Proceedings of the 10th Brazilian Congress of Thermal Sciences and Engineering -- ENCIT 2004
Braz. Soc. of Mechanical Sciences and Engineering -- ABCM, Rio de Janeiro, Brazil, Nov. 29 -- Dec. 03, 2004

Paper CIT04-0526

HYBRID SOLUTION FOR FLOW DEVELOPMENT IN IRREGULAR DUCTS

Roseane de Lima Silva
João Nazareno Nonato Quaresma
Chemical and Food Engineering Department, CT, Universidade Federal do Pará, UFPA
Campus Universitário do Guamá, 66075-110, Belém, PA; quaresma@ufpa.br

Carlos Antônio Cabral dos Santos
Solar Energy Laboratory, LES, Mechanical Engineering Dept., DTM, Universidade Federal da Paraíba, UFPB
Cidade Universitária, 58059-900, João Pessoa, PB; cabral@les.ufpb.br

Abstract. *The analysis of two-dimensional laminar flow in the entrance region of irregular ducts is made by the application of the so-called Generalized Integral Transform Technique (GITT) in the solution of the Navier-Stokes equations. The streamfunction-only formulation is adopted, where a splitting-up procedure is applied with a general filter that adapts to the irregular contour. The case of a wavy wall duct is more closely analyzed. Computation of results for streamfunction, vorticity and velocity fields are performed, and comparisons with those in the literature are made in order to validate the computational code developed. In addition, results for the product of the friction factor-Reynolds number are also calculated and compared with available ones in the literature for different Reynolds numbers and amplitude of the duct.*

Keywords. *Flow development, Irregular ducts, Wavy wall duct, Integral transform.*

1. Introduction

The fluid flow is found in several applications related to the project, manufacture and operation of industrial products. Among them, involving mainly, incompressible fluids, we have the flow of liquids in plants of chemical processing, air flow in heating and ventilation of rooms and cooling of electronic equipment. In these applications there is the need of the knowledge of certain physical parameters, as the friction factors.

The flow and heat transfer phenomena occurring in wavy wall duct have been studied in different engineering sectors. Corrugated surfaces are for example, utilized in compact heat exchangers (Kays, 1984). Most of studies performed on the fluid dynamic and thermal phenomena occurring in corrugated wall ducts consider corrugations having a periodical pattern which is described by simple functions such as rectangular, triangular or sinusoidal ones. However, due to the variety of thermal and fluid dynamic characteristics described in the literature under different conditions, the study of more complex corrugation profiles can be useful to better evaluate the convenience of assigning to the corrugated duct walls rather than flat profiles.

Some experimental and theoretical studies available in the literature of fluids dynamic and thermal phenomena dealing with ducts of wavy wall can be seen in Goldstein and Sparrow (1977), Asako et al. (1988), Sunden and Trollheden (1989), Xiao et al. (1989) and Wang and Chen (2002).

In this context, the present work is motivated by the application of the Generalized Integral Transform Technique (GITT) in the solution of the Navier-Stokes equations in hydrodynamic developing flow in a wavy wall duct. A general formulation in terms of streamfunction is adopted, such as that proposed by Pérez Guerrero (1995) and Pérez Guerrero et al. (2000). In this formulation the streamfunction is separated in two parts, where one of them represents a generic filter, which adapts to the irregular boundary of the general duct. A wavy wall duct is more closely studied, where computations for the streamfunction, vorticity and velocity fields are performed, as well as for the product of the friction factor-Reynolds number, with different values of the governing parameters of the flow, such as the Reynolds number and the amplitude of the wavy surface.

2. Mathematical formulation

We consider two-dimensional steady laminar flow of an incompressible Newtonian fluid in the inlet region of a duct of irregular geometry. Figure (1) shows the schematic representation of the problem.

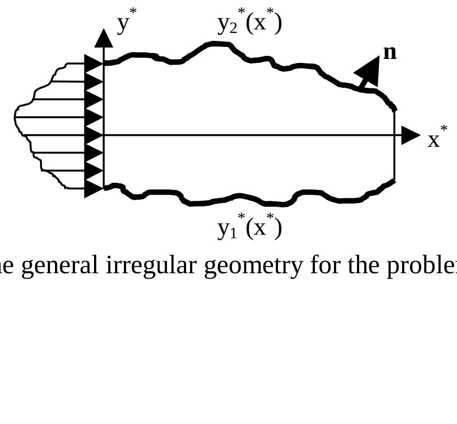

Figure 1. Definition of the general irregular geometry for the problem and coordinates system.

The flow is governed by the continuity and Navier-Stokes equations. Adopting the streamfunction formulation, the problem is written as

$$\frac{\partial\psi}{\partial y}\left(\frac{\partial^3\psi}{\partial x^3}+\frac{\partial^3\psi}{\partial x\partial y^2}\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial^3\psi}{\partial x^2\partial y}+\frac{\partial^3\psi}{\partial y^3}\right)=\frac{1}{\mathrm{Re}}\left(\frac{\partial^4\psi}{\partial x^4}+2\frac{\partial^4\psi}{\partial x^2\partial y^2}+\frac{\partial^4\psi}{\partial y^4}\right) \tag{1a}$$

where the boundary conditions are the non-slip and impermeability at the duct walls

$$\psi(x,-y_1(x))=k_1;\quad \frac{\partial\psi(x,-y_1(x))}{\partial \mathbf{n}}=0 \tag{1b,c}$$

$$\psi(x,y_2(x))=k_2;\quad \frac{\partial\psi(x,y_2(x))}{\partial \mathbf{n}}=0 \tag{1d,e}$$

where **n**, k_1 and k_2 represent the unit normal vector in the outward direction of the duct wall and the streamfunction values at the walls, respectively. The constant Q represents the volumetric flow for unit of length and is determined as (Pérez Guerrero, 1995)

$$\psi(0,y_2)=k_2=Q+k_1 \tag{1f}$$

In above equations the definition of streamfunction was introduced according to

$$u=\frac{\partial\psi}{\partial y};\quad v=-\frac{\partial\psi}{\partial x} \tag{2a,b}$$

