4Y_5Y_6$	$Y_1Y_2\|X_3X_4X_5X_6$

The mutation operator, on its turn, is used for changing a single gene value for a new random one. When an individual is represented by a bitmap, this operation consists of a random choice of a chromosome gene and the swapping of its value from 1 to 0 (or from 0 to 1, correspondingly). The goal of the mutation operator is to maintain the diversity of a population, always allowing a chromosome to cover a significantly large result space (Hinterding, 2000). It is usually applied at a low rate, as at high ones the results tends to be random.

4. Parametric airfoil shape

A parametric airfoils families were adopted, with the objective of generating a big number of airfoils shapes for the use with GA. These families are very appropriated because allow the generation of different types of shapes using a limited number of parameters. These parameters are used for the GA as the chromosomes of the individuals. According to Song and Keane, 2004 and Ray and Tsai, 2004, there are many functions proposed to evaluate the shape, like: analytical functions (PARSEC, NACA, etc); splines, B-splines and Bezier curves to via interpolation methods; and others.

The objective of the present work is to evaluate a method for optimize supercritical airfoils. The Parsec family of wing sections defined by 11 geometric parameters was used. According to Sobieczky, 1998, a blend of two or more airfoils schemes can be used to improve number of shapes representations and consequently the possibilities of geometries. To make a simple analysis, only the Parsec shapes were analyzed. This family is detailed bellow.

4.1. The Parsec Family

The PARSEC representation is particularly attractive as it uses a small number of design variables, all of which are related to some properties of the shape (Ray and Tsai, 2004). It parameterizes the upper and the lower airfoil surfaces using polynomials in coordinates X and Z as:

$$Z = \sum_{n=1}^{6} a_n X^{n-\frac{1}{2}} \tag{13}$$

where a_n are real coefficients. The parameters of a PARSEC representation include the leading-edge radius r_{le}, upper and lower crest heights Z_{UP}, Z_{LO} and location X_{UP}, X_{LO}, curvatures at the upper and lower crests Z_{XXUP}, Z_{XXLO}, trailing-edge thickness ΔZ_{TE} and ordinate Z_{TE}, and direction and wedge angle α_{TE}, β_{TE}. The parameters are schematically shown in Fig. 1.

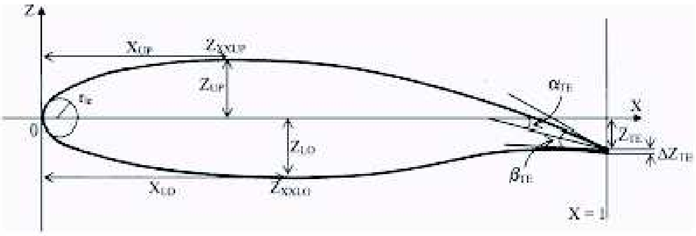

Fig. 3 Variables in the PARSEC representation scheme.

Figure 1: Variables in the Parsec representation scheme

5. A Multiobjective Function to Optimize the Aerodynamic Performance of Airfoils Using CFD

On the context of aviation design, in the conceptual project and in the detailed one, the aerodynamic analysis became very important. This importance leads to find out the optimum design, with the multiobjective optimization. The airfoil shape is one of the most optimized in the aviation design.

To perform the design optimization, many studies used several optimization methods like: GA, evolutionary algorithms, gradient-based methods and others. All show its capability to optimize on the aerodynamic design context, looking for single objective and multiobjective optimization, with or without constraints.

In the present paper, the optimization method using a GA with a Euler solver to evaluate the aerodynamic performance were carried out.

To classify the individuals on GA, it was used a fitness function defined by:

$$fitness = \begin{cases} 0, \text{ to t} > 20\% \text{ of chord} \\ 0, \text{ to camber} > 10\% \text{ of chord} \\ \text{fitness function} \end{cases} \tag{14}$$

where: t is the maximum thickness of the airfoil; *camber* are the maximum coordinate of the mean line (*camber* line) of airfoil; and three fitness functions were adopted. These functions are shown in detail the section results.

