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A NUMERICAL INVESTIGATION OF HEAT TRANSFER MECHANISMS IN GAS-SOLID FLUIDIZED BEDS USING THE TWO-FLUID GRANULAR TEMPERATURE MODEL

Jefferson L. M. A. Gomes
Imperial College London, Earth Science and Engineering Department, London SW7 AZ, UK
j.gomes@imperial.ac.uk

Christopher C. Pain
Imperial College London, Earth Science and Engineering Department, London SW7 AZ, UK
c.pain@imperial.ac.uk

Cassiano R. E. de Oliveira
Imperial College London, Earth Science and Engineering Department, London SW7 AZ, UK
c.oliveira@imperial.ac.uk

Anthony J. H. Goddard
Imperial College London, Earth Science and Engineering Department, London SW7 AZ, UK
a.goddard@imperial.ac.uk

Francisco B. S. Oliveira
Instituto Politécnico da UERJ, Nova Friburgo, RJ, Brazil
fbrunoso@iprj.uerj.br

Abstract. *The purpose of this work is to exploit numerical transport methods for modelling multiphase flow in complex geometries. A non-linear Petrov-Galerkin method together with a new non-linear bounded method which applies limiting in both space and time - the high resolution θ-method - are used to help discretize all the fluid transport equations. The methods described in this work are globally high order accurate in space and time and can thus resolve detailed spatially evolving fields. In this work, 2-D and 3-D fluidized beds are modelled through the Two-Fluid Granular Temperature Model (TFGTM). The conservative equations, i.e., continuity, momentum, thermal energy (for gas and solid phases) and fluctuation energy (for the granular phase), are solved, and the following space and time-dependent variables are calculated: solid volume fraction, granular temperature, velocities, pressure and temperatures. Numerical simulations of fluidized beds operating at standard pressure are conducted in the TFGTM framework using the finite-element CFD code, FLUIDITY. The heat exchange in the bubbles' wake is investigated by tracking the train of bubbles which rise along the heated wall. Large heat transfer coefficients were obtained in the rear wake region of bubbles due to relatively larger granular temperature there and intense particle circulation.*

keywords: *CFD, Fluidized bed, Two-Fluid Granular Temperature Model, Heat Transfer, High Resolution θ-Method*

1. Introduction

Gas-solid fluidized beds have excellent heat transfer rates with internal surfaces due to the strong mixing. These properties make fluidized beds an attractive proposition for many industrial processes (Kunii and Levenspiel, 1991). They are currently used in separation, classification, drying and mixing of particles, chemical reactions and regeneration processes. In this work, the complete field descriptions, provided by the two-fluid model, were used to obtain more accurate fluidized beds heat transfer predictions than those calculated from purely experimentally-obtained correlations. The modelling has been conducted using the finite-element based CFD code FLUIDITY (Pain et al., 2001; Pain et al., 2002; Pain et al., 2003b).

The two-fluid granular temperature model (TFGTM) assumes that both phases are an interpenetrating continuum. This requires additional closure laws to describe the rheology of the particulate phase. These closure laws are based, in this work, on the assumptions of the kinetic theory of granular flows. Comprehensive reviews about this subject can be found in Gidaspow, 1994. Ding and Gidaspow, 1990 developed a model to

predict the granular viscosity and stress for dry flows. They introduced the concept of granular temperature as a measure of the agitation of particles. Indeed, granular temperature provides a link between kinetic theory and traditional fluid mechanics.

Kuipers et al., 1992 numerically investigated the heat transferred from a heated wall into the bed, and compared some of the results with experiments and with penetration theory. In addition, the influence of the bubble wakes on thermal energy transfer was also studied. In both works, the influence of particle collisions on the heat transfer, and in particular on the diffusion coefficient were neglected. However, Schmidt and Renz, 2000 used correlations derived from the kinetic theory to calculate the effective thermal conductivity. These correlations were developed by Natarajan and Hunt, 1998 using the analogy of between gas molecules collisions and inelastic collisions in dense granular flows. Schmidt and Renz simulated a lab-scale fluidized bed, working in bubbling regime, with an immersed heated cylinder. They performed a few numerical simulations and calculated the heat transfer coefficients. Their results were compared with experimental results due to Sunderesan and Clark, 1995. In addition, some simulations using a standard correlation for the effective thermal conductivity, as used by Kuipers et al., 1992, were also performed and the results were compared with those obtained using a kinetic-based correlation.

