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INCREASING GREENHOUSE EFFICIENCY DUE TO TUBE SIZING AND LOCATION

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Abstract. *The objective of the present work is to report preliminary results on the investigation being made on heating crops inside greenhouses. The real problem involves Convective and Radiation Heat Transfer in presence of participating gases, a much complex situation that precludes closed form solutions. The analysis presented here consider pure Radiative transfer in presence of a transparent gas, a situation that has an analytical solution fully discussed herein, but also in presence of a semi transparent gas, a situation studied numerically using a CFD numerical package. Results indicate clearly that in both cases finding a tube position and its diameter is straight forward provided the Newton´s method is used. However, due to the existance of multiple solutions, the optimization search implemented was able to find an optimum solution but also indicated numerical problems due to a very ill conditioned matrix, requiring a different algorithm, currently under investigation. Furthermore, the results so far obtained for the participanting gas indicate that tube location and sizing are clearly affected by the nature of the gas and its effects need to be taken under consideration provided an accurate analysis of the problem is needed..*

Keywords. *Greenhouse efficiency, Numerical Simulation, Thermal Radiation, Inverse Problems*

1. Introduction

Due to the ever increasing fuel and heat production costs coupled to gradually more stringent environmental restrictions, many efficiency analyses in mechanical systems have been developed. Among others, thermal systems have been receiving greater attention lately perhaps due to their widespread applications (Boehm, 1987). In many cases, however, only approximated analyses are possible, as the physics underlying the process is not fully understood due to their inherent complexity. Nevertheless, interesting optimization problems have been introduced in which objective functions based on many arguments such as maximum (or minimum) temperatures, local heat fluxes, and entropy generated (Bejan, 1982, Bejan, 1988 and Braga, 2002) are studied. Most of these optimization studies dealt mainly with conductive-convective systems, neglecting thermal radiation for many different reasons. However, experience indicates (Siegel and Howell, 2002) that radiative heat fluxes are often of the same order as natural convection fluxes, besides being the most important heat transfer mechanism on high temperature devices. Due to modern engineering technology and the long known fact that thermal efficiency increases with temperature, analysis of such situations are becoming gradually more important and more sophisticated studies are becoming necessary.

One of the many areas in which radiative heat transfer is relevant is on greenhouse thermal designs, in which tubes are used in order to keep the crop at the desired temperature levels. Recently, Teitel & Tanny (1998) analyzed a very simplistic model for a greenhouse and concluded that the heating pipes should be installed as close as possible to the crop. Unfortunately, many simplifications were introduced on their investigation. For instance, their study neglected the radiation blocking coming from the crops toward the pipes and other surfaces (resulting on a very stringent criteria for tube sizing), the likely solar heating occurring mostly over one of the surfaces, the influence of gas participation on the radiation heat transfer, and the convective heat transfer. This paper addresses most of such previously neglected effects but it is focused on radiative heat transfer. On a coming article the combined radiation + natural convection heating on the greenhouse will be fully studied. The main objectives of the present paper are to introduce a deeper analysis of the influence of Radiative Heat Transfer in the problem of increasing efficency on greenhouses, considering not only the cost of the fuel needed to heat the ambient as well as the maximum heat fluxes that the crop may take to avoid surface burning, included herein as restrictions to the optimization procedure. In the near future, the unsteady analysis may also be considered.

2. Physical Model

Consider a rectangular greenhouse such as the one displayed on Fig. 1. Following Teitel & Tanny (1998), the aspect ratio L_2 / L_1 may shift from 1 (for roses) to 3 (for tomatoes). Both cases are analyzed here. At this stage, the enclosure is considered to be infinitely long. Surfaces 2 (left) and 4 (right) simulate the crop. The heating pipe, of diameter D, is located at a position defined as (x, y). Depending on the specific case to be considered later on, some or all of such variables are to be estimated following the current analysis. Instead of modelling all surfaces as black bodies as done by Teitel & Tanny (1998), the present work uses the gray body approximation (although a better model could be used as done by Braga, 2002. The lack of good spectral data on the crop emissivity turns this to be a not important feature). Results shown here were obtained considering that all plane surfaces have the same emissivity while the tube emissivity is chosen to be a design parameter, although always smaller than the others. As it is known from the literature (e.g. Siegel and Howell, 2002), the emissivities for nonconductors are usually higher than for conductors. The temperatures chosen are T_1 = 293 K = T_2 = T_4 and T_3 (the ambient) = 283 K. The tube temperature is chosen to be equal to 358 K. As already mentioned, the unsteady analysis to be made will drop such somewhat unreal specifications. External radiation (coming from the Sun, for instance) reaches the greenhouse at a specified angle. For simplicity, in the present work, surface 3 is a virtual one, although it could be modified to be a physical one, such as a glass panel, for instance.

