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CONVECTIVE HEAT TRANSFER IN MICROCHANNELS – A Review

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Abstract. *Fundamental differences between micro and macro heat transfer phenomenon are focused here. The trend of miniaturization has significantly aggravated the problem associated with overheating of integrated circuits, therefore new thermal control methods have become mandatory. Microelectricalmechanical systems (MEMS) are the subject of increasing active research however, it has been reported that the fluid behavior in such an extremely small channel deviates from the predictions based on the continuum hypotheses, thus Navier-Stokes-based fluid dynamics solvers are often inaccurate when applied to MEMS. The heat generated that is to be dissipated from a relatively low surface area requires effective thermal design. Although a substantial amount of experimental, analytical and numerical studies have been performed; there are still some points that are not yet understood. There is a need for us to overcome this lack of knowledge providing a more efficient and economical thermal control system. Important aspects of thermal enhancement of the microchannel heat sink are considered. To design effective microchannel heat sinks, key design parameters such as pressure required pumping for the cooling fluid; flow rate and temperature of the fluid are estimated. Molecular transport effects, such as viscous dissipation and thermal conduction, correlations between apparent viscosity and the friction factor for microchannel flow are also taken into account to better control the microchannel heat transfer.*

Keywords*. Microelectricalmechanical systems, heat transfer, microchannels*

1. Introduction

Devices having dimensions of the order of microns are being developed for use in cooling of integrated circuits, biochemical applications, microelectromechanical systems and cryogenics. The use of higher densities and frequencies in microelectronic circuits for computers are increasing day by day. The trend of miniaturization has significantly aggravated the problem associated with overheating of ICs. They require effective cooling due to heat generated that is to be dissipated from a relatively low surface area. This requires effective thermal design, increasing scales of circuit integration of electronic components accompanied by reducing feature size of IC chips tremendously increased the problem associated with the dissipation of the generated heat. With existing heat flux levels exceeding 100 W/cm^2, new thermal control methods have become mandatory. In particular miniaturization especially in computer technology has significantly increased the problems associated with overheating of ICs. These trends in thermal packaging are discussed by Bar-Cohen (1992) and in Cooling of Electronic System by Kakac et (eds. 1992). Hence the development of efficient cooling techniques for integrated circuit IC chips is one of the important contemporary applications of micro scale heat transfer. Microchannel heat sinks are the ultimate solution for removing these high amounts of heat. This pressing requirement has initiated extensive research in microchannel cooling. Experimental as well as analytic methods are reported for predicting the flow and temperature of the coolant, which is essential for chip thermal design.

2. Motivations and Application

Microchannel heat transfer has received much attention due to their ultimate potential for cooling high power microelectronic and application in biomechanical and aerospace industries. Heat dissipation from micro devices is now becoming a cutting edge field since the performance of the device used is primarily determined by flow and temperature fields. The main advantage of microchannel heat sinks is their extremely high heat transfer area per unit volume. Convection and conduction are two major heat transfer modes that have been investigated in micro-scale. The geometric parameters of the channel have a significant influence on the convective heat transfer characteristics. Hence, to design effective microchannel heat sinks, key design parameters like pressure required pumping the cooling liquid, flow rate, hydraulic diameter of the channel, temperature of the liquid and the channel wall, and the number of channels has to be considered. To make the system efficient and economical, these parameters have to be optimized.

Recent advances in micromechanics allow the fabrication of various micron scale devices. These micron scale devices can find their applications, both in commercial products and scientific investigations. However, for the last two decades, it has been reported that the fluid behavior in such an extremely small channel deviates from the predictions based on the continuum hypotheses, which have been accepted in the macro phenomena. This deviation in thought to be attributable to the rarefaction and compressibility effects mainly due to the tiny dimensions of microchannels.

Microelectricalmechanical systems (MEMS) are the subject of increasing active research in a widening field of disciplines. The small size of MEMS poses unique challenges in the design phase. In this range of machine scale, the dynamic behavior of the fluid motion against fluid and thermal drags cannot be predicted with the continuum theory. There is a need for us to obtain some fundamental understanding of heat and fluid flow in micromechanical devices, and to explore and control the natural phenomena in a length scale regime in which we have very little experience.

Liquid flow and heat transfer in microchannels are critical to the design and process control of various Microelectricalmechanical Systems (MEMS) and biomedical lab-on-a-chip devices. Experimental studies in the literature have showed that many microchannel flow and heat transfer phenomena cannot be explained by conventional theories of transport phenomena. For instance, the transition from laminar flow to turbulent flow starts much earlier (e.g., from Re=300); the correlations between the friction factor and the Reynolds number for microchannel flow are very different from that in classical theory of fluid mechanics; the apparent viscosity and the friction factor of a liquid flowing through a microchannel may be several times higher than that in the conventional theories. Therefore, fundamental understanding of liquid flow and convection heat transfer in microchannels is required. These special characteristics of flows and heat transfer in microchannels are results of micron-scale channel size and the interfacial electrokinetic and surface roughness effects near the solid-liquid interface, where the interface represents the wall of the microscale flow channel. These effects will significantly affect both the liquid flow and the convective heat transfer.

The use of convective heat transfer in microchannels to cool electronic chips has been proposed in the last two decades. Many analytical and experimental studies have been performed to have a better understanding of heat transfer at microscale. Both liquids and gases have been investigated. However, none of them has been able to come to a general conclusion. For example, there are controversial results in the literature about the boundary conditions, for liquids flows. It is not clear whether discontinuity of velocity and temperature exists on the wall or not.