In this work, the two-fluid granular temperature model was used to investigate the heat transfer in 2-D and 3-D gas-solid fluidized beds. In order to investigate the heat exchanged between the heated surface and the bed, the following fields were calculated from the set of conservative equations: solid volume fraction, granular temperature, gas and solid phase temperatures and velocity components for each phase. The thermal conductivity of the solid phase is obtained from a granular temperature-based function and interphase heat transfer is calculated from the Gunn correlation (Gunn, 1978). As the bubbles' wake plays an important role in the mixing and in the thermal diffusion, the rising bubbles were tracked and the heat transfer coefficient in the rear of the bubbles were calculated.

2. The Two-Fluid Granular Temperature Model

In order to model the granular flow, the two-fluid approach was used as previously described by Pain et al., 2003b (see also Ding and Gidaspow, 1990 for a full derivation of the energy fluctuation - granular temperature equation). In this approach, both phases are modelled as interpenetrating continua and corresponding mass, momentum, thermal energy and fluctuation energy balance equations are solved with interaction terms representing the coupling between the phases.

Closure laws are used to fulfill the remaining requirements for the balance as described by Pain et al., 2001. In summary, a modified form of the Ergun equation was used to calculate the gas-solid drag forces (Pain et al., 2003b). Solid pressure, granular bulk and shear viscosities, as described by Lun et al., 1984, were also used. The solid stress tensor has a kinetic contribution, arising from the momentum which is transferred through the system by particles moving in the flow, and a collisional contribution, arising from the momentum transferred by direct collisions as described by Pain et al., 2001. In addition, a radial distribution function, as described by Gidaspow and Huilin, 1998, was used to represent the particle-particle collisional probability function.

Collisional-energy dissipative parameters, such as restitution coefficient and friction coefficient, play an important role in dilute granular flows, as they exert a strong influence upon the momentum transferred from and within the particulate phase. The restitution coefficient is related to the energy dissipated during collisions between particles and between particles and the wall. The friction coefficient is the coefficient of proportionality relating the tangential and normal components of the impulse exerted by the wall on the particles when sliding occurs (for a full description of the granular rheology see Jenkins and Mancini, 1987).

The set of the TFGTM conservative equations (Table (1)) comprises a number of variables that are defined by specific functions, which are established either by fundamental equations or by empirical or experimental correlations. The functions that are used to help solving the continuity, momentum and fluctuation energy equations are described by Pain et al., 2002 (see also Gomes et al., 2003b).

In the thermal energy equation, the volumetric interphase heat transfer coefficient, α, is defined for uniformly-sized spherical particles as:

$$\alpha = \frac{6\varepsilon_f}{d_s} h_{fs} \tag{1}$$

Several empirical correlations were proposed to calculate the fluid-particle heat transfer coefficient, h_{fs}. In this work, the empirical correlation introduced by Gunn, 1978 was used:

$$Nu = \frac{h_{fs} d_s}{\kappa_f} = \left(7 - 10\varepsilon_f + 5\varepsilon_f^2\right)\left(1 + 0.7 Re_p^{1/5} Pr^{1/3}\right) + \left(1.33 - 2.4\varepsilon_f + 1.2\varepsilon_f^2\right) Re_p^{0.7} Pr^{1/3} \tag{2}$$

This equation is valid for $0.35 \leq \varepsilon_f \leq 1.00$ and for $Re_p \leq 10^5$. Equation (2) satisfies the four asymptotical relations which bound the Nusselt number from low and high Reynolds number and from low and high porosities

Table 1: Set of conservative equations (see the nomenclature at Table (2)).

Continuity equation	$\frac{\partial}{\partial t}\left(\varepsilon_k \rho_k\right) + \frac{\partial}{\partial x_i}\left(\varepsilon_k \rho_k v_{ki}\right) = 0$
Momentum equation	$\frac{\partial}{\partial t}\left(\varepsilon_k \rho_k v_{ki}\right) + \frac{\partial}{\partial x_j}\left(\varepsilon_k \rho_k v_{ki} v_{kj}\right) = -\varepsilon_k \frac{\partial p_f}{\partial x_i} + \varepsilon_k \rho_k g_i + \beta\left(v_{k'i} - v_{ki}\right) - \frac{\partial}{\partial x_i}\left(\tau_{kij}\right) - \Gamma_k v_{ki}$
Thermal energy equations	$C_{p_f} \rho_f \epsilon_f \frac{DT_f}{Dt} = -p_f\left(\frac{\partial}{\partial x_i}\varepsilon_f v_{g_i} + \frac{\partial}{\partial x_i}\varepsilon_s v_{s_i}\right) + \frac{\partial}{\partial x_i}\left(\varepsilon_f \kappa_f \frac{\partial T_f}{\partial x_i}\right) + \alpha\left(T_s - T_f\right) + \widehat{\Gamma}_{wg}$ $C_{p_s} \rho_s \epsilon_s \frac{DT_s}{Dt} = \frac{\partial}{\partial x_i}\left(\varepsilon_s \kappa_s \frac{\partial T_s}{\partial x_i}\right) + \alpha\left(T_f - T_s\right) + \widehat{\Gamma}_{ws} + S_f$
Granular energy equation	$\frac{3}{2}\left[\frac{\partial\left(\varepsilon_s \rho_s \Theta\right)}{\partial t} + \frac{\partial}{\partial x_j}\left(\varepsilon_s \rho_s v_{s_j} \Theta\right)\right] = \tau_{s_{ij}} \frac{\partial v_{s_i}}{\partial x_j} - \frac{\partial q_j}{\partial x_j} - \gamma - 3\beta\Theta$