To make this optimization possible, it was necessary the integrations between the steps described above: definition of geometry; grid generation; Euler solution; post-process of the result of Euler simulation; and the genetic algorithm implementation. The methodology is illustrated on Fig. 2.

To improve the computational cost, it was made an evaluation of the geometry before the mesh generator or the Euler solution start. The thickness and the camber condition were used in this evaluation.

6. Results

This section was divided in two subsections, the first one shows the validation of the present Euler Solver code, where the numerical results are compared with numerical and experimental results. The second subsection shows the results obtained with three different fitness functions, with the fitness evolutions and the optimized airfoil shapes.

6.1. Euler Solver Validation

The aerodynamic code was validated comparing the numerical results with experimental data. It cold to see on Fig. 3 the comparison between experimental Cp distribution of RAE-2822 transonic airfoil and the numerical result obtained with the implemented code. The figure shows that the numerical code is able predict the flow,

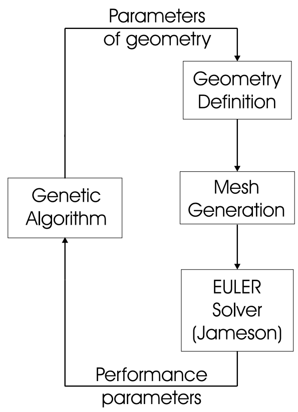

Figure 2: Methodology structure used on implementation.

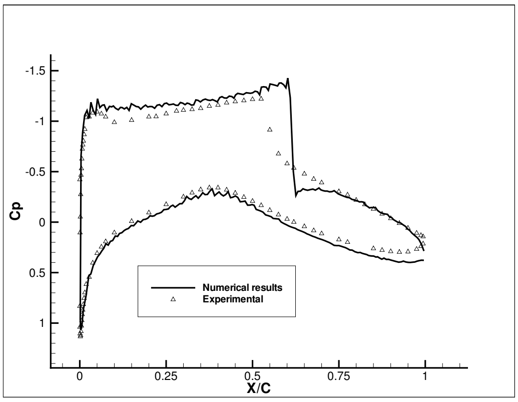

Figure 3: Numerical and experimental results for Cp distribution over RAE-2822 airfoil.

with some differences in the position and intensity of the shock wave. This differences are attributed to the viscous effects that were neglected in the present study.

The mesh used in optimization used about 11000 cell. A refinement near the airfoil were used to capture the shock wave and its effect on airfoil performance and the pressure distribution. On foil surface, the maximum length of the size of volumes were defined as 0.007 chord.

6.2. Optimization

To optimize the airfoil and to configure the Genetic algorithm, we restricted the random generation of geometry parameters. The limits of this values adopted are shown in Tab. 2.

Table 2: Parameters limits

Param.	r_{le}	X_{up}	Z_{up}	Zxx_{up}	X_{lo}	Z_{lo}	Zxx_{lo}	α_{te}	β_{te}	Z_{te}	dZ_{te}
Lower	0.001	0.3	0.05	-0.5	0.3	-0.05	0.55	8^o	10^o	0	0
top	0.007	0.5	0.12	0.1	0.45	0.1	1.35	17^o	11.5^o	0.05	0

The parameters that we want to maximize are: L/D, Cl^3/Cd^2 according to the fitness functions presented in the next sections; that are direct related to the power required, the trust required to a steady flight and range of airplane. These parameters express the size of engine needed and the consumption of airplane. To reduce the down force necessary to equilibrate the airplane, we want to minimize the Cm and to determine the flight speed, we want to restrict the Cl to 0.2.

The parameters of the genetic algorithm was: 70 individuals per generation; 70% of crossover probability; and 1% of mutation probability. The flight condition were defined as: $Ma = .75$; incidence of airfoil $\alpha = 0^o$.