3. Fluid and Heat Transfer Modeling for microscale applications

There are basically two ways of modeling a flow field. Either as the fluid really is-a collection of molecules-or as a continuum where the matter is assumed continuous and indefinitely divisible. The former modeling is subdivided into deterministic methods and probabilistic ones, while in the latter approach the velocity, density, pressure, etc., are defined at every point in space and time, and conservation of mass, energy and momentum lead to a set of nonlinear partial differential equations(Navier-Stokes). Fluid modeling classification is depicted schematically in Figure 1.

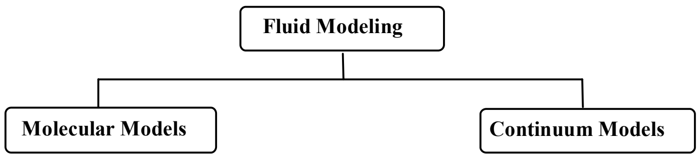

Figure 1. Fluid modeling classification.

Navier-Stokes-based fluid dynamics solvers are often inaccurate when applied to MEMS. This inaccuracy stems from their calculation of molecular transport effects, such as viscous dissipation and thermal conduction, from bulk flow quantities, such as mean velocity and temperature. This approximation of micro-scale phenomena with macro-scale information fails as the characteristic length of the gaseous flow gradients (L) approaches the average distance traveled by molecules between collisions (the mean path, λ). The ratio of these quantities is known as Knudsen number.

3.1. Knudsen Number

Knudsen number is defined in Kn=λ/D_h, , where

$$\lambda = \frac{\bar{k}T}{\sqrt{2}\pi P\sigma^2} = \frac{1}{\sqrt{2}\pi n\sigma^2}, \tag{2}$$

which is valid for and ideal gas model as rigid sphere. σ is the molecular diameter and k is Boltzman constant (1.38×10^{-23} J/K mol). Generally the traditional continuum approach is valid, albeit with modified boundary conditions as long as Kn < 0.1.

The Navier-Stokes equations are valid when λ is much smaller than a characteristic flow dimension L. As this condition is violated, the flow is no longer near equilibrium and the linear relation between stress and rate of strain and the no-slip velocity condition are no longer valid. Similarly, the linear relation between heat flux and temperature gradient and the no-jump temperature condition at a solid-fluid interface are no longer accurate when λ is not much smaller than L. The different Knudsen number regimes are shown in Fig. 2.

For the small values (Kn≤10^{-3}), the fluid is considered to be continuum, while for large values (Kn≥10), the fluid is considered to be a free molecular flow. For 10^{-3}< Kn <10^{-1} is the near continuum region.

Table 1: c_1 and c_2 for different heat inputs

Knudsen number (Kn=λ/L)	Regime
Kn <0.01	"Continuum" regime
0.01< Kn <0.1	Slip-flow
0.1< Kn <3	"Transition" regime
Kn >3	"Free-molecular" regime

Acceptable references include journal papers, numbered papers, dissertations, thesis, published conference proceedings, preprints from conferences, books, submitted papers (if the journal is identified), and private communications.

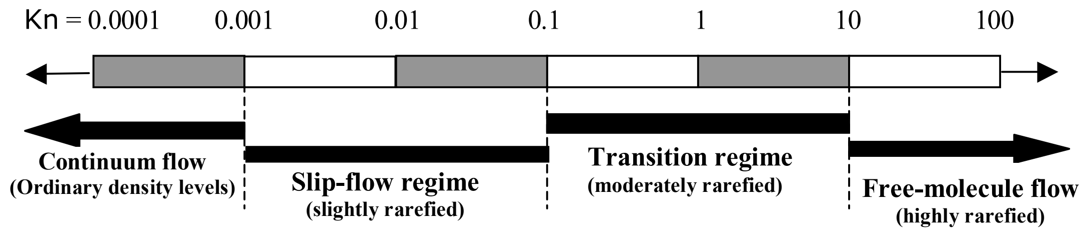

Figure 2. Knudsen number regimes.

The local value of Knudsen number in a particular flow determines the degree of rarefaction and degree of validity of the continuum model. The different Knudsen number regimes are determined empirically and are therefore only approximate for a particular flow geometry. The pioneering experiments in rarefied gas dynamics were conducted by Knudsen in 1909. Knudsen number can be expressed in terms of two important dimensionless numbers of fluid mechanics: Reynolds number and Mach number, where Mach number is a dynamic measure of fluid compressibility and may be considered as the ratio of inertial forces to elastic ones. From the kinetic theory of gases, the mean free path is related to the viscosity as follows

$$v = \frac{\mu}{\rho} = \frac{1}{2}\lambda \bar{V}_m , \tag{6}$$

where μ is the viscosity, and V_m is the mean molecular speed, which is higher than the speed of sound a_0, Eq.(7)

$$\bar{V}_m = \sqrt{\frac{8}{\pi\gamma}}\, a_0 . \tag{7}$$

One may start analyzing microchannel heat transfer and fluid flow for two dimensional flow. For steady two dimensional, incompressible flow with constant properties in Cartesian coordinates, Navier-Stokes equations, the continuity, momentum and energy equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 . \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right); \tag{9}$$

where ϕ is the viscous dissipation given by

$$\Phi = 2\mu\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 - \frac{1}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right]. \tag{12}$$

3.2 Boundary Condition

In heat transfer studies in macro channels, no-slip condition at a fluid-solid interface is enforced in the momentum equation and an analogous no-temperature-jump condition is applied in the energy equation. The notion underlying the no-slip/no-jump condition is that within the fluid there cannot be any finite discontinuities of velocity/temperature. The interaction between a fluid particle and a wall is similar to that between neighboring fluid particles, and therefore no discontinuities are allowed at the fluid-solid interface either. But strictly speaking those two boundary conditions are valid only if the fluid flow adjacent to the surface is in thermodynamic equilibrium. This requires and infinitely high frequency of collisions between the fluid and the solid surface. In practice, the no-slip/no-jump condition leads to fairly accurate predictions as long as Kn<0.001 (for gases). Beyond that, the collision frequency is simply not high enough to ensure equilibrium and a certain degree of tangential-velocity slip an temperature jump must be allowed.