(see Gunn, 1978 for further details). Moreover, the Nusselt number tends to 2 for a single particle in a stagnant fluid, i.e., the Nusselt number is a minimum at its limit condition:

$$\lim_{\varepsilon_f \to 1}\left(\frac{\partial Nu}{\partial \varepsilon_f}\right) = 0 \tag{3}$$

The use of this equation introduces some inaccuracies, however, as the volumetric heat transfer coefficient becomes larger, the temperature difference between the phases remains small, and this inaccuracy can be considered negligible. This is a simplified version of the wall-bed with stagnant gas heat transfer equation (see Kunii and Levenspiel, 1991) and is related to the heat flux from the surface into the particle-fluid layer with thickness of $d_s/2$ in contact with it. In this work, the wall-gas phase heat transfer is considered negligible.

The fluid phase effective thermal conductivity is defined as (Kuipers et al., 1992):

$$\kappa_f^* = \varepsilon_f \kappa_f = \left(1 - \sqrt{1 - \varepsilon_f}\right) \kappa_{gas} \tag{4}$$

the solid phase effective thermal conductivity is, however, based on the granular temperature, which uses kinetic theory to model the granular flow. In analogy with the gas kinetic theory, the energy stored in a gas, is diffused by the exchange of kinetic energy between its molecules, expressed by the thermal conductivity. Similarly, from the particle kinetic theory of dense flows, the kinetic energy is transferred by the collision between particles. Hence, the thermal conductivity of the solid phase is due to the collisions and the velocity fluctuations, both of which are strongly influenced by the change in the thermodynamic temperature. Several correlations were proposed to calculate the solid phase effective thermal conductivity. In this work, the expression proposed by Hunt, 1997 was used:

$$\kappa_s^* = \varepsilon_s \kappa_s = \varepsilon_s \rho_s C_s d_s \frac{\sqrt{\pi\Theta}}{32 g_0}. \tag{5}$$

3. Fluids Transport Methods

A non-linear Petrov-Galerkin method is applied here to discretize the momentum equations. This involves the usual 'linear' streamline upwind weighting of the equations and an additional non-linear diffusion term which operates in the direction of the gradient of the solution. This method is applied separately to each velocity component in the momentum equations (see Mallet, 1986).

A transient mixed finite element formulation is used to discretize the equations. In addition, the finite volume discretization of the continuity equations and field variables and a continuous Petrov-Galerkin (Johnson, 1987) discretization of the momentum equations are employed. Within each time step the equations are iterated upon using a projection-based pressure determination method until all equations balance simultaneously (see Pain et al., 2001). As a result the non-linear continuity equations are strictly satisfied, ensuring mass conservation.

In the mixed formulation, hexahedra 'brick' elements in 3-D and rectangular elements in 2-D are employed here which have a bi-linear variation of velocity and a piecewise variation of pressure, density and all other

Table 2: Nomenclature

v	velocity ($\mathrm{m.s^{-1}}$)	t	time (s)
x	spatial coordinate (m)	p	pressure (Pa)
g	gravitational force ($\mathrm{m.s^{-2}}$)	T	thermodynamic temperature (K)
C	heat capacity ($\mathrm{J.(kg.K)^{-1}}$)	q	flux of fluctuation energy ($\mathrm{kg.s^{-3}}$)
Greek Symbols			
ε	volume fraction	ρ	density ($\mathrm{kg.m^{-3}}$)
β	interphase momentum transfer coefficient ($\mathrm{kg.m^{-3}.s^{-1}}$)	τ	viscous stress tensor ($\mathrm{N.m^{-2}}$)
Γ	frictional force exerted on the wall by the phase ($\mathrm{N.s.m^{-4}}$)	κ	thermal conductivity ($\mathrm{W.(m.K)^{-1}}$)
α	volumetric interphase heat transfer coefficient ($\mathrm{W.m^{-3}.K^{-1}}$)	Ω	volumetric wall-phase heat transfer coefficient ($\mathrm{W.m^{-3}.K^{-1}}$)
Θ	granular temperature ($\mathrm{m^2.s^{-2}}$)	γ	collisional energy dissipation ($\mathrm{kg.m^{-1}.s{-}3}$)
Υ	thermal energy source ($\mathrm{J.s^{-1}}$)		
Subscripts			
k	phase	s	solid phase
f	fluid phase	w	wall
i , j	coordinates index	T	total
$susp$	suspension	$emul$	emulsion

advected quantities. That is this element has a single pressure associated with each element and a velocity node (collocation point) at the corners of the element with C^0 variation of velocity between elements (for a full description of the methods used here, see Pain et al., 2003a; Pain et al., 2003b).

