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REVISITING THE LAW OF THE WALL FOR TWO-PHASE TURBULENT BOUNDARY LAYERS

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Abstract. *This paper revisits a particular analytical procedure that has been used in the past for the deviation of a two-phase flow law of the wall. The procedure decomposes the total liquid turbulent stress into the sum of a bubble induced local stress component and a shear induced stress component so that a first integration of the local motion equation can be made. Here, a different formulation is proposed for the shear stress component, so that a new expression for the law of the wall is arrived at. The present findings are compared with the analytical and experimental data of other authors.*

keywords: *Two-phase flow, law of the wall, eddy viscosity.*

1. Introduction

The existence of a logarithmic mean velocity profile in the wall region of a single-phase turbulent flow is a well established fact. A particular interest in representing the near wall solution through an analytical expression relies on the specification of boundary conditions for numerical codes. More significantly though, is the extensive use of near wall logarithmic solutions for predictions of the skin-friction coefficient and of the heat transfer coefficient.

For a two-phase bubbly flow, early studies tended to believe that very close to wall the classical single-phase logarithmic law persisted. However, one needs to realize that because most of the pipes used in the early investigations had small diameters, the placement of standard local probes near to the wall was a problem that seriously compromised detailed studies. In fact, as early as 1981, Sato et al. observed an evident discrepancy between the velocity profile given by the single-phase logarithmic distribution and the velocity profile given by the bubbly flow logarithmic distribution. The emphasis of their work was to propose an eddy diffusivity model to account for the transfer of momentum and heat in a bubbly flow.

To overcome some of the experimental difficulties, some authors (Moursali et al. (1995), Marié et al. (1997)) turned their attention to the case of a turbulent boundary layer developing over a vertical, smooth, flat plate in the presence of milimetric bubbles. Their conclusion was that the single-phase logarithmic law of the wall is definitively altered when subject to the presence of milimetric bubbles. Then, with some scaling arguments and by splitting the void fraction profile into a rectangular step distribution, Marié et al. (1997) arrived at a modified law of the wall for bubbly flow.

More recently, in 2001, Troshko and Hassan developed a two-phase law of wall using arguments similar to those of Sato et al. (1981a). By splitting the total liquid turbulent stress into the sum of a bubble induced local stress component and a shear induced stress component, local motion equation are integrated to yield a logarithmic law. To account for non-linear interactions between the two stress components, an arbitrary proportionality coefficient was introduced that should be empirically determined.

The purpose of this work is basically to revisit the developments of Troshko and Hassan (2001) and, in doing so, to propose a new law of the wall for bubbly flow. The fundamental modification in the theory of Troshko and Hassan (2001) concerns the specification of the shear induced eddy viscosity. Although very subtle, this modification will lead to a much different expression for the law of the wall. The present new law of the wall is tested against the results of Sato et al. (1981b).

2. A short review on the state of the art.

Before we move to the current developments, let us present to the reader a short revision on the subject of finding a logarithmic law of the wall for bubbly flow.

Zun (1980) has very rightly pointed out that observations of peak void fraction values in the wall region suggest a transverse bubble migration from the core region to the wall for certain bubbly flow regimes. Early studies proposed that the Magnus and the Zhukovski forces could be the responsible for such behaviour. The influence of the liquid velocity gradient, the static pressure change across channel due to turbulence, for example, were other suggested physical mechanisms for this effect. In his work, Zun treated the non-equilibrium bubble transverse migration by combining the bubble dispersion and the circulation of liquid around the bubble provoked by the liquid velocity gradient.

Sato et al.(1981a) proposed a theory to describe the transfer processes of momentum and of heat in a two-phase bubbly flow in a channel. Basically, the turbulent structure of the liquid phase was considered to be divided into two components, one dependent only on the shear stress of the liquid phase and the other on the additional turbulence caused by the bubbles. The theory permitted the prediction of mean liquid velocity profiles and frictional pressure gradients for a given void friction profile. With some analogy arguments, the mean liquid temperature profiles and heat transfer coefficients could also be calculated provided the wall heat flux was known.