3.2.1 SLIP VELOCITY

In micro channels, the molecular mean free path λ, becomes comparable with flow dimensions and the interactions between the fluid and the wall become more significant than intermolecular collision in microchannels. When the gas molecules hit the surface, the molecules can be reflected either specular or diffuse. One can write the tangential momentum balance at the wall. The molecules have the same tangential momentum in case of specular reflection. In case of a diffuse reflection, one can write the tangential momentum balance at the wall to obtain the slip velocity as:

$$u_s = 2\frac{2-F_m}{F_m}\frac{\mu}{\rho u_m}\left(\frac{du}{dy}\right)_w , \tag{13}$$

where viscosity is given by Eq.(14), then slip velocity can be written as in Eq.(15)

$$\mu \cong \frac{1}{2}\rho u_m \lambda , \tag{14}$$

$$u_s = \frac{2-F_m}{F_m}\lambda\left(\frac{du}{dy}\right)_w , \tag{15}$$

where F_m represents the fraction of the tangential momentum of the molecules given to the surface and it is called the tangential momentum accommodation coefficient. In case of a idealized perfect smooth surface at the molecular level, molecules will be reflected specularly which means that the incident angle exactly equals the reflected angle and the molecules conserve their tangential momentum and thus exact no shear on the wall and F_m =0. For diffuse reflection F_m =1 which means that the tangential momentum is lost at the wall ($M_w = M_i$).

For real walls, some molecules reflect diffusively and some reflect specularly. In other words, a portion of the momentum of the incident molecules is lost to the wall and a typically smaller portion is retained by the reflected molecules. This coefficient depends on the fluid, the solid and the surface finish, and have been determined experimentally to be between 0.2-0.8, the lower limit being for exceptionally smooth surfaces while the upper limit is typically of most practical surfaces. For F_m =0, the slip velocity is unbounded.

3.2.2 TEMPERATURE JUMP

In case of rarefied gas flow, there is a finite temperature difference between the wall temperature and the fluid temperature at the wall. The temperature jump coefficient is proposed as:

$$c_j = \frac{T_s - T_w}{\left(\frac{\partial T}{\partial y}\right)_w}, \qquad (24)$$

and the thermal accommodation coefficient is defined as the fraction of molecules reflected diffusively by the wall that accommodated their energy to the wall temperature.

$$F_T = \frac{E_a - E_l}{E_a - E_w}, \qquad (25)$$

where E_a is the energy of the incoming stream, E_l is the energy carried away by the molecules leaving the surface, E_w is the energy of the molecules leaving the surface at the wall temperature. E_a-E_l is the net energy carried to the surface. The accommodation coefficient can be related to jump coefficient c_j assuming the ideal gas behavior.

For a perfect gas, the temperature jump coefficient is obtained as in Eqs.(26,27):

$$c_j = \frac{2-F_T}{F_T}\frac{1}{(\gamma+1)}\frac{k\sqrt{2\pi RT}}{c_V P} \quad \text{or} \quad c_j = \frac{2-F_T}{F_T}\frac{2\gamma}{(\gamma+1)}\frac{\lambda}{\Pr} \qquad (26, 27)$$

then the temperature jump can be obtained as

$$T_S - T_W = \frac{2-F_T}{F_T}\frac{2\gamma}{(\gamma+1)}\frac{\lambda}{\Pr}\frac{\partial T}{\partial y}, \qquad (28)$$

where y is measured from the wall.

For fully developed laminar flow in a duct with constant properties, the fully developed velocity profile for slip flow can be obtained from the momentum Eq.(11) and (19) for flow between parallel-plates and in a circular duct respectively as

$$u = \frac{3}{2}u_m \frac{\left[1-\left(\frac{y}{d}\right)^2 + 4Kn\right]}{1+8Kn}, \quad \text{and} \quad u = \frac{2u_m\left[1-\left(\frac{r}{R}\right)^2 + 4Kn\right]}{1+8Kn}. \qquad (29,30)$$

One can put the two-dimensional energy for fully developed laminar flow between parallel channels in a dimensionless form as in Eqs.(31).

$$\frac{3}{32}\frac{Gz\left(1-\eta^2+4Kn\right)}{(1+8Kn)}\frac{\partial\theta}{\partial X} = \frac{\partial^2\theta}{\partial\eta^2} + \frac{9Br}{(1+8Kn)^2}\eta^2, \qquad (31)$$

where

$$\theta = \frac{T-T_s}{T_i - T_s}, \quad \eta = \frac{y}{d}, \quad X = \frac{x}{L}, \quad \bar{u} = \frac{u}{u_m}, \quad Gz = \frac{\mathrm{Re}\cdot\Pr\cdot 4d}{L}, \qquad (32)$$

where ΔT is the temperature difference of the fluid between the wall and the inlet temperature (ΔT=T_i-T_s) and the boundary and the inlet conditions in terms of the dimensionless quantities become as $\theta=0$ at $\eta=1$, $\theta=1$ at $X=1$, $\partial\theta/\partial\eta = 1$ at $\eta = 0$.

5. Brinkman Number

For laminar fully-developed flow through conventional-sized channel, h depends on k and D_h. For microchannels, indication from the survey showed that h depends on **Re,** and hence ρ_h, u and μ. In addition, h depends on the microscopic geometry, fluid properties, local wall and fluid temperatures. Hence δ, c and ΔT should also be pertinent parameters to be considered. A general relationship between all the relevant quantities can be written as $\phi(h,k,\rho,u,\delta,D,\mu,c,\Delta T)$=0.