These definitions allow automatically to satisfy the continuity equation. The dimensionless groups employed in Eqs. (1) are defined as

$$x=x^*/b;\ y=y^*/b;\ y_1(x)=y_1^*(x^*)/b;\ y_2(x)=y_2^*(x^*)/b;\ u=u^*/u_0;\ v=v^*/u_0;\ p=p^*/\rho u_0^2;\ \mathrm{Re}=bu_0/\nu \tag{3a-h}$$

where b represents the half distance between the walls at the duct inlet.

3. Solution methodology

In the solution of Eqs. (1) by using the GITT approach, it is convenient to define a filter in order to homogenize the boundary conditions in the y direction, which later will be the one chosen for the eigenvalue problem. Therefore, the filter is written as

$$\psi(x,y)=\phi(x,y)+F(x,y) \tag{4}$$

where $\phi(x,y)$ represents the unknown potential to be determinate, and F(x,y) is the filter, which has the same values that $\psi(x,y)$ at the duct walls. The function F(x,y) is not a particular solution of $\psi(x,y)$ (Pérez Guerrero, 1995). Therefore, introducing Eq. (4) into Eqs. (1), results

$$\begin{aligned}&\frac{\partial\phi}{\partial y}\frac{\partial^3\phi}{\partial x^3}+\frac{\partial\phi}{\partial y}\frac{\partial^3\phi}{\partial x\partial y^2}-\frac{\partial\phi}{\partial x}\frac{\partial^3\phi}{\partial x^2\partial y}-\frac{\partial\phi}{\partial x}\frac{\partial^3\phi}{\partial y^3}+\frac{\partial\phi}{\partial y}\frac{\partial^3 F}{\partial x^3}+\frac{\partial\phi}{\partial y}\frac{\partial^3 F}{\partial x\partial y^2}-\frac{\partial\phi}{\partial x}\frac{\partial^3 F}{\partial x^2\partial y}-\frac{\partial\phi}{\partial x}\frac{\partial^3 F}{\partial y^3}+\\&\frac{\partial F}{\partial y}\frac{\partial^3\phi}{\partial x^3}+\frac{\partial F}{\partial y}\frac{\partial^3\phi}{\partial x\partial y^2}-\frac{\partial F}{\partial x}\frac{\partial^3\phi}{\partial x^2\partial y}-\frac{\partial F}{\partial x}\frac{\partial^3\phi}{\partial y^3}+\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial x^3}+\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial x\partial y^2}-\frac{\partial F}{\partial x}\frac{\partial^3 F}{\partial x^2\partial y}-\frac{\partial F}{\partial x}\frac{\partial^3 F}{\partial y^3}\\&=\frac{1}{\mathrm{Re}}\left[\frac{\partial^4\phi}{\partial x^4}+2\frac{\partial^4\phi}{\partial x^2\partial y^2}+\frac{\partial^4\phi}{\partial y^4}+\frac{\partial^4 F}{\partial x^4}+2\frac{\partial^4 F}{\partial x^2\partial y^2}+\frac{\partial^4 F}{\partial y^4}\right]\end{aligned} \tag{5a}$$

$$\phi(x,y_1)=k_1-F(x,-y_1);\quad \frac{\partial\phi(x,-y_1)}{\partial\mathbf{n}}=0 \tag{5b,c}$$

$$\phi(x,y_2)=k_2-F(x,y_2);\quad \frac{\partial\phi(x,y_2)}{\partial\mathbf{n}}=0 \tag{5d,e}$$

Through the methodology presented by Pérez Guerrero (1995) and Pérez Guerrero et al. (2000), the filter F(x,y) should be in a such way that reproduces the value of the streamfunction at the duct walls along the length. This function can be built by considering that at each position along the duct a velocity profile is fully developed, which adapts to the irregularity of the duct.

In order to introduce this filter, a relationship between the coordinates system (x,y) and a new transformed system (η,x) is given as

$$\eta = y - y_3(x)\,; \quad y_0(x) = \frac{1}{2}\left[y_1(x) + y_2(x)\right]; \quad y_3(x) = \frac{1}{2}\left[y_2(x) - y_1(x)\right] \tag{6a-c}$$

or in terms of the original coordinates

$$F(x,y) = \frac{3}{4}Q\left[\left(\frac{y-y_3}{y_0}\right) - \frac{1}{3}\left(\frac{y-y_3}{y_0}\right)^3\right] + \frac{Q}{2} + k_1 \tag{7}$$

where y_3 represents the distance between the axes y and η and y belongs to interval $[-y_1(x),\ y_2(x)]$, and $\eta \in [-y_0(x),\ y_0(x)]$. It is defined like this a new variable ξ, which will allow to determine the coefficients of the integral transformation. Therefore the domain $\xi \in [-1,1]$ is defined as

$$\xi = \frac{\eta}{y_0} = \frac{y-y_3}{y_0} \tag{8}$$

Thus, the filter can be rewritten in the form

$$F(\xi) = \frac{3}{4}\left[\xi - \frac{\xi^3}{3}\right] + \frac{Q}{2} + k_1 \tag{9}$$