4. 2-D Numerical Simulations

In this section[1], a numerical investigation of the heat transferred the wall to the bed was performed in a gas-solid fluidized system. By tracking the rising bubbles along the wall region, the heat transfer coefficient was calculated and the contribution from the bubbles' wake was investigated.

4.1. Initial and Boundary Conditions

In the 2-D simulations, glass beads, with a diameter of 500 μm (ρ_s=2660 $\mathrm{kg.m^{-3}}$,κ_s=1.0 $\mathrm{W.m^{-1}.K^{-1}}$, $\mathrm{C_{p,s}}$=737 J $.\mathrm{kg^{-1}.K^{-1}}$), are fluidized by air, which enters uniformly in the domain with an inlet velocity of 75 $\mathrm{cm.s^{-1}}$ and at 20^{o}C and atmospheric pressure. The simulated fluidized bed is 67.00 cm high and 18.50 cm wide. Both phases are initially at room temperature and the r.h.s. wall temperature is 100^{o}C.

The boundary conditions can be summarized as follows: at the top of the domain, no normal flow and zero shear stress conditions were applied to the solid phase, whereas for the gas phase, zero shear stress was applied. This allows the gas to leave the domain unhindered, the solid phase, however must not leave the bed. In the vertical walls, a shear stress was prescribed, obtained from Blasius' equation (Schlichting and Kestin, 1968) with a length scale equal to the static height of the bed. Wall boundary conditions for the solid phase described in Jenkins, 1992 (see also Pain et al., 2001) were also used in the simulations described in this work. A summary for the boundary and initial conditions are outlined at Table (3).

In order to achieve a high numerical resolution in the wall region, an exponential mesh was used in this region (14.50≤X≤18.50 cm), whereas in the remainder of the domain a regular mesh was applied. Exponential mesh was also applied to the y-axis beginning from the free surface. The resulting grid has 3360 nodes in 3239 elements and the simulation was performed over 25 seconds.

4.2. Investigation of Heat Transfer Mechanisms Around the Bubbles

The main heat transfer contribution in granular flows is due to the solid phase. However, in highly packed systems, the heat transfer decreases due to the low granular temperature produced in such systems. Voidages, and bubbles in particular, enhance the heat transfer by increasing the free mean path between the particles

[1] Animations concerning this and the following sections are available in http://amcg.ese.imperial.ac.uk/jeffanim.

Table 3: Initial and boundary conditions used in the numerical simulation reported in this section.

(a)	Inlet gas velocity	$v_f\,(x, y = 0, t)$=0.75 m.s^{-1}
(b)	Initial ε_s	0.50
(c)	Initial gas and solid phase velocities	$v_s\,(x, y, t = 0)$=0.0 m.s^{-1} $v_f\,(x, y \neq 0, t = 0)$=0.0 m.s^{-1}
(d)	Initial gas and solid temperatures	$T_f\,(x, y, t = 0)$=20.0 ^{o}C $T_s\,(x, y, t = 0)$=20.0 ^{o}C
(e)	Solid flow in the top	$v_s\,(x, y = L, t)$=0.0 m.s^{-1}
(f)	Solid stress in the top	$\tau_s\,(x, y = L, t)$=0.0 N.m^{-2}
(g)	Particle-particle restitution coefficient	e_{pp}=0.97
(h)	Wall-particle restitution coefficient	e_{wp}=0.90
(i)	Friction coefficient	$\overline{\mu}$=0.10

which increases the particle collision rate. In addition, the circulation produced by bubbles replaces hot particles by cold particles (see Gomes et al., 2001; Gomes et al., 2003a).