In a follow up paper, Sato et al. (1981b) conducted measurements in a circular pipe to corroborate their previous theoretical results. Comparisons were made for the predictions of velocity and of temperature profiles. The overall agreement between data and theory was found to be good.

In the same year, van der Welle (1981) proposed an empirical correlation for the turbulence viscosity in a two-phase flow. The correlation was also based on the assumption that the turbulent field could be divided into two independent components: one due to the momentum exchange of the liquid phase, the other due to the movement of the dispersed phase. Tests of the correlation against data from various authors were performed, showing a standard deviation of 22 per cent.

Beyerlein et al. (1985) early reported that the bubble concentration profile in a vertical upward flow is wall-skewed. To predict this type of behaviour, the authors incorporated into the equations of motion a lateral force due to the relative velocity of the two phases and the eddy diffusivity of the liquid. A good agreement was noted between the theoretical predictions and the experimental data.

The three dimensional turbulence structure and the phase distribution in bubbly two-phase flows were investigated by Wang et al. ((1987). Using both single- and three-sensor hot-film anemometer probes, measurements of local void fraction, liquid velocity and Reynolds stresses were made. For up flows the authors found that the bubbles tend to migrate to the wall whereas for down flows the bubbles migrated to the pipe center. These two distinct trends result respectively in a void fraction peaking at the wall and in a "coring" phenomena that, can be predicted by considering the turbulence structure of the continuous phase and the lateral lift force acting on the bubbles. Measurements of the Reynolds stress components showed nearly flat profiles in the core region, but an anisotropic structure near the wall. The presence of bubbles in a liquid flow is normally observed to increase the level of turbulence. Wang et al., however, observed that for the higher flow rates the bubbles suppressed turbulence.

The modelling of the skin-friction and of the heat transfer in bubbly up flows in pipes was considered by Marié (1987). Two main considerations were used by the author: the persistence of the logarithmic region and the existence of a similarity between the modifications caused by the bubbles and those that would be caused by a grid in a single phase flow. Therefore, the resulting laws were supposed to be valid just for low gas concentrations. The model was shown to work well for up to 0.2-0.3 void fractions.

The near wall flow structure of a two-phase turbulent boundary layer was studied by Moursali et al. (1995) in the case of a vertical flat plate. Graphs of void fraction distribution, wall shear stress and liquid mean velocity profiles were presented for different mean bubble diameter. An important result was the realization that a significant fraction of the bubbles was deflected toward the wall depending on their size. This migration together with a marked deceleration of the bubbles in the near wall region proved to be the two main mechanisms that are responsible for the so-called void peaking phenomenon. The presence of the dispersed phase was found to increase the skin-friction coefficient. This increase is reflected on a modification of the classical law of the wall, and on a depression of the wake.

The kinematics and the turbulent structure of a bubbly boundary layer at low air concentrations were detailed studied by Marié et al. (1997). These authors were positive in saying that in the presence of milimetric bubbles the logarithmic law of thew wall is modified. Then, through simple analytical considerations and dimensional analysis, a modified law was proposed. The wall friction calculated on the basis of the new law was shown to fit well the experimental data. The authors also presented longitudinal turbulence intensity profiles and showed that turbulence is increased by two main mechanisms: a modification of the wall production and the creation of pseudo-turbulence in the external layer. The mixing length calculated from the data was compared with some other models proposed in literature.

The effects of bubble size and of two-phase flow rates on the wall shear stress were investigated by Liu (1997) experimentally. Using a flush-mounted hot film sensor, the time varying fluctuations of the wall shear stress were measured in a air-water bubbly flow in a vertical channel. The reported experiments were unique in the sense that a special bubble generator was capable of decoupling the bubble size effect from the inlet conditions. Thus, the experiments were carried out under various fixed gas and liquid fluxes, with only the bubble size being a variable. The data show that the wall shear stress is strongly influenced by the wall structure of the flow, while both the liquid phase velocity and the wall concentrated bubbles are the dominant parameters on both the magnitude and the fluctuation intensity of the wall shear stress in the regime of bubbly flow. The findings were compared with the data of other authors as well as with other models for the prediction of the wall shear stress.