In the light of applying the GITT approach in the solution of the PDE system given by Eqs. (5), due to homogeneous characteristics of the boundary conditions in the y direction, it is more appropriate to choose this direction for the process of integral transformation. By considering the relation given by Eq. (8), the auxiliary eigenvalue problem is taken as

$$\frac{d^4 Y_i(\xi)}{d\xi^4} = (\mu_i y_0)^4 Y_i(\xi) \equiv \beta_i^4 Y_i(\xi) \tag{10a}$$

$$Y_i(-1)=0; \quad \frac{dY_i(-1)}{d\xi}=0 \tag{10b,c}$$

$$Y_i(1)=0; \quad \frac{dY_i(1)}{d\xi}=0 \tag{10d,e}$$

Problem (10) is analytically solved, to furnish

$$Y_i(\xi) = \begin{cases} \dfrac{\cos(\beta_i\xi)}{\cos(\beta_i)} - \dfrac{\cosh(\beta_i\xi)}{\cosh(\beta_i)}, & i=1,3,5,... \\[2ex] \dfrac{\sin(\beta_i\xi)}{\sin(\beta_i)} - \dfrac{\sinh(\beta_i\xi)}{\sinh(\beta_i)}, & i=2,4,6,... \end{cases} \tag{11a,b}$$

where the eigenvalue β_i is defined as

$$\beta_i = \mu_i y_0 \tag{12}$$

The normalization integral is defined as

$$N_i = y_0 \int_{-1}^{1} Y_i^2(\xi)\, d\xi\,, \quad i = 1,2,3,... \tag{13}$$

which is computed as

$$N_i(x) = 2y_0(x) = N(x) = N \tag{14}$$

The eigenfunctions satisfy the following orthogonality property:

$$\int_{-y_1}^{y_2} Y_i Y_j dy = \begin{cases} 0, & \text{for} \quad i \neq j \\ 2y_0, & \text{for} \quad i = j \end{cases} \tag{15a,b}$$

The eigenvalue problem defined by Eqs. (10) allow the definition of the following integral transform pair:

$$\bar{\phi}_i(x) = \frac{1}{N(x)} \int_{-y_1}^{y_2} Y_i(x,y)\phi(x,y)dy\,, \quad \text{transform} \tag{16}$$

$$\phi(x,y) = \sum_{i=1}^{\infty} Y_i(x,y)\bar{\phi}_i(x)\,, \quad \text{inverse} \tag{17}$$

We can now to accomplish the transformation of the original partial differential system given by Eqs. (5). For this purpose, Eq. (5a) is multiplied by Y_i and is then integrated over the domain $[-y_1(x), y_2(x)]$ in y, after that the inverse formula given by Eq. (17) is employed, resulting in the following coupled ordinary differential system for the calculation of the transformed potentials $\bar{\phi}_i$:

$$\bar{\phi}_i^{(iv)} = -\mu_i^4 \bar{\phi}_i + \frac{L_i}{N} + \frac{Re}{N} \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} \Big\{ \bar{\phi}_j \bar{\phi}_k A_{ijk} + \bar{\phi}_j \bar{\phi}_k' B_{ijk} + \bar{\phi}_j \bar{\phi}_k'' C_{ijk} + \bar{\phi}_j \bar{\phi}_k''' D_{ijk} +$$
$$\bar{\phi}_j' \bar{\phi}_k E_{ijk} + \bar{\phi}_j' \bar{\phi}_k' F_{ijk} + \bar{\phi}_j' \bar{\phi}_k'' G_{ijk} \Big\} + \frac{1}{N} \sum_{j=1}^{\infty} \Big\{ \bar{\phi}_j H_{ij} + \bar{\phi}_j' I_{ij} + \bar{\phi}_j'' J_{ij} + \bar{\phi}_j''' K_{ij} \Big\} \tag{18}$$

where the coefficients that depend on x are calculated from

$$A_{ijk} = \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial^3 Y_k}{\partial x^3} dy + \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial^3 Y_k}{\partial x \partial y^2} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} \frac{\partial^3 Y_k}{\partial x^2 \partial y} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} \frac{\partial^3 Y_k}{\partial y^3} dy \tag{19}$$

$$B_{ijk} = 3\int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial^2 Y_k}{\partial x^2} dy + \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial^2 Y_k}{\partial y^2} dy - 2\int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} \frac{\partial^2 Y_k}{\partial x \partial y} dy \tag{20}$$

$$C_{ijk} = 3\int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial Y_k}{\partial x} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} \frac{\partial Y_k}{\partial y} dy \tag{21}$$

$$D_{ijk} = \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} Y_k dy\,; \quad E_{ijk} = -\int_{-y_1}^{y_2} Y_i Y_j \frac{\partial^3 Y_j}{\partial x^2 \partial y} dy - \int_{-y_1}^{y_2} Y_i Y_j \frac{\partial^3 Y_k}{\partial y^3} dy \tag{22,23}$$