As it is well known, both the radiative and the convective heat transfer depend on physical parameters such as the temperatures, the thermal properties, but also on the geometry. Hot fluid (water, for instance) being pumped throughout the tube heats the greenhouse walls directly by radiation and indirectly by convection. As previously mentioned, the present work deals only with radiation and the geometric aspects are handled using configuration factors for diffuse surfaces (specular surfaces could also be handled, as done by Braga, 2002 but those are simply not feasible for practical greenhouses). This is discussed next.

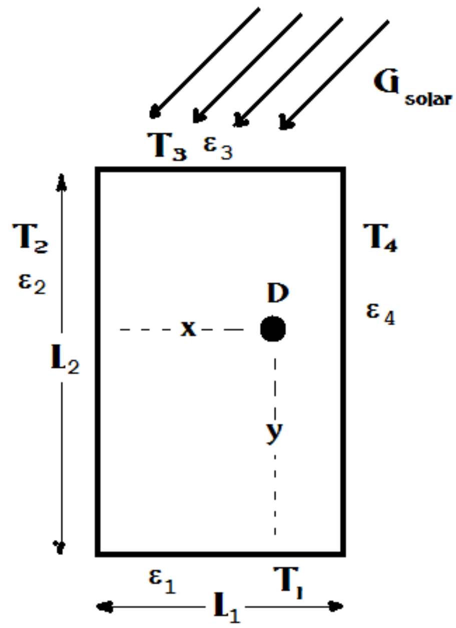

Figure 1. Greenhouse Geometry

3. Radiative Geometric Configuration Factors

Configuration factors take care of the relative geometry of all surfaces involved in the balance of energy, that is, the First Law of Thermodynamics. In the current situation, a rectangular cavity, most of such factors are found in standard heat transfer books but not those associated to the tube surface. A careful analysis of Fig. 1 indicates that there are two particular tube locations to be considered. In the first one, the tube is located to the left of the cavity diagonal linking the corner of surfaces 2 and 3 and the corner of surfaces 1 and 4. This region will be named as region A. In the second, the tube is located to the right of the same diagonal, region B. If the heating tube is located in region A, clearly it will not affect the radiative heat exchange between surfaces 3 and 4, but it will definitely alter the exchange between surfaces 1 and 2.

Before proceeding, it is instructive to notice that the configuration factor between the pipe and surface 1 (or 2, for that matter) is found in the literature (see, for instance Siegel & Howell, 2002). However, the configuration factor between 1 and 2 in presence of the tube in a location such as A was not found, even under the assumption of a very long enclosure, the situation considered herein, and that is manageable using Hottel's crossed-string method. As this is the most critical geometrical issue in the present analysis, brief comments are due next.

Suppose that the configuration factor from surface 1 (bottom surface) to surface 2 (left surface), named as F_{12}, is to be determined. Drawing tangents to the tube, one obtains pseudo-surfaces such as 1', 1", 2' and 2" as shown in Figure 2. Clearly, the length of surface 1, L_1, is given by $L_{1'} + L_{1''}$. Similarly, it may be seen that $L_2 = L_{2'} + L_{2''}$. Such lengths most certainly depend on the tube location, indicated by the pair (x, y) in Fig. 1, the diameter, D, and the cavity aspect ratio and it is not difficulty to obtain such auxiliary lengths using simple planar geometry.