Troshko and Hassan (2001) developed a new formulation for the law of the wall considering the total liquid turbulent stress to result from the summation of the bubble induced local stress and the shear induced stress. Both stress components were estimated through the Boussinesq turbulent viscosity approximation. The non-linear interaction between the shear and the bubble induced turbulence fields was accounted by a proportionality coefficient. The authors conclude through a numerical simulation that the new law performs better than the classical single-phase law.

3. The bubbly flow law of the wall

The analysis of Troshko and Hassan (2001) will be presented next. Then, it will be repeated with a different expression for the shear induced viscosity. This procedure will lead to the modified law of the wall presented here.

Consider an incompressible, isothermal, two-phase, turbulent boundary layer in a Cartesian coordinate system.

The x-component of the liquid momentum equation can be written as (Troshko and Hassan (2001))

$$\frac{\partial(\rho_l\alpha_l U_l U_l)}{\partial x}+\frac{\partial(\rho_l\alpha_l U_l V_l)}{\partial y}=-\alpha_l\frac{\partial P}{\partial x}+F_x+\rho_l\alpha_l g_x+\frac{\partial}{\partial x}\Big(\rho_l\alpha_l\Big(2\nu_l\frac{\partial U_l}{\partial x}-u^2\Big)\Big) + \frac{\partial}{\partial y}\Big[\rho_l\alpha_l\Big(\nu_l\Big(\frac{\partial U_l}{\partial y}+\frac{\partial V_l}{\partial x}\Big)-uv\Big)\Big] \tag{1}$$

where U_l and V_l are the longitudinal and transversal components of the mean liquid velocity, u^2 and uv are the Reynolds stress components, F_x and g_x are the inter-facial force density and gravity projections and α_l is the local liquid void fraction.

To find a local solution for the fully turbulent region the standard procedure is to consider that there exists a region in the flow where the turbulence effects dominate on their own.

The immediate consequence is that Eq. 1 is reduced to the much simpler form

$$\frac{\partial}{\partial y}\Big(-\alpha_l uv\Big)=0. \tag{2}$$

To integrate the above equation, the specification of a turbulence model is necessary. The simplest possible way is to introduce the turbulent viscosity concept. Upon a simple integration, this leads to

$$\alpha_l\nu_t\frac{\partial U_l}{\partial y}=\frac{\tau_w}{\rho_l}=U_\tau^2. \tag{3}$$

where $\tau_w=[\alpha_l\rho_l\nu_l(\partial U_l/\partial y)]_{y=0}$ is the two-phase wall shear stress; U_τ is the friction velocity.

The total turbulent viscosity, following Sato et al. (1981) and according to Troshko and Hassan (2001), can be written as

$$\nu_t=\nu_t^{out}+\nu_t^{in}, \tag{4}$$

where ν_t^{out} and ν_t^{in} are respectively the turbulent shear stress and bubble induced viscosities.

Troshko and Hassan (2001) comment that this linear behaviour is only acceptable for boundary layer void fractions below 10%. The two component of the turbulent viscosity were considered by these authors to have the form

$$\nu_t^{out}=\varkappa y U_\tau, \tag{5}$$

$$\nu_t^{in} = \varkappa_l \alpha_{g\ max} U_R y, \tag{6}$$

where $\varkappa$ denotes the single-phase value of the von Karman constant, $\varkappa_l$ is a non-linearity empirical coefficient, α_g is the local gas fraction, $\alpha_{g\ max} = \max(\alpha_g | 30 \le y^+ \le 200)$ and U_R is the slip velocity.

To find a logarithmic behaviour, Troshko and Hassan argue that $\alpha_l \nu_t$ must be proportional to y. Then, since the void fraction is an unknown, they assume that $\alpha_l \cong 1 - \alpha_{g\ max}$.