In this work, the heat transfer coefficient is calculated from the Fourier's law applied to the wall boundary conditions:

$$H_T = (\varepsilon_f \kappa_f) \left| \frac{\left(\frac{\partial T_f}{\partial n}\right)\Big|_w}{T_w - T_{susp}} \right| + (\varepsilon_s \kappa_s) \left| \frac{\left(\frac{\partial T_s}{\partial n}\right)\Big|_w}{T_w - T_{susp}} \right| \tag{6}$$

where T_{susp} is the temperature of the suspension along the heated surface. Generally the temperature of the fluid and solid phases are very similar, therefore the simple averaging of these temperature fields next to the wall was used to calculate T_{susp}. Therefore, the heat transfer coefficient was calculated from the temperature gradient between the suspension and the wall, accounting for a thermal boundary layer at the wall.

A number of researchers worldwide (Pain et al., 2002; Kuipers et al., 1992) have reported the enhancement of the heat and mass transfer in the bubbles' wake, due to the intense mixing in this region. The rising bubbles yield a large solid particle replacement in the rear of the bubble which enhance solid circulation. Hence, fresh and 'cold' particles are dragged into the rear of the bubbles and rise the temperature gradient between 'cold and hot'particles. Figure (1) shows the solid volume fraction and heat transfer coefficient (r.h.s) calculated along the heated wall. As bubbles rise in the vicinity of the wall, the heat transfer coefficient is enhanced in the rear of the bubble region, as can be clearly observed in the two large bubbles presented in Figs. (1) (a-d). In the simulation performed here, the time-averaged heat transfer coefficient is 427 W.m^{-2}.K^{-1}.

5. 3-D Numerical Simulations

A 3-D numerical simulation was performed with similar geometry (18.50×18.50×67.00 cm), boundary and initial conditions (Table (3)) as applied to the 2-D device, in which the temperature was maintained at 100 ^{o}C on one of the walls. This simulation was performed over 25 seconds and the grid has 4576 nodes in 3720 elements.

As the bubbles rise along the heated wall, hot particles are displaced and thermal energy is diffused through the bed due to the strong mixing produced by the random motion of these bubbles. The train of rising bubbles helps making the temperature more homogeneous throughout the domain as the dynamics evolves in time. Such property can be observed in the snap-shot shown in Fig. (5) regarding the gas phase temperature at 20.59 seconds of numerical simulation. Although the gas temperature increases sharply in the vicinities of the heated wall, in the remaining of the domain, the temperature rises smoothly. Two main reasons are responsible for such behavior: proximity of the heat source and the dynamics associated with the flow in the wall region, i.e., rising bubbles colliding with falling particles. Both reasons can be related to the packet renewal theory (Khan and Elkamel, 2002) in which fresh packets of particles replace hot packets in the emulsion phase due to the action of bubbles and gas streams in contact with the heated surface flowing through the packets.

The surface map of four snapshots of the heat transfer coefficient along the heated surface are shown. As described by Gomes et al., 2003b, the thermal diffusion mainly occurs in the emulsion phase, therefore large H_T is expected in regions with relatively large solid concentration. Such behavior can be observed in the set of solid volume fraction and heat transfer coefficient snapshots presented in Figures 3 and 4.