$$F_{ijk} = -2\int_{-y_1}^{y_2} Y_i Y_j \frac{\partial^2 Y_k}{\partial x \partial y} dy\,; \quad G_{ijk} = -\int_{-y_1}^{y_2} Y_i Y_j \frac{\partial Y_k}{\partial y} dy\,, \quad H_{ij} = Re\, a_{ij} - b_{ij} \tag{24-26}$$

$$I_{ij} = Re\, c_{ij} - d_{ij}\,, \quad J_{ij} = Re\, e_{ij} - f_{ij}\,, \quad K_{ij} = Re\, g_{ij} - h_{ij}\,, \quad L_i = Re\, i_i - j_i \tag{27-30}$$

$$a_{ij} = \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial^3 F}{\partial x^3} dy + \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y} \frac{\partial^3 F}{\partial x \partial y^2} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} \frac{\partial^3 F}{\partial x^2 \partial y} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} \frac{\partial^3 F}{\partial y^3} dy$$
$$+ \int_{-y_1}^{y_2} Y_i \frac{\partial^3 Y_j}{\partial x^3} \frac{\partial F}{\partial y} dy + \int_{-y_1}^{y_2} Y_i \frac{\partial^3 Y_j}{\partial x \partial y^2} \frac{\partial F}{\partial x} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial^3 Y_j}{\partial x^2 \partial y} \frac{\partial F}{\partial x} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial^3 Y_j}{\partial y^3} \frac{\partial F}{\partial x} dy \tag{31}$$

$$b_{ij} = \int_{-y_1}^{y_2} Y_i \frac{\partial^4 Y_j}{\partial x^4} dy + 2\int_{-y_1}^{y_2} Y_i \frac{\partial^4 Y_j}{\partial x^2 \partial y^2} dy \tag{32}$$

$$c_{ij} = -\int_{-y_1}^{y_2} Y_i Y_j \frac{\partial^3 F}{\partial x^2 \partial y} dy - \int_{-y_1}^{y_2} Y_i Y_j \frac{\partial^3 F}{\partial y^3} dy + 3\int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial x^2} \frac{\partial F}{\partial y} dy + \int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial y^2} \frac{\partial F}{\partial y} dy$$
$$+ 3\int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial x^2} \frac{\partial F}{\partial y} dy + \int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial y^2} \frac{\partial F}{\partial y} dy - 2\int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial x \partial y} \frac{\partial F}{\partial x} dy \tag{33}$$

$$d_{ij} = 4\int_{-y_1}^{y_2} Y_i \frac{\partial^3 Y_j}{\partial x^3} dy + 4\int_{-y_1}^{y_2} Y_i \frac{\partial^3 Y_j}{\partial x \partial y^2} dy\,, \quad e_{ij} = 3\int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x}\frac{\partial F}{\partial y} dy - \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial y}\frac{\partial F}{\partial x} dy \tag{34,35}$$

$$f_{ij} = 6\int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial x^2} dy + 2\int_{-y_1}^{y_2} Y_i \frac{\partial^2 Y_j}{\partial y^2} dy\,, \quad g_{ij} = \int_{-y_1}^{y_2} Y_i Y_j \frac{\partial F}{\partial y} dy\,, \; h_{ij} = 4\int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x} dy \tag{36,37}$$

$$i_i = \int_{-y_1}^{y_2} Y_i \left(\frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial x^3} + \frac{\partial F}{\partial y}\frac{\partial^3 F}{\partial x \partial y^2} - \frac{\partial F}{\partial x}\frac{\partial^3 F}{\partial x^2 \partial y} - \frac{\partial F}{\partial x}\frac{\partial^3 F}{\partial y^3} \right) dy\,, \quad j_i = \int_{-y_1}^{y_2} Y_i \left(\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} \right) dy \tag{38,39}$$

Analyzing the ODE system given by Eq. (18) we observe that the integral coefficients depend on the axial position x. This would imply in a high computational cost if the coefficients would be numerically calculated, once they need to be reevaluated along the solution procedure for the ordinary differential equation system. A strategy to overcome this is analytically to calculate them through the software of symbolic computation Mathematica (2000). The calculation procedure is made in such a way that the same ones are calculated only once time and stored, after that they are multiplied by functions that take into account the dependence of the irregular domain each time that the differential system is solved.

It is necessary to truncate the infinite series in a number of terms sufficiently high to guarantee the relative error for obtaining the original potentials. Therefore, the ordinary differential equation system can be rewritten as

$$\begin{aligned} \frac{d^4\bar{\phi}_i}{dx^4} = -\mu_i^4\bar{\phi}_i + \frac{L_i}{N} + \frac{Re}{N}\sum_{j=1}^{NTV}\sum_{k=1}^{NTV} \Big\{ & A_{ijk}\bar{\phi}_j\bar{\phi}_k + B_{ijk}\bar{\phi}_j\frac{d\bar{\phi}_k}{dx} + C_{ijk}\bar{\phi}_j\frac{d^2\bar{\phi}_k}{dx^2} \\ & + D_{ijk}\bar{\phi}_j\frac{d^3\bar{\phi}_k}{dx^3} + E_{ijk}\frac{d\phi_j}{dx}\phi_k + F_{ijk}\frac{d\phi_j}{dx}\frac{d\phi_k}{dx} + G_{ijk}\frac{d\phi_j}{dx}\frac{d^2\phi_k}{dx^2} \Big\} \\ & + \frac{1}{N}\sum_{j=1}^{NTV}\left\{ H_{ij}\bar{\phi}_j + I_{ij}\frac{d\bar{\phi}_j}{dx} + J_{ij}\frac{d^2\bar{\phi}_j}{dx^2} + K_{ij}\frac{d^3\bar{\phi}_j}{dx^3} \right\} \end{aligned} \tag{40}$$

where NTV is the order of truncation of the infinite series.