Substitution of Eqs. 5 and 6 onto Eqs. 2 and 3, followed by an integration, results

$$\frac{dU_l}{\beta U_\tau} = \frac{dy}{\varkappa y}. \tag{7}$$

where the two-phase flow scaling coefficient is

$$\beta = \Big[(1 + \frac{\varkappa_l \alpha_{g\ max} U_R}{\varkappa U_\tau})(1 - \alpha_{g\ max}) \Big]^{-1}. \tag{8}$$

The integration of equation 7 furnishes

$$U_+^x = \frac{1}{\varkappa} \ln(y_+^x) + B^x, \tag{9}$$

where the wall variables y_+^x and U_+^x are defined through the new velocity scale $U_\tau^x = \beta\, U_\tau$ and B^x is an additive function that has to be determined from the experimental data. Note that as $\alpha_{g\ max}$ tends to zero Eq. 9 reduces to the classical single-phase law of the wall.

In the present work, however, we propose a new derivation for Eq. 9. Instead of considering Eq. 5 to hold, let it be

$$\nu_t^{out} = \varkappa^2 y^2 \frac{\partial U_l}{\partial y}, \tag{10}$$

Then, Eq. 2 becomes

$$\frac{\partial}{\partial y} \Big((1 - \alpha_{g\ max}) \Big[\varkappa^2 y^2 \frac{\partial U_l}{\partial y} + \varkappa_l \alpha_{g\ max} U_r y \Big] \frac{\partial U_l}{\partial y} \Big) = 0. \tag{11}$$

The integration of Eq. 11 furnishes

$$(1 - \alpha_{g\ max}) \Big[\varkappa^2 y^2 \frac{\partial U_l}{\partial y} + \varkappa_l \alpha_{g\ max} U_r y \Big] \frac{\partial U_l}{\partial y} = U_\tau^2, \tag{12}$$

that is,

$$\Big(\varkappa y \frac{\partial U_l}{\partial y} \Big)^2 + (\frac{\varkappa_l}{\varkappa} \alpha_{g\ max} U_r) \Big(\varkappa y \frac{\partial U_l}{\partial y} \Big) = \frac{U_\tau^2}{(1 - \alpha_{g\ max})}. \tag{13}$$

Solving the above second order algebraic equation, we find again Eq. 7, but with

$$\beta = \frac{\varkappa_l \alpha_{g\ max} U_r}{2 \varkappa U_\tau} \Big(\sqrt{1 + \frac{(2 \varkappa U_\tau)^2}{(\varkappa_l \alpha_{g\ max} U_r)^2 (1 - \alpha_{g\ max})}} - 1 \Big). \tag{14}$$

This expression clearly has a distinct asymptotic behaviour from Eq. 8 as $\alpha_{g\ max}$ tends to zero. This distinct behaviour must be reflected by the law of the wall, Eq. 9.

4. Validation

To validate the present formulation, the data of Sato et al. (1981) will be used.

The bubbles slip velocity is evaluated from (Ishii and Zuber(1979))

$$U_r = [4g\sigma\Delta\rho/\rho_l^2]^{1/4}(1-\alpha_{g\ max})^{3/4}, \tag{15}$$

where σ is the surface tension and $\Delta\rho$ is the density difference of the phases.

As remarked by Troshko and Hassan (2001), due to the void peaking near the wall, slip velocity calculated through 15 is minimal in the boundary, in accordance with the data of Marié et al. (1997).

For the non-linear coefficient, $\varkappa_l$, Troshko and Hassan (2001) propose the following expression:

$$\varkappa_l = 4.9453\, exp(-40.661U_\tau), \tag{16}$$

where the friction velocity is given in m/s.

Tables 1 and 2 show the physical properties and the flow conditions used in the present validation, where y_0 denotes the viscous sublayer non-dimensional thickness, B denotes the additive parameter in the single-phase law of the wall, and $\varkappa$ stands for the von Karman constant.

Table 1: Physical properties of fluids.

ρ_{water} $[kg/m^3]$	ρ_{air} $[kg/m^3]$	g $[m/s^2]$	σ $[N/m]$
1000	1.225	9.81	0.04

Table 2: Flow properties.