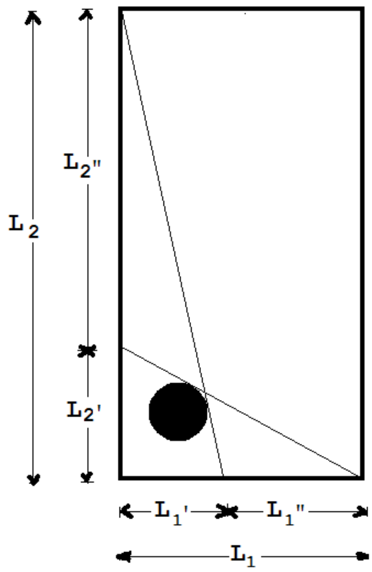

Figure 2. Auxiliary Surfaces for Configuration Factors

Using the reciprocity rule for the configuration factors, one may write:

$$L_{1''}F_{1''2''} = L_{2''}F_{2''1''} \tag{1}$$

in which $F_{1''2''}$ is given by Hottel´s crossed-string method (taking care of the arc along the pipe connecting the two tangent points as it may be seen in Fig. 2).

$$L_{1'}F_{1'\to(2'+2'')} = L_{1'}F_{1'2} = L_2F_{21'} \tag{2}$$

In the above expression, $F_{21'}$ is given by Hottel´s crossed-string method. Similarly, by the reciprocity rule it comes that:

$$L_{1'}F_{1'2'} = L_{2'}F_{2'1'} \tag{3}$$

in which $F_{2'1'}$ is given by Hottel´s crossed-string method. Then by the configuration factors algebra (Braga, 2003):

$$F_{1'2''} = F_{1'2} - F_{1'2'} \tag{4}$$

$$F_{(1'+1'')\to2''} = F_{12''} = (L_{1'} \times F_{1'2''} + L_{1''} \times F_{1''2''}) / L_1 \tag{5}$$

$$L_1F_{(1'+1'')\to2'} = L_1F_{12'} = L_{2'}F_{2'1} \tag{6}$$

in which $F_{2'1}$ is given by Hottel´s crossed-string method. Finally, it may be seen that:

$$F_{(1'+1'')\to(2'+2'')} = F_{12} = F_{12''} + F_{12'} \tag{7}$$

A spreadsheet was prepared to handle all configuration factors (and is available freely upon request). As it may be expected, the validity of all calculations was guaranteed using the summation rule:

$$\sum_{j=1}^{5} F_{ij} = 1 \tag{8}$$

4. Mathematical modeling:

Defining as usual the radiosity, J, as the total radiant energy leaving a surface, herein considered as a gray diffuse one, E_b as the total, hemispherical emissive power of a black body, ε as the total, hemispherical emissivity of a gray surface, Q as the net radiation transfer from a surface which area is indicated by A, and H_i is the external (i.e. from the sun) heat flux reaching the i-surface, the energy equation may be written as (e.g. Braga, 2003):

$$\frac{Q_i}{A_i} + H_i = J_i - \sum_{j=1}^{5} F_{ij} J_j \tag{9}$$

in which summation goes from 1 to 5 (including the pipe surface). The radiosities are obtained from:

$$J_i - (1-\varepsilon_i) \sum_{j=1}^{5} F_{ij} J_j = \varepsilon_i E_{bi} + (1-\varepsilon_i) H_i \tag{10}$$

As it may be noticed, Eq. (10) defines, for the current project, a set of 5 equations with 5 unknowns, defined by the radiosities J_i. For the problem under consideration, as all surface temperatures are specified, a set of linear equations are to be solved by standard methods. More interesting situations could be those in which some heat fluxes are considered. However, for a standard greenhouse, such situations have no practical meaning and were, therefore, disregarded.

Once the radiosities are found, the heat flux at all surfaces may be calculated, using Eq (9). In order to evaluate the effectiveness of the proposed thermal system in relation to others, a greenhouse thermal efficiency may be defined. For the present considerations, efficiency is defined as the ratio between the total heat reaching both the left and the right crops and the total heat input into the cavity, that is, the sum of the energy transferred by the pipe to the external heating. Clearly, other definitions are available but the one chosen here avoids larger than unity efficiencies and is directly affected by the thermal participants, if you will. So,