6.2.1. Fitness function 1

The first fitness function adopted is given by Eq. 15.

$$F_i = \frac{\frac{L}{D} + \frac{Cl^3}{Cd^2} + P1 + P2}{(|Cl - 0.2| + 0.01) \cdot (|CM| + 0.01) \cdot (I1 + I2 + 0.01)} \tag{15}$$

where: F_i is the fitness value for the proposed solution i; $\frac{L}{D}$ is the aerodynamic efficiency; $\frac{Cl^3}{Cd^2}$ is the aerodynamic influence in the required power; $P1$ and $P2$ are defined according to the adverse gradient of pressure over the airfoil surface (upper and lower surface, respectively); Cl is the lift coefficient; CM is the pitch moment coefficient; $I1$ and $I2$ are the fraction of supersonic flow over the airfoil surface (upper and lower surface, respectively); 0.2 defines the desired restriction; and, finally, the constants 0.01 avoid the division by zero.

The fitness evolution is shown in Fig. 4 - left. The individual has adapted with forty generations. The Fig. 4 - right shows the optimized profile with the Mach isolines distribution. In this figure it can be seen big acceleration of flow on upper surface and a strong shock.

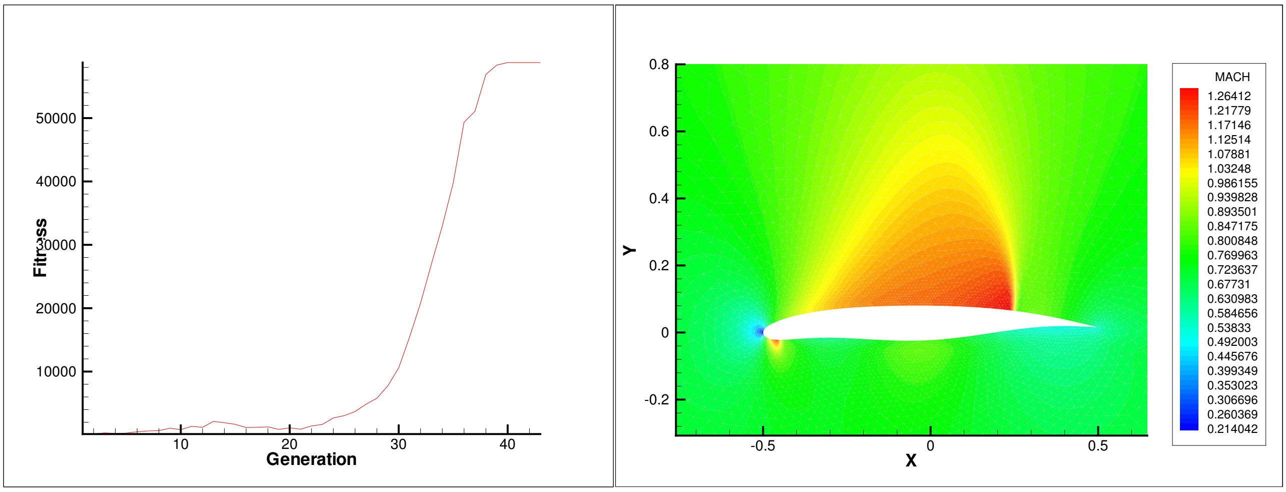

Figure 4: Fitness evolution - left and the optimized profile with Mach isolines for the first fitness function - right.

6.2.2. Fitness function 2

The second fitness function adopted was

$$F_i = \frac{\frac{L}{D} + P1 + P2}{(|Cl - 0.2| + 0.01) \cdot (|CM| + 0.01) \cdot (I1 + I2 + 0.01)} \tag{16}$$

the parameters shown here were defined in section 6.2.1. The fitness evolution is shown in Fig. 5 - left. The optimization converged with thirty generations. The Fig. 5 - left shows the evolution of optimization. This airfoil has a reduction of sonic flow over the upper surface comparing with the first result, but the shock wave are still present.