$\alpha_{g\ max}$	u_τ $[m/s]$	y_0	B	$\varkappa$
0.181	0.0463	11	5	0.4

The additive parameter in the two-phase law of the wall, B^x is calculated by the procedure introduced by Marie et al. (1997). Thus

$$B^x = y_0\Big(1-\beta\Big) + \beta B. \tag{17}$$

The resulting calculated parameters are shown in Table 3.

Table 3: Calculated parameters.

u_r $[m/s]$	$\varkappa_l$	$B^x_{TroshkoandHassan}$	$B^x_{Present}$	$\beta_{TroshkoandHassan}$	$\beta_{Present}$
0.171517	0.752641	7.75469	7.14362	0.540884	0.64273

The logarithmic profiles introduced by Troshko and Hassan (2001) and by the present formulation are shown in Fig. 1 as compared with the data of Sato et al.(1981). Here, the following expression was used for data reduction,

$$U^+ = \frac{\beta}{\varkappa}\ln(y^+) + \Big(\beta B^x + \frac{\beta}{\varkappa}\ln\beta\Big), \tag{18}$$

where U^+ and y^+ are the standard single-phase wall variables.

Please, note that in the limiting case of α tending to zero, β tends to one and Eq. 18 coincides identically with the single-phase law of the wall.

The dependence of the flow on the second phase is, therefore, incorporated directly in the angular and linear coefficients of Eq. 18.

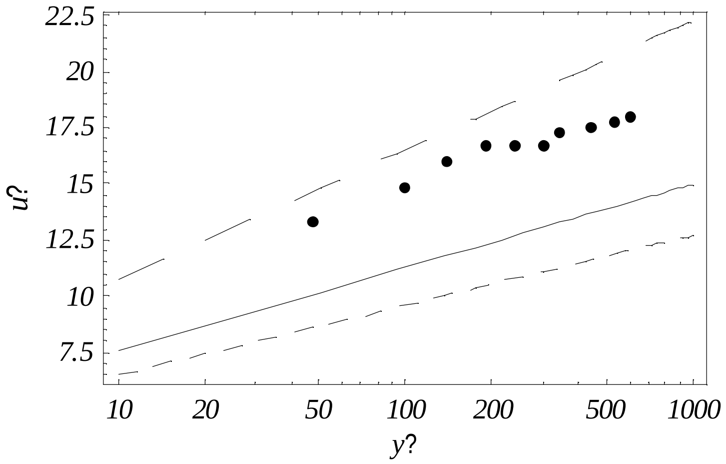

Figure 1: Two-phase law of the wall. Dots, data of Sato et al (1981b); top dashed line, single-phase law of the wall; solid line, present formulation; bottom dashed line, Troshko and Hassan (2001).

The results showed in Fig. 1 are quite different from the results presented by Troshko and Hassan (2001) in their Fig. 2. The data of Sato et al. (1981) shows that both the angular and the linear coefficients of the single-phase law of the wall decrease as a second phase is introduced in the fluid. For this reason, it is just natural that any advanced theory try to introduce a multiplying parameter, in our case β, that can reasonably account for this behaviour.

The problem with Fig. 1 is that all presently calculated values of β appear to be too small. In fact, it looks to the present author that the values for $\varkappa_l$ introduced by Eq. 16 seem to be too large. Please, bear in mind that $\varkappa_l$ is an artificial parameter introduced in the analysis to account for so-called non-linearity effects. This parameter is determined by Troshko and Hassan directly from the data of Sato et al. (1981), working effectively as a fitting curve parameter for a particularly given data set.

5. Final remarks

The present work has performed a preliminary analysis of the law of the wall for two-phase flows. In the curse of the research, a new expression has been derived for the law of the wall, which has been compared with the expression of Troshko and Hassan (2001). According to the present calculations, the agreement of both formulations as implemented through the expressions of $\varkappa_l$ advanced by Troshko and Hassan (2001) has been very poor. The calculation were made very judiciously, always furnishing the same results. Presently, our findings are being compared with the data of other authors so that a better understanding of the problem can be achieved.

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