The Buckingham Π Theorem provides five independent dimensionless groups, therefore Eq.(32) may be written as a correlation in the form presented by Eq.(33)

$$\mathrm{Nu} = C(\mathrm{Re})^a(\mathrm{Pr})^b\left(\frac{\delta}{D}\right)^c(\mathrm{Br})^d, \qquad (33)$$

where Brinkman number, Br, is defined by Eq.(34)

$$\mathrm{Br} = \frac{\mu u_m^2}{k\Delta T}, \qquad (34)$$

where ΔT is the wall-fluid temperature difference at a particular axial location.

The Brinkman number also emerges from the dimensionless general energy equation as the product of Eckert and Prandtl numbers. The negative values of Br means that the fluid is being cooled. It measures the relative importance of viscous heating (work done against viscous shear) to fluid conduction along the microchannel dimension. Although Br is usually neglected in low-speed and low-viscosity flows through conventionally-sized channels of short lengths, in flows through conventionally-sized long pipelines, Br may become important. For flows in microchannels, the length-to-diameter ratio can be as large as for flows through conventionally-sized long pipelines, Br may become important in microchannels also.

Table 2: Developed Conditions, Laminar Flow Nusselt Values (Tw=constant).

Br = 0	Nu_T	Br = 0.01	Nu_T
Kn = 0.0	4.3627	Kn = 0.0	6.4231
Kn = 0.6	3.2519	Kn = 0.06	3.9769
Kn = 0.8	2.9487	Kn = 0.08	3.4903

The table shows the effects of the Knudsen number and the Brinkman number on heat transfer in a tube flow. As it can be seen from Table 1, Nusselt number increases with the increase of the Brinkman number, and Nusselt number decreases with the increase in Knudsen number, since the increasing temperature jump decreases heat transfer.

3. Literature Survey

Due to the extraordinary advantage for practical applications, microfabrication or nanotechnology, originally emerging from the technology developed for integrated circuits, has expanded rapidly into such fields microelectronics, microscale sensing and measurement, spacecraft thermal control, miniheaters and micro heat exchangers, micro motors, micro valves, microelectromechanical systems. Major research initiatives are being launched to miniaturise electronic components and to improve our fundamental understanding of phenomena at the microscale where in certain effects, which are generally ignored at the macroscale, become predominant.

For the development of micro devices, a number of heat transfer and fluid flow issues at microscale are being resolved. The literature survey below gives a brief description of the analyses performed in single-phase forced convection in microchannels mostly in the last 15-20 years

One of the work by Tuckerman and Pease(1982) demonstrated that the water-cooled microchannel heat sink is capable of dissipating a heat flux of 790 W/cm² without a phase change. They noted that convective heat transfer coefficient, h, for laminar flow through microchannels might be higher than for turbulent flow through conventionally sized microchannels. Since then there has been an unprecedented upsurge of research in convection through microchannels.

Samalam (1989) has modelled the convective heat transfer of water flowing through microchannels etched in the back of silicon wafers. The problem was reduced to a quasi-two-dimensional non-linear differential equation under certain reasonably simplified and physically justifiable conditions, and was solved exactly. The optimum channel dimensions (width and spacing) were obtained analytically for a low thermal resistance. The calculations show that optimising the channel dimensions for low aspect ratio channels is much more important than for large aspect ratios. However, a crucial approximation that the fluid thermophysical properties are independent of temperature was made, which could be a source of considerable error, especially in microchannels with heat transfer.

Aul and Olbricht (1990) reported experimental studies of low-Reynolds number, pressure-driven core-annular flow in a straight capillary tube. The annular film was thin compared to the radius of the tube, and the viscosity of the film fluid was much larger than the viscosity of the, core fluid. The photographs showed that the film was unstable under all conditions investigated. It was found that the film fluid collects in axisymmetric lobes that are periodically spaced along the capillary wall. Eventually, the continued growth of the lobes results in the formation of a fluid lens that breaks the inner core.

Pfahler et al. (1990a-b, 1991) have experimentally investigated the fluid flow in rectangular microchannels with cross-section ranging in area from 80 to 7200μm^2. Their objective was to determine the length scales at which the continuum assumptions break down, and to estimate the adequacy of the N-S equations for predicting the fluid flow behavior. They found that in the relatively large flow channels, their observations are in agreement with the predictions of the Navier-Stokes equations. However, in the smallest of their channels, a significant deviation from the N-S predictions was observed.

New experimental measurements for pressure drop and heat transfer coefficient were made by Rahman (1993). Tests were performed on channels of different depths and using water as the working fluid. The fluid flow rate as well as the pressure and temperature of the fluid at the inlet and outlet of the device were measured. These measurements were used to calculate local and average Nusselt number and coefficient of friction in the device for different flow rates, channel size and configuration.

Designing small-scale fluid flow devices needs clarification of fluid dynamics on the order of 0.1-100 µm. Makihara et al. (1993) have described the flow of liquids in 4.5-50.5 µm micro-capillary tubes and a method of measuring it. They found that the measured values agree with the theoretical values calculated by the Navier-Stokes equations.

Beskok and Karniadakis (1994) numerically simulated the time-dependent slip flow in complex microgeometries. The numerical scheme was based on the spectral element method that they develop for flows in macrogeometries. A higher order velocity slip condition was used in the analysis. The method was verified by comparing it to the analytical solutions for simple cases. They noted the importance of the accommodation coefficient. Although the Knudsen number is small, a small value of the momentum accommodation coefficient will result in large slip velocities at the wall. Compressibility effects were also addressed especially for the cases where severe pressure drops occur.