6.2.3. Fitness function 3

The third fitness function is given by Eq. 17. The parameters are defined in section 6.2.1. The optimized airfoil obtained with Mach isolines is shown in Fig. 6 - right. The Fig. 6 - left show the fitness evolution of optimization. The optimization converged with thirty five generations.

$$F_i = \frac{\frac{L}{D}}{(|Cl - 0.2| + 0.01) \cdot (|CM| + 0.01)} \tag{17}$$

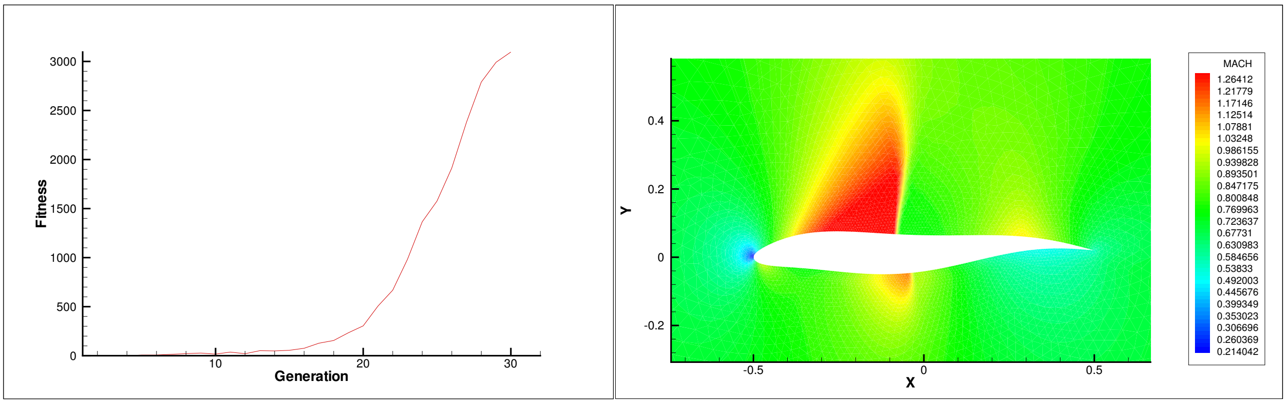

Figure 5: Fitness evolution - left and the optimized profile with Mach isolines for the second fitness function - right.

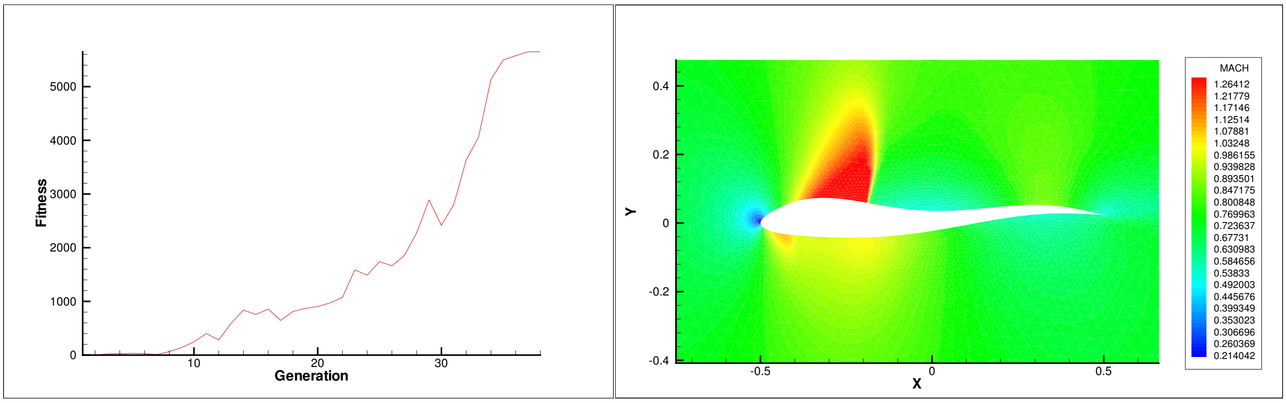

Figure 6: Fitness evolution - left and the optimized profile with Mach isolines for the third fitness function - right.

6.2.4. Comparison between results

The results of the coefficients obtained with each fitness function are shown in Tab. 3. In this table, the column **fit. RAE** show the fitness function obtained by the performance parameters of RAE-2822 airfoil for the flow condition. The column % shows the percentage comparison of the RAE fitness with our optimized airfoil solution.