$$\eta = \frac{Q_2 + Q_4}{Q_{tube} + H} \tag{11}$$

Clearly, for any set of variables (x,y,D) a different solution wiil result to the system of equations defined by Eq. (10) and a corresponding greenhouse efficiency defined by Eq. (11) will be obtained. In order to deal with a more interesting and realistic situation, the problem to be solved here is stated in terms of finding the solution of the mentioned problem that are able to satisfy the specified heat fluxes at both crops and the tube, that is, to some constraints. Two set of problems were analyzed. In the first one, a solution to the problem was sought. However, it was noticed that the problem would offer several different solutions, that is, it was observed that the restrictions were satisfied at essentially the practical level for several different tube positions and diameters. Therefore, it was studied how to find the optimum solution, understood as the one indicating the highest possible efficiency. This defines the second problem. As it comes, both problems may be understood as standard Inverse Problems, the second one being also an optimization one. They are discussed next.

4.1 First Inverse Problem:

Suppose one decides to use a tube for heating purposes and needs to find out where it should to be installed and which should be its size. The most complete situation is when one desires to estimate a convenient tube location and diameter, defined by the pair (x,y) and the diameter D, that is, whenever three unknowns are to be determined. In order to properly pose the problem, one may consider that the heat fluxes at both crops (i.e. surfaces 2 and 4, on Figure 1) are limited by some values and also that the heat flux from the pipe is set at some specified value (alternatively, a desired efficiency level could also be specified). That is, the following restrictions are to be satisfied:

$$Q_2\big|_{actual} = Q_2\big|_{max} \text{ or } R_1(x,y,D) = Q_2\big|_{actual} - Q_2\big|_{max} = 0 \tag{12}$$

$$Q_4\big|_{actual} = Q_4\big|_{max} \text{ or } R_2(x,y,D) = Q_4\big|_{actual} - Q_4\big|_{max} = 0 \tag{13}$$

and at last,

$$Q_{tube}\big|_{actual} = Q_{tube}\big|_{max} \text{ or } R_3(x,y,D) = Q_{tube}\big|_{actual} - Q_{tube}\big|_{max} = 0 \tag{14}$$

Having three unknowns (x, y, D) and three equations, Newton´s method may be used in order to determine the solution. Although there are other options, results obtained for this first problem, to be presented in a later section, considered the use of equations (12) and/or (13) for most problems (for instance, for the situation in which there are only two unknowns). Equation (14) is used only for cases with three unknowns, for no specific reason (besides the fact that it is not one that can be easily implemented). In many instances, the physical problem is symmetrical with respect to a vertical axis crossing the center of the greenhouse cavity (for example, whenever there is no external heating and both crops are similar). In such situation, the tube x-location is $L_1/2$. For conciseness, one may consider that $\vec{X}=(x,y,D)^t$ for the most general case. For all cases studied, convergence was always smooth and was based on the following algorithm:

$$\vec{F}(X)=0\approx\vec{F}(X_o)+\left.\frac{d\vec{F}}{d\vec{X}}\right|_{\vec{X}_o}\Delta\vec{X} \tag{14}$$

The solution for Eq. (14) is readily indicated as:

$$\Delta\vec{X}=-\frac{\vec{F}(X_o)}{\left.d\vec{F}/d\vec{X}\right|_{\vec{X}_o}} \tag{15}$$

and it gives the increments for the next iteration. Convergence is achieved using some convenient criteria ($\approx 10^{-4}$). In many cases, non-unique solutions were obtained as it will be shown.

4.2 Second Inverse Problem:

As previously mentioned, the overall project aims at analyzing the more complete situation in which Radiation + Convection and the gas participation are all relevant. Clearly, this is a much difficulty problem and, consequently, a more expensive one to be implemented and studied. Having noticed the existance of non-unique solutions, it became important to investigate the possibility of finding a particular tube location and sizing in which maximum efficiency is attained, and externally imposed restrictions are conveniently handled. Due to the computational costs involved, it became evident that the investigation should proceed with the simpler situation considered in the present paper and its analytical solution, even considering Natural Convection to be less important. The analytical solution obtained for the pure radiative problem with a transparent medium, previously discussed, allowed a quicker investigation. The results are given below.