In another study of Beskok et al.(1995), they focused on the competing effects of compressibility and rarefaction for Knudsen numbers up to 0.3. The higher order velocity slip and temperature jump boundary conditions were modified for the numerical stability purposes and yield

$$u_g - u_w = \frac{2-\sigma_V}{\sigma_V}\left[\mathrm{Kn}\left.\frac{\partial u}{\partial n}\right|_S + \frac{\mathrm{Kn}^2}{2}\left.\frac{\partial^2 u}{\partial n^2}\right|_S + \frac{\mathrm{Kn}^3}{6}\left.\frac{\partial^3 u}{\partial n^3}\right|_S + \ldots\right] \quad (35)$$

$$T_g - T_w = \frac{2-\sigma_T}{\sigma_T}\left(\frac{2\gamma}{\gamma+1}\right)\frac{1}{\mathrm{Pr}}\left[\mathrm{Kn}\left.\frac{\partial T}{\partial n}\right|_S + \frac{\mathrm{Kn}^2}{2}\left.\frac{\partial^2 T}{\partial n^2}\right|_S + \frac{\mathrm{Kn}^3}{6}\left.\frac{\partial^3 T}{\partial n^3}\right|_S + \ldots\right] \quad (36)$$

Viscous heating and thermal creep were found to be important mechanisms at microscale. Viscous heating can result in considerable temperature gradients. They concluded that compressibility is important for pressure driven flows and rarefaction is important for shear driven flows.

Rectangular microchannels with hydraulic diameter ranging from 133 to 367 micrometers were used in the experiments of Peng et al.(1994a-b). Steady state and fully developed flow conditions were investigated, indicating that transition range is not as wide as in conventional channels. Transition starts at Re=200 to 700 and the transitional Re decreases with decreasing hydraulic diameter. Flow friction also decreases with decreasing D_h until the aspect ratio equals to 0.5. Experimental Nusselt number values for laminar flow were smaller than predicted and were dependent on the Reynolds number, $\mathrm{Nu} \propto \mathrm{Re}^{0.62}$, unlike conventional channels. Heat transfer coefficient for both laminar and turbulent flows changes significantly for different values of hydraulic diameter and aspect ratio. An optimum value of aspect ratio as 0.75 for laminar and 0.5-0.75 for turbulent flows was reported. For all their test conditions, their dimensionless distance was such that the flow was fully-developed. A representative h was calculated based on the downstream end of the microchannel, defined as

$$h = q''/\left(T_w - T_{f,in}\right), \quad (37)$$

where T_W is the value measured at the downstream end. They compared their data in the laminar regime with the Sieder-Tate equation. Since the Sieder-Tate (1936) equation did not correlate well quantitatively with their experimental data, they proposed a new correction and compared with their data. However neither their new correlation nor the Sieder-Tate equation qualitatively agree with the unusual behaviour of $\mathrm{Nu/Pr}^{1/3}$ receding with increasing Re, exhibited by their experimental data.

Peng et al. (1995a) also tested rectangular-shaped microgrooves built into stainless steel plates with methanol as a working fluid to determine the effects of aspect ratio, fluid velocity and temperature and channel configuration. It was observed that, due to the large heat fluxes, liquid temperature along the channel changes significantly, which causes a significant thermophysical property variation.

Experiments were performed by Peng and Peterson (1995b) and (1996) to understand the physics of the single-phase convective heat transfer in microchannels of varying size. The influence of the flow characteristics, thermofluid conditions and channel geometry was analyzed. Their experiments showed a Nusselt number reduction with increasing Re in the laminar flow regime, for which they did not give an explanation. In the turbulent flow regime, Nu increased with increasing Re. Liquid inlet temperature and velocity also has a significant influence on the heat transfer coefficient. In general, a small inlet temperature and a large velocity result in higher heat transfer coefficients.

Kleiner et al. (1995) theoretically and experimentally investigated forced air-cooling, which employs microchannel parallel plate fin heat sinks and tubes. Optimization was performed and design trade-off studied. Tube sizes were observed to have a significant impact on optimum heat sink design. Air-cooled heat sinks are used for micro channel heat sinks with heat load less than 100 $\mathrm{W/cm^2}$.

Yu et al. (1995) experimentally investigated the flow of dry nitrogen gas and water in microtubes with diameters 19, 52, and 102 μm, for Re range 250-20,000, and for Pr range 0.7-5.0. They found a reduction in the friction factor in the turbulent regime, and that h was enhanced. The Reynolds analogy was found inapplicable in channels whose dimensions are of the order of the turbulent length scale, though the fluid can still be treated as a continuum. Their theoretical scaling analysis indicated the turbulent momentum and energy transport in the radial direction to be significant in the near-wall zone. They developed an analogy by considering the turbulent eddy interacting with the walls as a frequent event, thereby causing a direct mass and thermal energy transfer process between the turbulent lumps and the wall, similar to the eddies bursting phenomenon. This phenomenon significantly alters the laminar sublayer region in turbulent flows through microtubes. Since even a small eddy diffusivity in the laminar sublayer region can contribute significantly to the heat transfer rate while having a negligible effect on momentum transfer, an eddy can carry heat to a greater distance; hence, the increased hand lower friction factors in turbulent flows through microtubes.

A heat transfer analysis was performed by Gui and Scaringe (1995) based on the data from Rahman and Gui's (1993) experiment where they used water and refrigerants to determine the cooling capacity of a silicon chip and obtain 106 $\mathrm{W/m^2}$ heat dissipation. They found the laminar-to-turbulent transition Reynolds number as 1400 instead of 2300 for macro dimensions. They ascribed this to the surface roughness. Their analytical values were always smaller than the experimental results. They listed the reasons for more efficient heat transfer as: the reduced thermal boundary layer thickness, entrance effects-higher heat transfer at the channel inlet, pre-turbulence at the inlet and surface roughness.