Table 3: Comparison of optimization results

func.	Cl	Cm	L/D	Cl^3/Cd^2	$fitness$	$fit.RAE$	%
1	0.7066	-0.38435	102.31	7398.2	58760	4.12E+10	0.000143
2	0.4649	-0.28020	24.76	285.1	3096	6794	45.6
3	0.1937	-0.16412	6.8	8.95	5648	3096	182.42
RAE	0.4422	-0.23391	177.02	13855			

Comparing the results, is possible to see that $fitness_1$ function take a Cl greater than desired and the $fitness_2$ function take a Cl near to the RAE-2822, but the $fitness_3$ function could restrict the Cl to 0.2 as desired. The minimization of Cm was performed better by $fitness_3$ function. The maximizations of $\frac{L}{D}$ and $\frac{Cl^3}{Cd^2}$ were performed by $fitness_1$ function. The results obtained show that the Cl restriction "fights" with the maximizations. The comparison between the fitness of generated airfoils and the RAE-2822 shows that the $fitness_3$ function gives the better Cl and Cm. The $fitness_1$ function gave the better maximization terms.

The Cp distributions of the optimized airfoils obtained with each fitness function and the RAE-2822 Cp are shown in Fig. 7. The Cp curves show that every airfoils have supersonic flow over its surface and strongs shock waves, despite the maximization of aerodynamic efficiency.

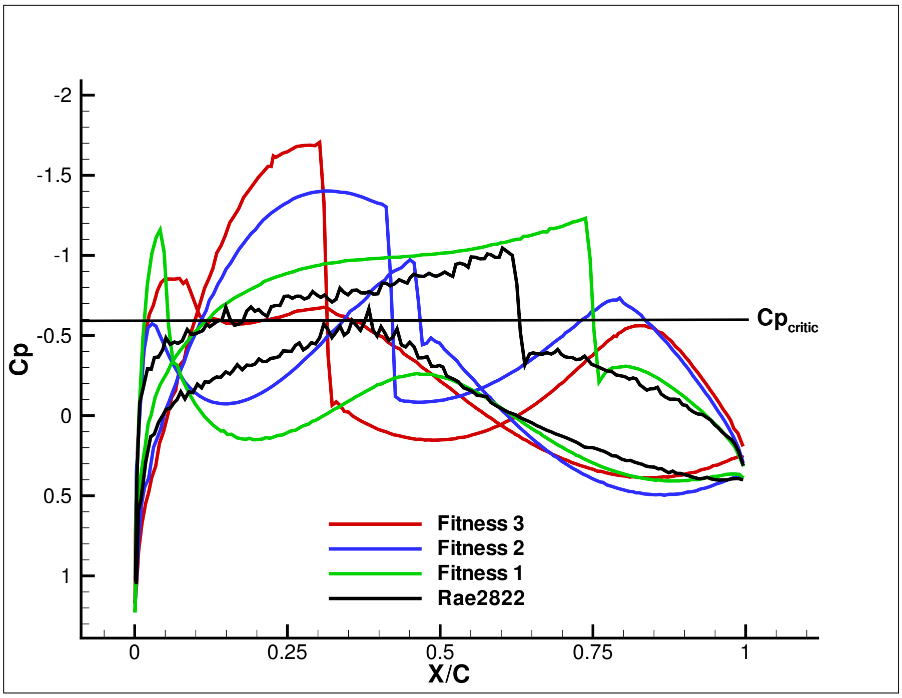

Figure 7: *Cp* distribution of foil.

7. Conclusions

In the current work a optimization study of a transonic airfoil was presented. A compressible formulation (Euler equations) was adopted. The numerical code for flow simulation was based in Jameson's method applied in a unstructured triangular mesh. The airfoil optimization was performed by a genetic algorithm. A family of parametric shapes (PARSEC) for super-critical airfoils was used to define the individuals used in the numerical simulations. The performance parameters used in the optimization depends on the fitness function adopted. The results show that the optimization by using Genetic Algorithm requires a good fitness function, which has to consider all the parameters for optimization and their relationship.

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