In short, the second inverse problem one may be interested is the one in which it is sought to find the optimum (hopefully, maximum) value for Eq. (11) subjected to restrictions given by Eq. (12) and (13) but generically indicated as below:

$$g(x,y,D)=0 \tag{16}$$

$$h(x,y,D)=0 \tag{17}$$

One way of solving such problem is through the use of Lagrangian multipliers, although there are other techniques. Being an exploratory investigation, it was decided to apply those multipliers. Introducing λ_1 and λ_2 as such parameters, the new objective function is now:

$$F(x,y,D)=\eta(x,y,D)+\lambda_1 g(x,y,D)+\lambda_2 h(x,y,D) \tag{18}$$

Both λ_1 and λ_2 are taken to be held constant during the various steps of any search but gradually reduced towards zero, tentatively, after a minimum is found. Due to the specifics of the current problem, ill conditioned is often found, sometimes precluding achieving any solution. Experience collected herein indicates that an adequate search procedure is obtained provided the parameters λ_1 and λ_2 are somehow related. Among other options, results shown here were obtained with:

$$\lambda_2=\frac{g(x,y,D)}{h(x,y,D)}\lambda_1 \tag{19}$$

In many instances, it was noticed that whenever initial values of g(x,y,D) and h(x,y,D) were quite large, due to a poor location and/or tube sizing estimates, the optimization procedure suffered from oscillations. Better results were always achieved whenever λ_1 were select in order that the modified objective function indicated by Eq. (18) and the original one, indicated by Eq. (11), would not be significantly different. As the problem under investigation may have up to three unknowns (although only results for up to two simultaneous independent variables will be presented herein), for the most general situation, the 3x3 Hessian matrix is to be calculated:

$$H(X) = \begin{pmatrix} \frac{\partial^2 F(X)}{\partial X_1^2} & \frac{\partial^2 F(X)}{\partial X_1 \partial X_2} & \frac{\partial^2 F(X)}{\partial X_1 \partial X_3} \\ \frac{\partial^2 F(X)}{\partial X_2 \partial X_2} & \frac{\partial^2 F(X)}{\partial X_2^2} & \frac{\partial^2 F(X)}{\partial X_2 \partial X_3} \\ \frac{\partial^2 F(X)}{\partial X_1 \partial X_3} & \frac{\partial^2 F(X)}{\partial X_2 \partial X_3} & \frac{\partial^2 F(X)}{\partial X_3^2} \end{pmatrix} \tag{20}$$

The numerical discretization of the second order derivatives are done following standard procedures (Abramowitz & Stegun, 1970). As the problem is strongly non-linear, its solution must be found by an iterative procedure. At iteration i+1, the new estimates are obtained from Newton's method:

$$X^{i+1} = X^i - \left[H(X^i)\right]^{-1} \nabla F(X^i) \tag{21}$$

in which $\nabla F(X^i)$ is the gradient of F at X^{i-1}. As it is known, solution to Eq (21) is not always smooth as a near singular Hessian matrix can result in large changes in the predicted values, resulting on a unstable numerical procedure.

4.3 Semi Transparent Medium

Energy transfer between surfaces is significantly altered in presence of semi transparent medium, due to absorption, emission and scattering of radiation (Siegel & Howell, 2002). Due to non homogeneous air condition inside greenhouses, resulting from the mixture of what is generally called greenhouse gases, powder, small particles and others, it was decided to investigate how the tube size and location would be affected by changes in the radiation intensity in the medium. For simplicity, a non-scattering medium was considered, although this may be corrected. Due to the objectives of the present investigation, it was decided to use a specialized CFD package named FLUENT, available at the Mechanical Engineering Department of PUC-Rio, to solve the Radiative Transfer Equation. In spite of the fact that other methods are available, it was decided to use the method of discrete ordinates, MDO, mainly because it could handle both the transparent medium case (in which an analytical solution was available, as indicated by the previous discussion, allowing assessment and numerical error control) and the translucent medium.

In essence, MDO substitute the Radiative Transfer Equation by a set of equations for an intensity of radiation angularly averaged over a finite number of ordinate directions. Integrals over a range of angles are replaced by sums over those ordinate directions. The results to be shown were obtained considering a total of 576 directions (12 theta divisions times 12 Phi divisions times 4, due to symmetry), a number sufficiently small to give results close to the analytical ones for the case of transparent media, matching engineering level of accuracy and cost of computation. The triangular mesh used for the present simulations handled 150 points along the vertical walls, 30 points along the horizontal walls and 20 points along the tube surface. Such numbers allowed good agreement to the analytical results. Details of the accuracy results will be available elsewhere. The results were obtained considering the absorption coefficient for the gas was considered to be equal to 0.2, although results with higher values were also obtained.