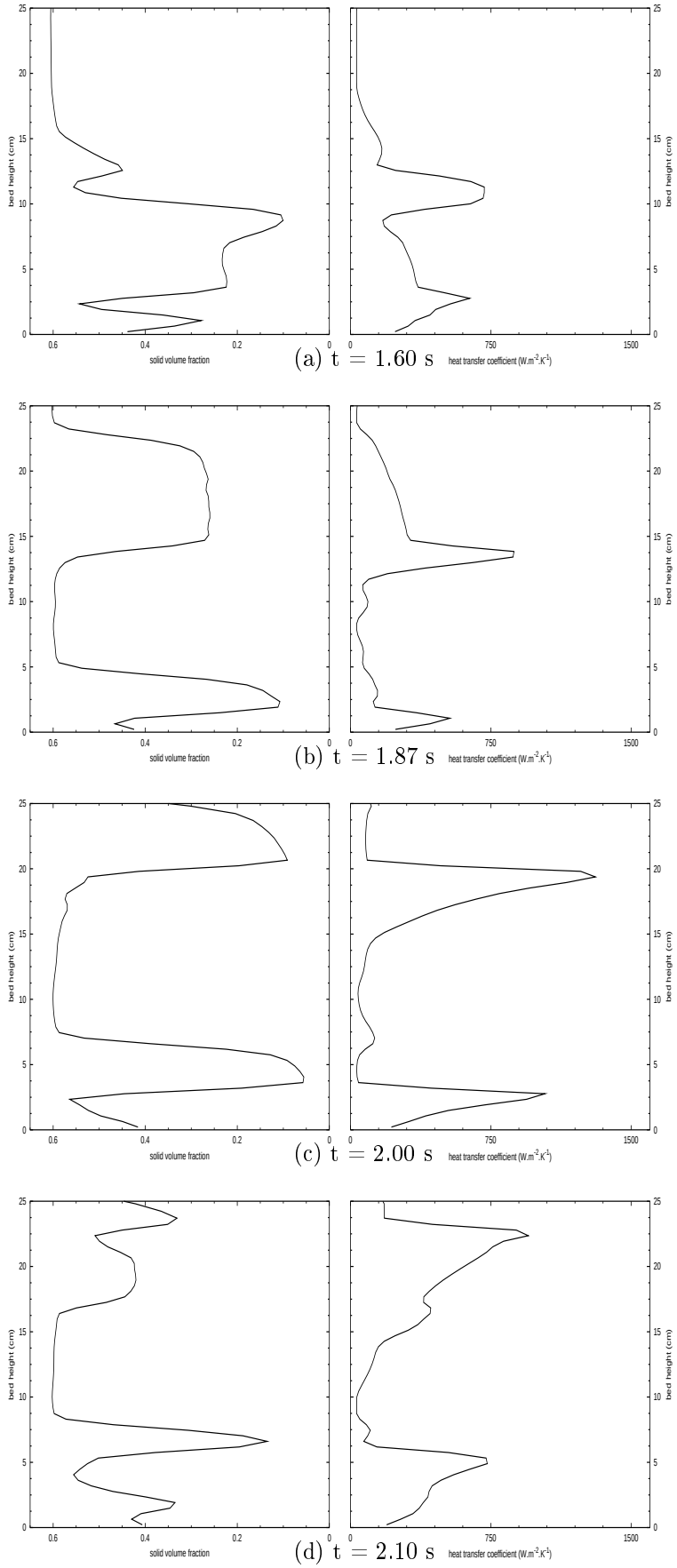

Figure 1: Flow past a heated wall: solid volume fraction (left-hand side) and heat transfer coefficient against bed height at (a) 1.60, (b) 1.87, (c) 2.00 and (d) 2.10 seconds. Note that the solid volume fraction axis increases from right to left.

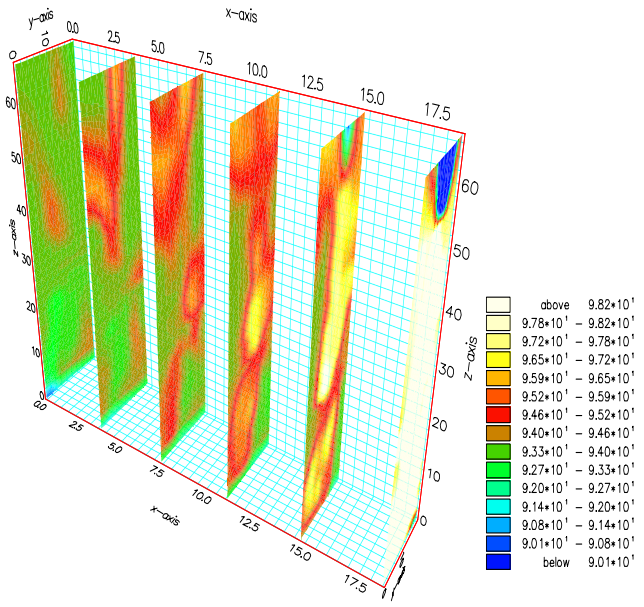

Figure 2: Flow past a heated wall - 3-D simulation: gas phase temperature distribution at 20.59 seconds.

By integrating the local heat transfer coefficient along the heated surface area and by averaging it over the time domain, a time-averaged heat transfer coefficient (Eqn. (6)) of 449 W.m^{-2}.K^{-1} is obtained. However, using the empirical heat transfer correlation reported by Kunii and Levenspiel, 1991, H_T is 360 W.m^{-2}.K^{-1}. The time-averaged heat transfer coefficient obtained from 2-D (Section 4) and 3-D simulations are very similar suggesting that the important dynamics are captured by the numerical simulations. However long term simulations including further investigations during fully steady state regime should be conducted.

6. Conclusions

In this work, the two-fluid granular temperature model was used to study the heat transfer mechanisms of fluidization of monosized particles by air. The set of governing and constitutive equations were solved by a multiphase finite element method code, FLUIDITY. The thermal effective conductivity of the solid phase was calculated using a granular temperature based correlation which makes the thermal energy analysis consistent with the overall hydrodynamics model.

In order to investigate the heat transfer in the bubbles' wake, 2-D and 3-D simulations were performed and the rising of bubbles along a heated wall was studied. Fresh particles were dragged in the rear of uprising bubbles, replacing hot particles and increasing the temperature gradient. Such replacement, which improved the solid circulation in the bubbles' wake, enhanced the heat transfer rates.