Choquette et al. (1996) performed analyses to obtain momentum and thermal characteristics in microchannel heat sinks. A computer code was developed to evaluate the performance capabilities, power requirements, efficiencies of heat sinks, and for heat sink optimization. Significant reductions in the total thermal resistance were found not to be achieved by designing for turbulent flows, mainly due to the significantly higher pumping power requirements realised, which offset the slight increase in the thermal performance.

Gaseous flow in microchannels was experimentally analyzed by Shih et al. (1996) with helium and nitrogen as the working fluids. Mass flow rate and pressure distribution along the channels were measured. Helium results agreed well with the result of a theoretical analysis using slip flow conditions, however there were deviations between theoretical and experimental results for nitrogen. In between the slip and the transition flows where most MEMS application can be found, direct simulation Monte Carlo (DSMC) offers an alternative. The advantage of DSMC is that it makes no continuum assumption. Instead it models the flow as it physically exists: a collection of discrete particles, each with a position, a velocity, an internal energy, a species identity, etc. these particles are allowed to move and interact with the domain boundaries an small time steps during the calculation. Intermolecular collisions are all performed on a probabilistic basis. Macro quantities, such as flow velocity and temperature, are then obtained by sampling the microscopic state of all particles in the region of interest. It is shown that DMCS has ability to calculate microflows in any of the four Knudsen number regions without modification. This is particularly valuable in simulating flows with different regimes.

A heat convection problem considering the laminar fully developed hydrodinamically gaseous flow solved in a cylindrical microchannel with constant heat flux boundary condition by Ameel et al. (1997). In this work, two simplifications are adopted reducing the applicability of the results. Both the thermal accommodation coefficient and the momentum accommodation coefficient were assumed to be unity. This second assumption, while reasonable for most fluid-solid combinations, produces a solution limited to a specified set of fluid-solid conditions. The fluid was assumed to be incompressible with constant properties, the flow was steady and two-dimensional, and viscous heating was not included in the analysis. They used the results from a previous study of the same problem with uniform temperature at the boundary by Barron et al. (1997). Discontinuities of both velocity and temperature at the wall were considered. The fully developed Nusselt number relation was given by

$$\mathrm{Nu} = \frac{48(2\beta-1)^2}{(24\beta^2-16\beta+3)\left[1+\frac{24\gamma(\beta-1)(2\beta-1)^2}{(24\beta^2-16\beta+3)(\gamma+1)\mathrm{Pr}}\right]} \tag{40}$$

where $\beta = 1\ 4\ \mathrm{Kn}$.

It was noted here that, for Kn=0, in other words the no-slip condition, the above equation gives Nu=4. 364, which is the well-known Nusselt number for conventionally sized channels. Kakac and Yener (1995). The Nusselt number was found to be decreasing with increasing Kn. Over the slip flow regime, Nu was reduced about 40%. A similar decay was also observed for the gas mixed mean temperature. They determined that the entrance length increases with increasing rarefaction, which means that thermally fully developed flow is not obtained as quickly as conventional channels. The following formula shows the relationship between the entrance length and the Knudsen number $x_e^* = 0.0828 + 0.14\mathrm{Kn}^{0.69}$.

Kavehpour et al. (1997) solved the compressible two-dimensional fluid flow and heat transfer characteristics of a gas flowing between two parallel plates with both uniform temperature and uniform heat flux boundary conditions. They compared their results with the experimental results of Arkilic (1994) for Helium in a 52.25x1.33x7500 mm channel.

Mala et al. (1997) have investigated possible importance of the interfacial effects of the electrical double layer (EDL) at the solid-liquid interface (which is formed due to the electrostatic charges on the solid surface) on convective heat transfer and liquid flow in microchannels. They have solved the momentum and energy equations numerically for a steady hydrodynamically-developed and thermally-developing flow, considering the electrical body force resulting from the double layer field. They found that the EDL modifies the velocity profile and reduces the average velocity, thereby increasing the pressure drop and. reducing the heat transfer rate. They reported that the EDL thickness ranges from a few nanometers to several hundreds of nanometers, and calculated the effect on a microchannel separation distance of 25 μm. As this is an order of magnitude smaller than the channels used in the reported experimental investigations, the true influence of EDL on the convective heat transfer is uncertain. Moreover, the EDL effects do not exist if the walls of the microchannel are conducting materials, which is the case for the reported experimental observations. Hence, the EDL effects cannot explain the unusual behavior of convective heat transfer and flow transitions observed in the experiments.

Mala et al. (1997) found that with water as working fluid, the difference between the measured pressure drop per channel length and correlation from conventional theory was small for micro tube diameter more than 15mm. Experiments conducted on the EDL field. They found that the EDL results in a reduced velocity of flow than in conventional theory, affecting the temperature distribution and reducing the Re number. It is seen that without the EDL, a higher heat transfer is predicted.

Randall et al. (1997) studied the original problem for thermally developing heat transfer in laminar flow through a circular tube considering the slip-flow condition. They extended the original problem to include the effect of slip-flow, which occurs in gases at low pressures or in microtubes at ordinary pressures. A special technique was developed to evaluate the eigenvalues for the problem. Eigenvalues were evaluated for Knudsen numbers ranging between 0 and 0.12. Simplified relationships were developed to describe the effect of slip-flow on the convection heat transfer coefficient.

Adams et al. (1998) have experimentally investigated the single-phase turbulent forced convection of water flowing through circular microchannels with diameters of 0.76 and 1.09 mm. Their data showed that the Nusselt numbers for the microchannels are higher than those predicted by the traditional correlations for turbulent flows in the conventionally-sized channels, such as the Gnielinski correlation. Their data suggested that the extent of enhancement in the convection increased as the channel diameter decreased and the Reynolds number increased. To accommodate this enhancement, the Gnielinski correlation was modified from a least squares fit of a combination of their experimental data and the data reported for small diameter channels. This modified correlation is applicable when the diameter is in the range 0.102 - 1.09 mm, the Reynolds number is in the range 2600 - 23000, and the Prandtl number is in the range 1.53 - 6.43.