5. Solution Procedure

The solution procedure chosen consisted of the following steps:

i. Direct Problem (or specification of a desired goal). In order to assess and evaluate the solution method proposed herein, it was necessary to find the solution of a direct (to oppose to the inverse) problem. The direct problem starts when a specific set of data is chosen (from now on, named the reference solution), that is, the location indicated by the pair (x.y) and the tube sizing, indicated by D is selected. Surface emissivities, greenhouse aspect ratio and temperature levels are also specified and kept constant throughout the entire simulation. For the chosen set of independent variables (either one or two, chosen from the three possible options: x, y and D), the thermal efficiency, indicated by Eq. (11), is determined from the solution of Eqs. (9) and (10). Among other data, the desired heat exchanged at each crop (and, eventually, at the tube) are selected.
ii. Inverse Problem. After the selection of an initial set of estimates, roughly 50-100% off the correct values, the procedure is followed until some adequate convergence criteria is achievable. At each step, either the Jacobian that appears in Eq. (15) (first problem) or the Hessian matrix given by Eq. (20) (second problem) is computed and from that the new estimates are obtained. No convergence difficulty was observed for the first problem. For the second problem, a suitable set of values for λ_1 and λ_2 are chosen.

iii. If the convergence criterion is attended, the procedure for the first problem is stopped. For the second problem, after convergence, a new set of values for λ_1 and λ_2 is determined and step ii is re-initialized. If the increment $X^{i+1} - X^i$ is considered to be small enough, the global iterative search is stopped. In many instances, no convergence was achievable, most likely due to the ill conditioning of the Hessian matrix.

iv. For both problems, results are compared to the "correct" set of data (specified for the direct problem) for assessment and error control.

Results for all problems discussed herein are shown next.

6. Results

In this paper, it was decided to indicate two sets of results: one neglecting the influence of external heating, and the other considering it. Again, three cavity aspect ratios were investigated: 5:1, 3:1 and 1:1. Some of the results are given in the tables below.

6.1 First Problem

As previously mentioned, in this first problem, the goal was to obtain the right solution for the variables in order to meet some criteria. Table 1 below indicates some of the available results. The first column indicates the set of unknown variables chosen from the possible set (x - y - D). For instance, the first case, defined by (101) indicates that the first (x) and the last (D) variables were set free, while the second (y) was fixed (i.e. was not a variable) during that particular situation. All but the last two cases shown in such table were obtained considering external heating equal to zero.

Table 1. Results for First Problem. Tube Emissivity = 0,8, others = 1,0; Aspect Ratio = 5:1

	Initial							Final							
Case of Study	x	y	D	R1	R2	R3	η	x	y	D	R1	R2	R3	η	Number of Iter
101	0,600	2,800	0,080	36,28	45,47	---	39,8%	0,400	2,800	0,160	0	0	---	63,27%	3
011	0,400	1,200	0,080	46,06	48,51	---	35,8%	0,400	1,673	0,163	0	0	---	62,37%	4
011	0,400	2,805	0,158	1,10	1,17	---	63,0%	0,400	2,800	0,160	0	0	---	63,27%	2
110	0,800	0,600	0,080	-1,85	13,13	---	29,1%	0,393	2,326	0,080	0	0	---	39,84%	6
111	0,402	1,201	0,158	0,97	1,17	-2,59	59,0%	0,400	1,200	0,160	0	0	0	59,34%	1
111	0,800	2,800	0,080	33,33	47,92	-103,48	40,3%	0,411	3,822	0,159	0	0	0	59,34%	6
111	0,300	4,500	0,300	-4,93	-59,86	137,45	48,6%	0,789	3,004	0,200	0	0	0	49,15%	6
101	0,700	2,500	0,100	64,45	46,17	---	35,8%	0,880	2,500	0,197	0	0	---	57,27%	3

As it may be observed, these results are quite good, indicating that the numerical methodology developed is able to handle properly all situations tested. It may be also seen that multiple solutions are available. Consider, for instance, the results indicated in the second and the third lines. Starting from very different initial estimates, the numerical procedure finds two sets of results, both meeting the imposed restrictions and having the same thermal efficiency (to the significant digits shown here). Something similar is shown in the 5th and the 6th cases studied.