Therefore, to solve the system by efficient numerical algorithms for boundary value problems, such as the subroutine DBVPFD from the IMSL Library (1991), which offers an automatic adaptive scheme for local error control of the results for the transformed potentials, it is necessary rewritten the system as a first order one, in the form

$$\chi_i = \bar{\phi}_i\,; \quad \frac{d\chi_i}{dx} = \chi_{NTV+i} = \frac{d\bar{\phi}_i}{dx}\,; \quad \frac{d\chi_{NTV+i}}{dx} = \chi_{2NTV+i} = \frac{d^2\bar{\phi}_i}{dx^2} \tag{41a-c}$$

$$\frac{d\chi_{2NTV+i}}{dx} = \chi_{3NTV+i} = \frac{d^3\bar{\phi}_i}{dx^3}\,; \quad \frac{d\chi_{3NTV+i}}{dx} = \frac{d^4\bar{\phi}_i}{dx^4} \tag{41d,e}$$

Therefore by making use of Eqs. (41), the system can be rewritten as

$$\frac{d\chi_i}{d\tau} = \frac{\chi_{NTV+i}}{\left(\frac{d\tau}{dx}\right)}\,; \quad \frac{d\chi_{NTV+i}}{d\tau} = \frac{\chi_{2NTV+i}}{\left(\frac{d\tau}{dx}\right)}\,; \quad \frac{d\chi_{2NTV+i}}{d\tau} = \frac{\chi_{3NTV+i}}{\left(\frac{d\tau}{dx}\right)} \tag{42a-c}$$

$$\begin{aligned} \frac{d\chi_{3NTV+i}}{d\tau} = \Bigg\{ -\mu_i^4\chi_i + \frac{L_i}{N} + \frac{Re}{N}\sum_{j=1}^{NTV}\sum_{k=1}^{NTV} \Big[& A_{ijk}\chi_j\chi_k + B_{ijk}\chi_j\chi_{NTV+k} + C_{ijk}\chi_j\chi_{2NTV+k} \\ & + D_{ijk}\chi_j\chi_{3NTV+k} + E_{ijk}\chi_{NTV+j}\chi_k + F_{ijk}\chi_{NTV+j}\chi_{NTV+k} \\ & + G_{ijk}\chi_{NTV+j}\chi_{2NTV+k} \Big] + \frac{1}{N}\sum_{j=1}^{NTV}\Big[H_{ij}\chi_j + I_{ij}\chi_{NTV+j} \\ & + J_{ij}\chi_{2NTV+j} + K_{ij}\chi_{3NTV+j} \Big] \Bigg\} \Big/ \left(\frac{d\tau}{dx}\right) \end{aligned} \tag{43}$$

where $\tau = 1 - e^{-cx}$, which is related to c a parameter of scale compression for the case of infinite duct, and $0 \le \tau \le 1$.

In the analysis of the irregular duct, the boundary conditions in the x direction necessary for the solution of the ordinary differential equation system, Eq. (43), are given in two ways: it is considered a truncated duct and the version for considering the infinite duct. At the duct entrance, for the two versions it is considered that u = 0 and v = 0. At the duct outlet, each case is separately analyzed. In the version of the truncated duct, we consider that $\partial\omega/\partial x = 0$ and v = 0, where ω is the vorticity. For the infinite duct, $u = u_\infty(y)$ and v = 0, where $u_\infty(y)$ is the velocity profile for the fully developed flow. These boundary conditions in terms of streamfunction are written as:

- For the truncated duct, in terms of the vector solution χ, we have

$$\chi_i(0) = 0, \quad \chi_{NTV+i}(0) = 0\ , \quad \chi_{NTV+i}(x_{out}) = -\frac{1}{N(x_{out})}\left[M_i + \sum_{j=1}^{NTV}\chi_j(x_{out})N_{ij}\right] \tag{44a-c}$$

$$\chi_{3NTV+i}(x_{out}) = -\frac{1}{N(x_{out})}\left\{ O_i + \sum_{j=1}^{NTV}\left[\chi_j(x_{out})P_{ij} + \chi_{NTV+j}(x_{out})Q_{ij} + \chi_{2NTV+j}(x_{out})R_{ij} \right]\right\} \tag{44d}$$

where

$$M_i = \int_{-y_1}^{y_2} Y_i \frac{\partial F}{\partial x}dy\ ; \quad N_{ij} = \int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x}dy \tag{45a,b}$$

$$O_i = \int_{-y_1}^{y_2} Y_i\left(\frac{\partial^3 F}{\partial x^3} + \frac{\partial^3 F}{\partial x \partial y^2}\right)dy\ ; \quad P_{ij} = \int_{-y_1}^{y_2} Y_i\left(\frac{\partial^3 Y_j}{\partial x^3} + \frac{\partial^3 Y_j}{\partial x \partial y^2}\right)dy \tag{45c,d}$$

$$Q_{ij} = \int_{-y_1}^{y_2} Y_i\left(3\frac{\partial^2 Y_j}{\partial x^2} + \frac{\partial^2 Y_j}{\partial y^2}\right)dy\ ; \quad R_{ij} = 3\int_{-y_1}^{y_2} Y_i \frac{\partial Y_j}{\partial x}dy \tag{45e,f}$$