As the heat exchanged between the surface and the bed is mainly due to the solid phase, the granular temperature may play an important role in the heat transfer coefficient calculation. Indeed, in highly packed regions the heat transfer coefficient is small due to low granular temperature, however, in dilute regions, i.e., in regions where the solid volume fraction is very low, the granular temperature is also low due to the low collisional probability. In the bubbles wake, however, a large solid concentration combined with large granular temperatures and shear stresses result in large wall heat transfer coefficients.

The time-averaged heat transfer coefficients obtained from 2-D and 3-D numerical simulations were in good agreement. For 2-D simulations, time-averaged heat transfer coefficient in the order of 427 W.m^{-2}.K^{-1} were found, whereas for 3-D simulations, the heat transfer coefficient was 449 W.m^{-2}.K^{-1}. Although, there was not any reliable experimental data available for further validation, the model introduced here and elsewhere Pain et al., 2003b; Pain et al., 2003a and validated, in a 2-D framework, against experiments by Gomes *et al* Gomes et al., 2003b may confidently be used to model heat transfer in granular flows.

7. Acknowledgements

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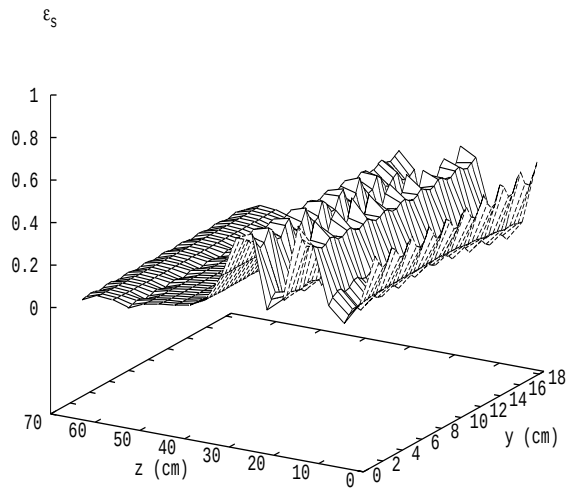
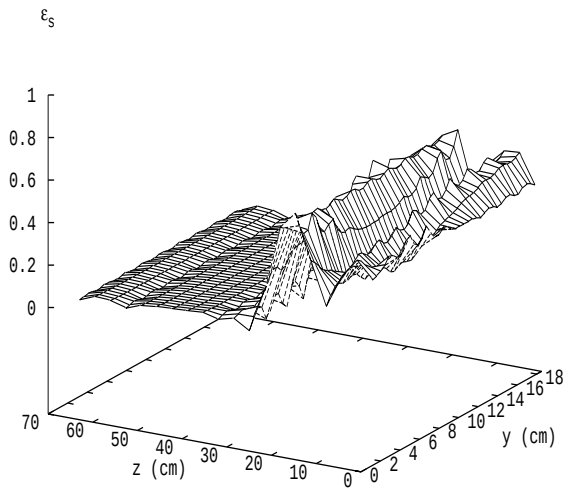
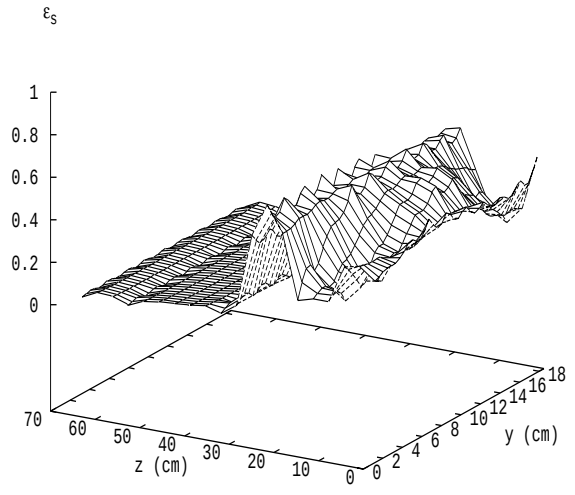