Tso and Mahulikar (1998a,b) proposed the use of the Brinkman number to explain the unusual behavior of laminar liquid flow in microchannels.by investigating the effect of viscosity variation on the Brinkman number. In another paper (1998b), they investigated the effect of the Brinkman number on determining the flow regime boundaries in microchannels, and found that Br plays a more important role in the laminar-to-transition boundary than in transition-to-turbulent boundary.

Xu et al. (1999) investigated liquid flow in microchannels both experimentally and analytically. Laminar water flow in microchannels with diameters between 50 and 300 micrometers and Reynolds numbers between 50 and 1500 was considered. It was found that the results deviated from Navier-Stokes predictions for diameters less than 100 μm. They also found that this deviation was not dependent on the Reynolds number.

Mahulikar (1999) has shown the Brinkman number of coolant flow in microchannels to correlate the forced convective heat transfer in the laminar and transition regimes and hence explained the unusual behavior of convective heat transfer in microchannels. In addition to Reynolds number, Brinkman number also determines the flow regime boundaries from laminar to transition and from transition to fully turbulent flow. The transition range varies due to the difference in the extent of the role played by the Brinkman number in determining the two flow regime boundaries, relative to Reynolds number. In Fan et al.(1999), numerical simulation on the gaseous micro channel using the DMSC is carried out. Results of DSMC are presented. Mean streamwise velocity at the walls is found to increase to make up for the density drop. Velocities at the walls are found to be nonzero and increase in the streamwise direction. The results of DSMC are validated by analytic solution in slip regime, and the compared.

Iwai and Suzuki (1999) numerically investigated the effects of rarefaction and compressibility on heat transfer for a flow over a backward-facing step in a microchannel duct. Velocity slip is applied as boundary condition and temperature jump at the heated wall. Compressibility effects are significant for microchannel flows with flow separation and reattachment. They become more important as Kn becomes larger. Compressibility increases Nu due to the increase of temperature difference between fluid and the wall since the thermal energy is converted into the kinetic energy. They also stated that there was not a significant effect of temperature jump on Nu distribution under the simulation conditions.

Convective heat transfer analysis for a gaseous flow in microchannels was performed by Hadjiconstantinou (2000). A Knudsen number range of 0.06-1.1 was considered. In this range, flow is called transition flow. Since the continuum assumption is not valid, the DSMC technique was applied. He considered the constant heat flux boundary condition for two-dimensional flow, where channel height varied between 0.03125 and 1 micrometers.

Larrode et al. (2000) solved the heat convection for gaseous flows in circular tubes in the slip flow regime with uniform temperature boundary condition. The effects of the rarefaction and surface accommodation coefficients were considered. They defined a fictitious extrapolated boundary where velocity does not slip by scaling the velocity profile with the new slip radius:

$$\rho_S^* = \frac{1}{1+48\beta_V \mathrm{Kn}} \tag{42}$$

where β_v is a function of momentum accommodation coefficient, α_m is defined as $\beta_v = 2 - \alpha m / \alpha m$. Therefore, the velocity profile is converted to the one used for the continuum flow.

Kim et al. (2000) modeled the microchannel heat sink as a porous structure while studying the forced convective heat transfer through the microchannel. From the analytical solution, variables of engineering importance were identified as Darcy number and the effective thermal conductivity ratio.

Weilin et al. (2000) conducted experiments to investigate heat transfer characteristics of water flowing through trapezoidal silicon microchannels with hydraulic diameter ranging from 62 to 169 μm. The experimentally determined Nusselt number is lower than that given by numerical results. This may be due to the effect of surface roughness of microchannel walls. Based on a roughness – viscosity mode, a modified Nu relation, which accounts for the roughness –viscosity effects was proposed to interpret the experimental results.

Tunc and Bayazitoglu (2000 and 2001) studied the convective heat transfer for steady state, laminar, hydrodynamically developed flow in microtubes with uniform temperature and uniform heat flux boundary conditions are solved by the integral transform technique. Temperature jump condition at the wall and viscous heating within the medium are included. The effect of viscous heating is investigated for both of the cases where the fluid is being heated or cooled. Prandtl number analysis has shown that, as we increase the Prandtl number the temperature jump effect diminishes which gives a rise to the Nusselt number.

Yu and Ameel (2001) studied laminar slip-flow forced convection in rectangular microchannels analytically by applying a modified generalized integral transform technique to solve the energy equation, assuming hydrodynamically fully developed flow. Results are given for fluid mixed mean temperature, and both local and fully developed mean Nusselt numbers. Heat transfer is found to increase, decrease, or remain unchanged, compared to non-slip-flow conditions, depending on two dimensionless variables that include effects of rarefaction and the fluid/wall interaction. The transition point at which the switch from heat transfer enhancement to reduction occurs is identified for different aspect ratios.

Toh et al. (2002) in this work, three-dimensional fluid flow and heat transfer phenomena inside heated microchannels is investigated. The steady, laminar flow and heat transfer equations are solved using a finite-volume method. The numerical procedure is validated by comparing the predicted local thermal resistances with available experimental data. The friction factor is also predicted in this study. At lower Reynolds numbers the temperature of the water increases, leading to a decrease in the viscosity and hence smaller frictional losses.