- For the infinite duct, in terms of the vector solution χ, we have

$$\chi_i(0) = 0, \quad \chi_{NTV+i}(0) = 0\ , \quad \chi_i(1) = 0, \quad \chi_{NTV+i}(1) = 0 \tag{46a-d}$$

4. Results and discussion

We analyze the wavy wall duct whose geometry is shown in Fig (2). The functions that describe this geometry in dimensionless terms are given as

$$y_1(x) = 1 + f(x); \quad y_2(x) = 1 + f(x)\ ; \quad f(x) = \alpha \cdot \sin\left[\pi(x-3)\right] \tag{47a-c}$$

where $\alpha = a^*/b$ is the dimensionless amplitude of the wavy surface, and the value of the axial coordinate at the duct outlet x_{out} was taken as $x_{out} = 20$. In the present analysis, the interval used for the axial coordinate x was $3 \le x \le 15$, which corresponds to six complete sinusoidal waves.

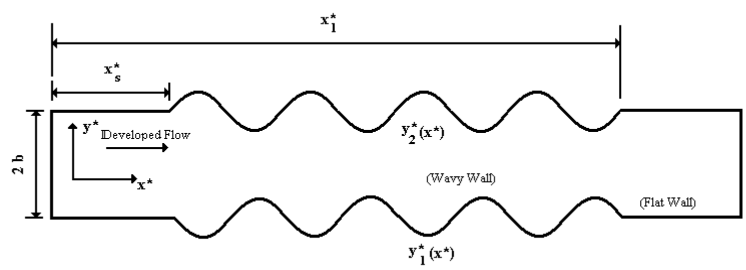

Figure 2. Wavy wall duct analyzed.

Tables (1) and (2) shows the convergence of the streamfunction along the line y = 0.5 for Re = 10 and α = 0.1 and Re = 100 and α = 0.2, respectively. An excellent convergence is observed for low truncation orders NTV = 6 at the duct inlet for Re = 10 and α = 0.1, satisfying the requested tolerance. For the case of Re = 100 and α = 0.2, it is observed that the convergence with NTV = 18. The results obtained with NTV = 30 shown that the original potential remained unaffected, therefore guaranteed the precision in the convergence analysis.

Table 1. Convergence behavior of the streamfunction at y = 0.5 for Re = 10 and α = 0.1.

Infinite Duct							
	NTV						
x	6	10	14	18	22	26	30
0	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875
3	0.7117	0.7117	0.7117	0.7117	0.7117	0.7117	0.7117
5	0.7092	0.7092	0.7092	0.7092	0.7092	0.7092	0.7092
7	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
9	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
11	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
13	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
15	0.6861	0.6861	0.6861	0.6861	0.6861	0.6861	0.6861
20	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875
Truncated Duct							
	NTV						
x	6	10	14	18	22	26	30
0	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875
3	0.7117	0.7117	0.7117	0.7117	0.7117	0.7117	0.7117
5	0.7092	0.7092	0.7092	0.7092	0.7092	0.7092	0.7092
7	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
9	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
11	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
13	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096	0.7096
15	0.6861	0.6861	0.6861	0.6861	0.6861	0.6861	0.6861
20	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875

Table 2. Convergence behavior of the streamfunction at y = 0.5 for Re = 100 and α = 0.2.

Infinite Duct							
	NTV						
x	6	10	14	18	22	26	30
0	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875
3	0.7321	0.7326	0.7327	0.7327	0.7327	0.7327	0.7327
5	0.7312	0.7380	0.7390	0.7391	0.7391	0.7391	0.7391
7	0.7470	0.7523	0.7531	0.7532	0.7532	0.7532	0.7532
9	0.7539	0.7588	0.7595	0.7595	0.7593	0.7593	0.7593
11	0.7577	0.7624	0.7630	0.7631	0.7631	0.7631	0.7631
13	0.7599	0.7646	0.7652	0.7652	0.7652	0.7652	0.7652
15	0.7167	0.7204	0.7211	0.7212	0.7212	0.7212	0.7212
20	0.7147	0.7157	0.7159	0.7159	0.7159	0.7159	0.7159
Truncated Duct							
	NTV						
x	6	10	14	18	22	26	30
0	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875	0.6875
3	0.7322	0.7326	0.7327	0.7327	0.7327	0.7327	0.7327
5	0.7312	0.7380	0.7390	0.7391	0.7391	0.73912	0.7391
7	0.7470	0.7523	0.7531	0.7532	0.7532	0.7532	0.7532
9	0.7539	0.7588	0.7595	0.7595	0.7595	0.7595	0.7595
11	0.7577	0.7624	0.7630	0.7631	0.7631	0.7631	0.7631
13	0.7599	0.7645	0.7652	0.7652	0.7652	0.7652	0.7652
15	0.7167	0.7204	0.7211	0.7212	0.7212	0.7212	0.7212
20	0.7135	0.7157	0.7159	0.7159	0.7159	0.7159	0.7159

Figure (3) shows the streamlines for Re = 10 and α = 0.1. It is observed a strong influence of the amplitude. For low amplitude, it is not verified the appearance of recirculations near the duct wall, due to low convective effects in relation to diffusive ones attached to the Reynolds number. In Fig. (4), it is verified the appearance of this recirculation pattern near the duct wall for Re = 100 and α = 0.2, where it is evidenced the influence of the amplitude, consequently due to higher convective effects associated with higher Reynolds number.