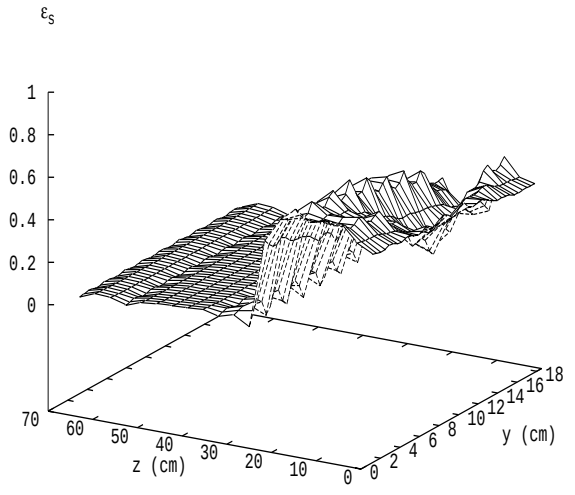
(a) t = 0.2 s



(b) t = 2.59 s

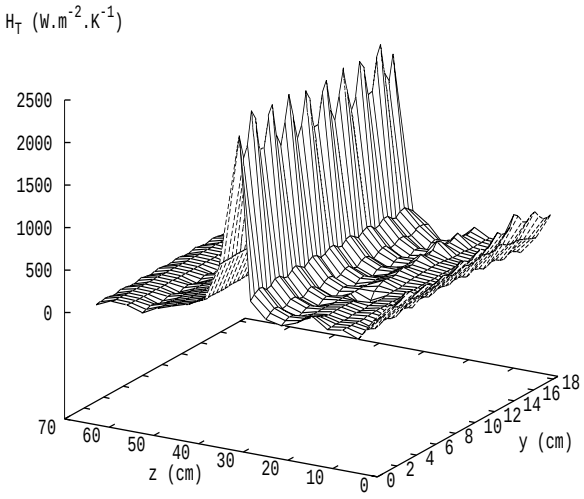


(c) t = 13.79 s



(c) t = 20.36 s

Figure 3: Flow past a heated wall - 3-D simulation: surface map of the solid volume fraction along the heated surface at (a) 0.20 s, (b) 2.59 s, (c) 13.79 s and (d) 20.36 s.



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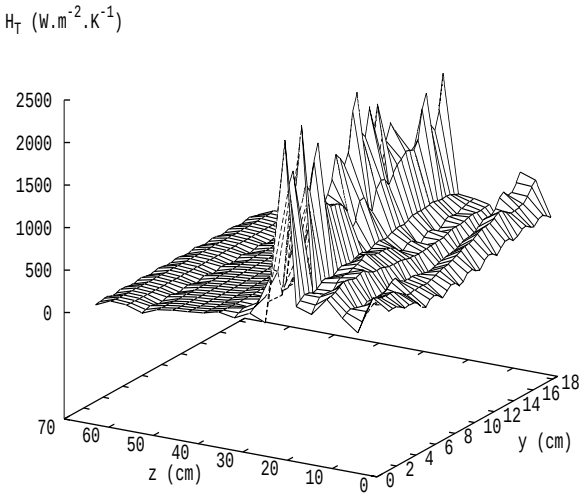
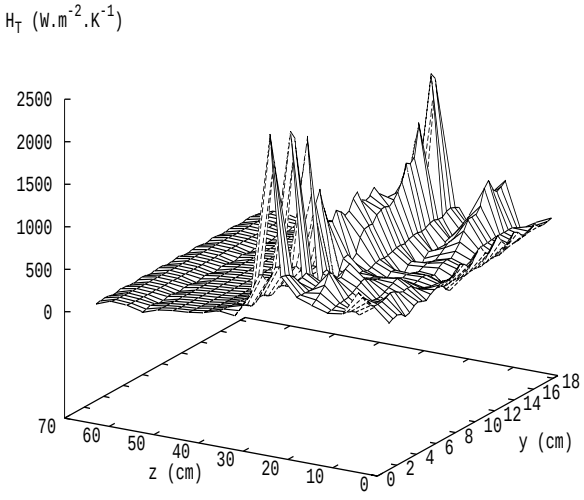
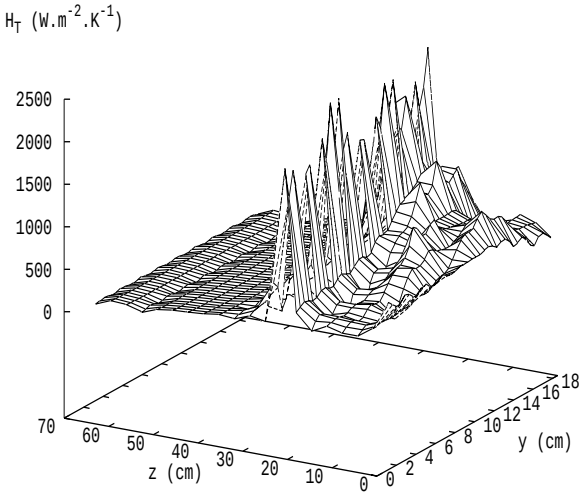


(b) t = 2.59 s



(c) t = 13.79 s



(c) t = 20.36 s

Figure 4: Flow past a heated wall - 3-D simulation: surface map of the heat transfer coefficient along the heated surface at (a) 0.20 s, (b) 2.59 s, (c) 13.79 s and (d) 20.36 s.

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