In Weilin and Mudawar (2002) and Ryu et al. (2003) the three-dimensional fluid flow and heat transfer in a rectangular microchannel heat sink with water as the cooling fluid are analyzed numerically using based on the finite difference method. The heat flux and Nu number have higher values near the channel inlet and vary around the channel periphery, approaching zero in the corners. Flow Reynolds number affects the length of the flow developing region. For a relatively high Reynolds number of 1400, fully developed flow may not be achieved inside the heat sink. Increasing the thermal conductivity of the solid substrate reduces the temperature at the heated base surface of the heat sink, especially near the channel outlet.

Maynes and Webb (2003) studied thermally fully developed, electro-osmotically generated convective transport for a parallel plate microchannel and circular microtube under imposed constant wall heat flux and constant wall temperature boundary conditions. The result is a combination of unique electro-osmotic velocity profiles and volumetric heating in the fluid due to the imposed voltage gradient. The exact solution for the fully developed, dimensionless temperature profile and corresponding Nusselt number have been determined analytically for both geometries and both thermal boundary conditions. The fully developed temperature profiles and Nusselt number are found to depend on the relative duct radius (ratio of the Debye length to duct radius or plate gap half-width) and the magnitude of the dimensionless volumetric source.

An experimental investigation has been performed on the laminar convective heat transfer and pressure drop of water in 13 different trapezoidal silicon microchannels by Wu and Cheng (2003). It is found that the values of Nusselt number and apparent friction constant depend greatly on different geometric parameters. The laminar Nusselt number and apparent friction constant increase with the increase of surface roughness and surface hydrophilic property. These increases become more obvious at larger Reynolds numbers. The experimental results also show that the Nusselt number increases almost linearly with the Reynolds number at low Reynolds numbers (Re <100), but increases slowly at a Reynolds number greater than 100. Based on experimental data, dimensionless correlations for Nu and the apparent friction constant are obtained for the flow of water in trapezoidal microchannels having different geometric parameters, surface roughness and surface hydrophilic properties. Finally, an evaluation of heat flux per pumping power and per temperature difference is given for the microchannels used in this experiment.

4. Conclusions

A number of heat and fluid transport issues at the microscale that were surveyed can be summarized as:

- Convective heat transfer in microchannels is significantly enhanced, depending on the values for Knudsen number, the Prandtl number, the Brinkman number and the aspect ratio, heat transfer can be significantly different from conventionally sized channels.
- The convection of liquids flowing through microchannels has been extensively experimented to obtain the characteristics in the laminar, transition, and turbulent regime. The observations indicate significant departures from the thermofluid correlations for the conventionally-sized tubes, which has not been explained.
- The liquid flow in the microchannel dimensions investigated experimentally is still in the continuum regime. Hence, the conventional Navier-Stokes equations are applicable.
- Velocity slip and temperature jump affect the heat transfer in opposite ways: a large slip on the wall will increase the convection along the surface. On the other hand, a large temperature jump will decrease the heat transfer by reducing the temperature gradient at the wall. Thus, neglecting temperature jump will result in overestimation of the heat transfer coefficient.
- For the reported experiments, h is representative of the entire length of the microchannels, calculated either at the downstream end of the microchannels, or based on the bulk mean wall-fluid temperature difference.
- The status of the correlations of the single-phase forced flow convection in the laminar regime have not been reported for the parameters obtained locally and along the flow.
- For the laminar heat transfer regime in microchannels, Nu is proportional to $Re^{0.62}$, while the fully-developed turbulent heat transfer is predicted by the Dittus-Boelter correlation by modifying only the empirical constant coefficient from 0.023 to 0.00805.
- In the laminar and transition regimes in microchannels, the behavior of convective heat transfer coefficient is very strange and complicated compared with the conventionally-sized situation. In the laminar regime, Nu recedes with increasing Re, which has not been explained. Hence, the transition and laminar convection are strange compared with the conventionally-sized situations.
- In microchannels, the flow transition point and range are a function of the heating rate or the wall temperature conditions. The transitions are also a direct result of the large liquid temperature rise in the microchannels, which causes significant liquid thermophysical property variations and hence, significant increases in the relevant flow parameters, such as the Reynolds number. Hence, the transition point and range are affected by the liquid temperature, velocity, and geometric parameters of the microchannel.
- The unusual behavior of Nu receding with increasing Re in the laminar regime in microchannels may alter the status of thermal development and hence the conventional thermal entry length, since the variation of the heat transfer coefficient along the flow is a variation of the boundary condition. A criterion for a thermally-developed flow applicable to the T1, H1, H2 boundary conditions is available in the literature.
- But the thermal entry length applicable to any variation of the boundary condition has not been explained.
- The Nu in the laminar and transition regimes channels is correlated with Br, in addition to Re, Pr, and a geometric parameter of the microchannels. The role of Br in the laminar regime is supported by an analysis of the experimental data.
- From an analysis of the experimental data, Br is discovered to determine the flow regime boundaries from laminar-to-transition and from transition-to-turbulent, in addition to Re. The Re has a higher role relative to Br in determining both the flow regime boundaries. The role of Br relative to Re in determining the laminar-to-transition boundary is higher than its relative role in determining the transition-to-turbulent boundary.
- Viscous heat generation is a result of the friction between the layers. Since the ratio of surface area to volume is large for microchannels, viscous heating is an important factor. It is especially important for laminar flow, where considerable gradients exist. The Brinkman number, Br, is defined to represent this effect. They observed decrease in Nu for Br > 0 and an increase for Br<0. This is due to Br increases or decreases the driving mechanism for convective heat transfer, which is the difference between wall temperature and average fluid temperature.

- Prandtl number is important, since it directly influences the magnitude of the temperature jump. Looking at the temperature jump equation, as Pr increases, the difference between wall and fluid temperature at the wall decreases. Therefore, they observed greater Nu values for large Pr.

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