Table 2 indicates some of the results obtained for a different aspect ratio (3:1) and considering that the emissivity of the greenhouse walls are equal to 0,8 and the tube emissivity equal to 0,6. The convergence rate is again quite smooth.

Table 1. Results for First Problem. Tube Emissivity = 0,6, others = 0,8; Aspect Ratio = 3:1

	Initial							Final							
Case of Study	x	y	D	R1	R2	R3	η	x	y	D	R1	R2	R3	η	Number of Iter
101	0,200	0,750	0,240	-40,61	-61,80	---	60,0%	0,500	0,750	0,100	0	0	---	37,34%	3
011	0,500	0,200	0,020	-49,29	188,22	---	47,3%	0,500	0,750	0,100	0	0	---	37,34%	5
111	0,220	0,150	0,240	-25,86	-40,70	132,33	44,8%	0,500	0,750	0,100	0	0	0	37,34%	5

Next, it was decided to investigate the effect of the tube emissivity and the greenhouse aspect ratio in the tube location. The cavity walls emissivities were all taken equal to 0,8 and no external heating were considered. In order to proceed with the analysis, the following set of restrictions was taken:

$$
\begin{aligned}
&Q_1 = -20\ \mathrm{W/m^2} \\
&Q_2 = -20\ \mathrm{W/m^2} \\
&Q_{tube} = 100\ \mathrm{W/m^2}
\end{aligned}
\tag{22}
$$

In this situation, the greenhouse efficiency is always equal to 40%, as given by Eq. (11). Due to the symmetry of the problem in this situation, the horizontal tube location was always equal to 0,5, that is, the tube is to be installed at the cavity center line. Table 3 indicates the results.

Table 3. Results for First Problem. Tube y-position and diameter

L2/L1	ε = 0,4	ε = 0,6	ε = 0,8
1	none	none	none
3	0,9415 - 0,1544	0,903 - 0,102	0,886 - 0,078
5	0,7619 - 0,1544	0,751 - 0,104	0,745 - 0,078

As it may be seen, no solution was obtained for a square greenhouse. Furthermore, it should be observed how the tube y-location changes significantly depending on the aspect ratio (a reasonable effect) but not the tube diameter. For a give aspect ratio, the y-location does not change as much as the tube diameter as functions of the tube emissivity. This is a much to be expected result as long as the tube heat flux is specified. Provided other variables and parameters are fixed, the tube heat flux is strongly dependent on the surface area (i.e. the diameter) and the tube emissivity. As the tube surface temperature is set at some specified value it may be concluded that the larger the area the smaller the emissivity to attend the prescribed condition. This explains the trend observed on the data shown.

6.2 Second Problem

During the early attempts to study this problem, it was noticed the possibility of multiple solutions depending on the physical and geometrical parameters. Table 1 showed a few such cases. Observing such situation, a search for the most efficient tube location and diameter was made using Lagrangian multipliers, as previously mentioned. Table 4 gives one of the optimum solutions found for a group of parameters. In this table, the column "Reference" indicates the set of parameters used to define the direct problem (step i of the solution procedure). The third column indicates the initial data and the last column indicates the optimum results. As it is shown, the greenhouse thermal efficiency is not that different from one to the other, at least for this case. However, the optimization procedure was successful.

It must be pointed out that in all cases studied, the Hessian matrix was always ill conditioned. In order to avoid handling such matrices, a new search algorithm based on genetic algorithms (Davalos & Rubinsky, 1996) is being developed. As it is known, such algorithms are adaptive search procedures loosely based on the Darwinian notion of evolution and have a definitely advantage over those based on the Hessian matrix as they do not require singular or near singular matrix inversions. The present authors hope to present a more robust search algorithm in a sequel to this article.