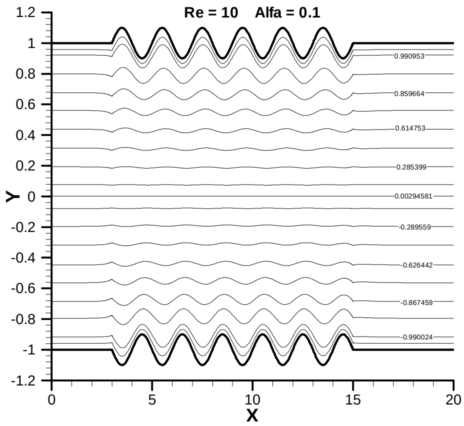

Figure 3. Streamlines for Re = 10 and α = 0.1.

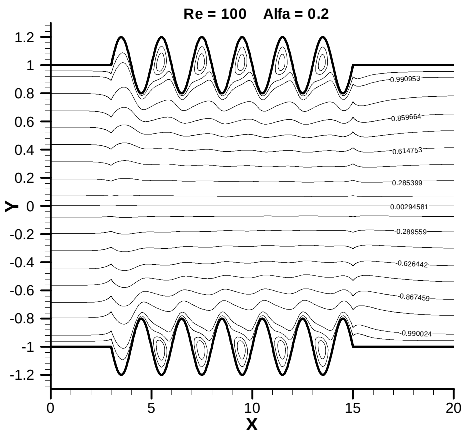

Figure 4. Streamlines for Re = 100 and α = 0.2.

Figure (5) and (6) show a comparison of the present results for the product of the friction factor-Reynolds number against those of Wang and Chen (2002), where it is verified an excellent agreement among the two sets of results. Also, it is shown the influence of the Reynolds number and of the amplitude in this parameter. For higher Reynolds number, i.e., higher convective effects, the product fRe tends to diminish, while the amplitude makes an increasing in the product fRe, due mainly the appearance of recirculation zones in the flow.

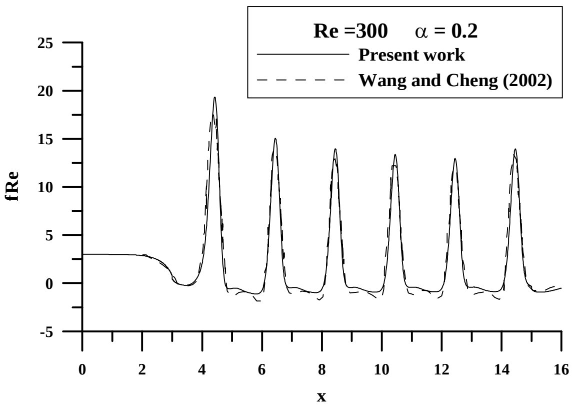

Figure 5. Comparison of the product of the friction factor-Reynolds number for Re = 300 and α = 0.2.

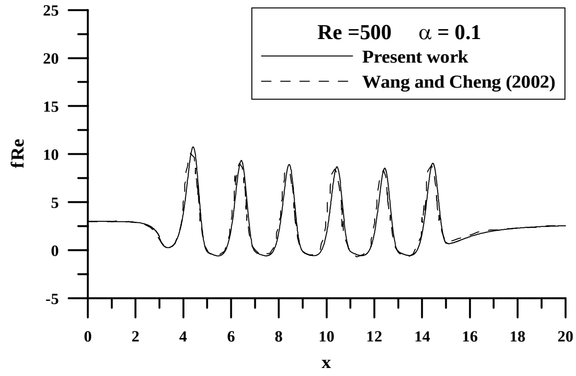

Figure 6. Comparison of the product of the friction factor-Reynolds number for Re = 500 and α = 0.1.

Figure (7) illustrates vorticity wall for different Reynolds numbers and different values of amplitudes. It is observed a higher vorticity with increasing Reynolds number and increasing amplitudes. It is still verified that for low Reynolds number (Re = 10 and α = 0.3), the values of the vorticity are near of vorticity values for Re = 10 and α = 0.1.

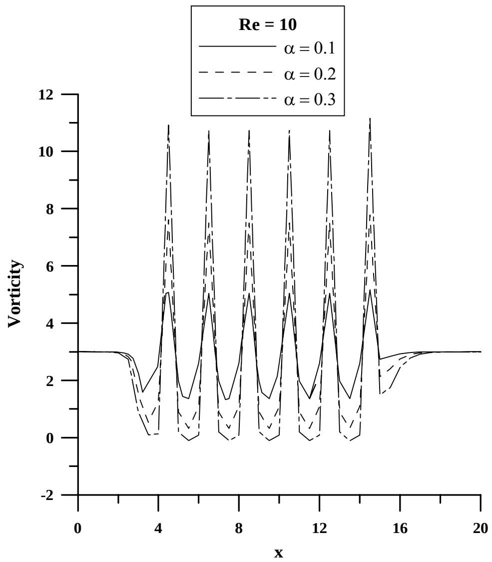

Figure 7. Vorticity at the duct wall for Re = 10 and different values of amplitude.

In Figs (8) to (10), it is observed the development of longitudinal velocity profile for different Reynolds number and different amplitudes values in the wavy wall duct. The evidence more markedly in these figures are the distortions of the profiles for higher Reynolds number and values of amplitude. This behavior was expected, since for these situations the convective effects are more markedly.