Table 4. Results for the greenhouse restrictive optimization procedure.
Surface Emissivities = 1,0; Aspect Ratio = 5:1, Case of Study 011.
Initial λ_1 = 1/256, Final λ_1 =1/1450

	Reference	Initial	Final
x	0,5	0,5	0,5
y	1,5	3,5	2,51
D	0,1800	0,0800	0,1754
Q1	-99,43	-31,01	-99,43
Q2	-99,43	-31,01	-99,43
Qtube	292,0	130,6	285,0
η	68,10%	47,50%	69,77%
R1	0	68,42	0
R2	0	68,42	0

6. 3 Semi Transparent Medium

Following the current investigation, it was desired to study the influence of a semi transparent medium on the physics of the problem. This part of the study was performed using the computational package FLUENT, a very powerful simulation for CFD problems. Details of this study will be shown elsewhere but it is imperative to show some of the results and effects. Table 5 displays the comparative results for the y-location of a tube inside a 5:1 greenhouse cavity in presence of a transparent and a semi transparent medium. Figure 3 displays the temperature profile inside the semi-transparent gas. The shift in the tube location is clearly indicated by comparison between data on that table and it is visualized in that figure. The tube location along the y-coordinate was found after the iterative procedure indicated on step ii at the Solution Procedure section above (x location and tube diameter were considered as constants, to reduce the computational costs). No attempt to find the optimum solution was considered.

Table 5: Numerical Estimative of the tube y-location.
Results for a transparent (a = 0) and a semi-transparent (a = 0,2) medium.

	Q2 [W/m2]	Q4 [W/m2]	y - location
Reference	-30,322	-30,271	1,500
a = 0	-30,338	-30,346	3,608
a = 0,2	-30,325	-30,252	1,214

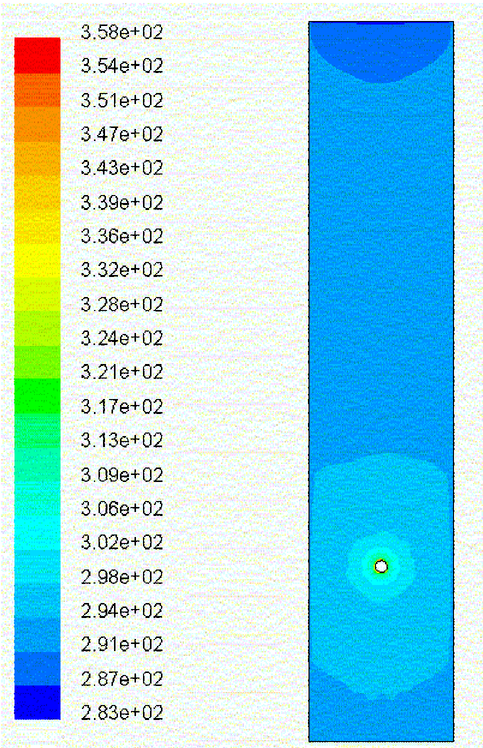

Figure 3. Tube location and Temperature Profile for the Semi Transparent Case

7. Concluding Remarks

The present paper describes the work under development to fully understand the heat transfer inside a greenhouse thermal cavity used to grow different crops. The most complete problem involves Radiation in presence of a semi transparent gas and Convection Heat Transfer, a situation that precludes a throughfull optimization analysis. Therefore, it was decided to investigate initially the situation of a pure Radiative Cavity with a transparent medium inside. For this situation, an analytical solution was obtained and implemented on a spreadsheet. This allowed a somewhat extensive investigation on the effects of several parameters, either physical such as surface emissivities, aspect ratios, temperature levels or numerical, associated to the optimization search (such as the Lagrangian multipliers). In order to prepare for a more complete investigation, the study turned to the numerical CFD package, FLUENT, that allowed the analysis of a participant gas. After an extensive numerical investigation on mesh sizes, convergent criterion, optimum number of directions for the Method of Discrete Ordinates and others, a good agreement between analytical and numerical results were obtained. After that, the investigation on the effects of the participant medium was conducted. The results so far obtained indicate that the gas may affect considerably the locatization of the tube and its diameter. Further investigation on these effects is currently being conducted, coupled to the analysis of the simultaneous Convective Heat Transfer. It is hoped that the combined effects will render the simulation more efficient and enlighten the physics of the heat transfer mechanisms inside greenhouses. Finally, a new optimization procedure based on Genetic Algorithms for instance, seems to be necessary to pursue the search for the optimum solution.

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