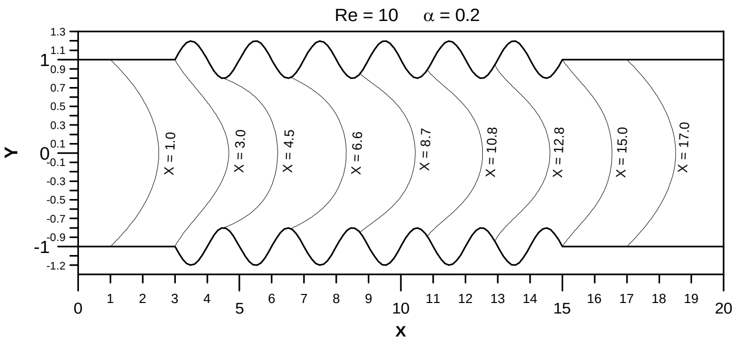

Figure 8. Development of the longitudinal velocity profile for Re = 10 and α = 0.2.

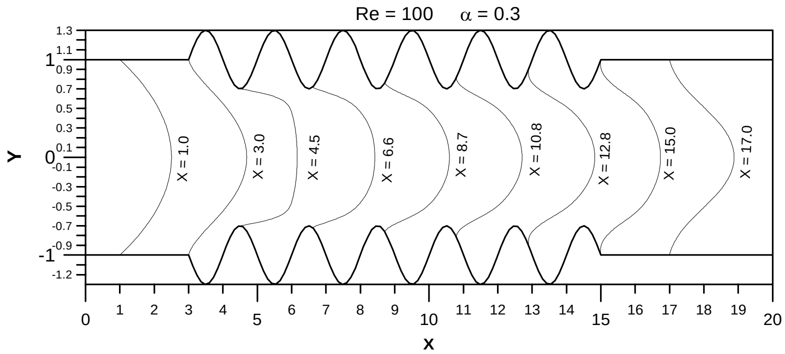

Figure 9. Development of the longitudinal velocity profile for Re = 100 and α = 0.3.

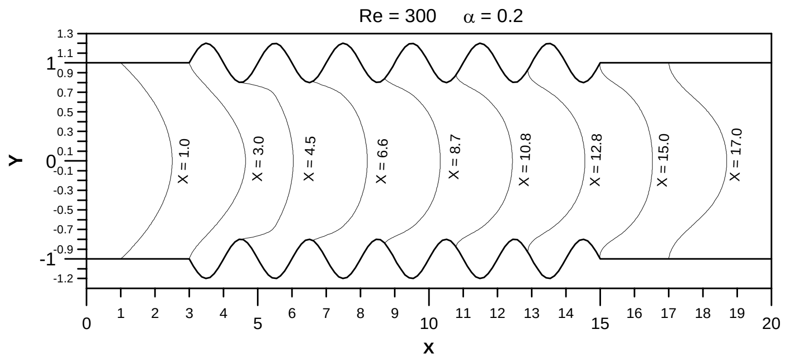

Figure 10. Development of the longitudinal velocity profile for Re = 300 and α = 0.2.

5. Conclusions

The results demonstrated the applicability of the Generalized Integral Transform Technique (GITT) as an appropriate tool to solve flow problems in irregular ducts involving the Navier-Stokes equations.

The strong influence of the Reynolds number and of the amplitude of the wavy surface in the results suggests that these types of ducts may be employed to intensify heat transfer rates. Therefore, the analysis of energy equation through the GITT approach will be the next natural step to elucidate some questions involving this geometric configuration.

6. References

Asako, Y., Nakamura, H. and Faghri, M., 1988, “Heat Transfer and Pressure Drop Characteristics in a Corrugated Duct with Rounded Corners”, International Journal of Heat and Mass Transfer, Vol. 31, pp. 1237-1245.

Goldstein, L. and Sparrow, E. M., 1977, “Heat/Mass Characteristics for Flow in a Corrugated Wall Channel”, Journal of Heat Transfer, Vol. 99, pp. 187-195.

IMSL Library, 1991, MATH/LIB, Houston, TX.

Kays, W. M. and London, A. L., 1984, “Compact Heat Exchangers”, McGraw-Hill, New York.

Mathematica, 2000, Standard Version 4.1, Champaign, Illinois.

Pérez Guerrero, J. S., 1995, “Integral Transformation of the Navier-Stokes Equations for Laminar Flow in Channels of Arbitrary Two-dimensional Geometry”, D.Sc. Thesis (in Portuguese), PEM/COPPE, Rio de Janeiro.

Pérez Guerrero, J. S., Quaresma, J. N. N. and Cotta, R. M., 2000, “Simulation of Laminar Flow inside Ducts or Irregular Geometry Using Integral Transforms”, Computational Mechanics, Vol. 25, pp. 413-420.

Sunden, B., and Trollheden, S., 1989, “Periodic Laminar Flow and Heat Transfer in a Corrugated Two-Dimensional Channel”, International Communications in Heat Mass and Transfer, Vol. 16, pp. 215-225.

Wang, C.-C. and Chen, C.-K., 2002, “Forced Convection in a Wavy-wall Channel”, International Journal of Heat and Mass Transfer, Vol. 45, pp. 2587-2595.

Xiao, Q., Xin, R. C. and Tao, W. Q., 1989, “Analysis of Fully Developed Laminar Flow and Heat Transfer in Asymmetric Wavy Channels”, International Communications in Heat and Mass Transfer, Vol. 16, pp. 227-236.