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STUDY OF THE HEAT TRANSFER ON SYSTEM WITH THERMAL PROTECTION

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Abstract. *The heat transfer with phase change and mass loss occurs in many engineering problem, and is of great technological importance. Problems of this type are inherently nonlinear due to the moving boundary. A phenomenon that has been gainning the researchers attention in the last decades is the heating caused by the friction between the air and space vehicle structure in the atmospheri ,reentry producing a surface recession due to the material loss. This phenomenon is known as ablation in the aerospace area. In this work an analysis of the transient ablation problem in a rectangular prism that was modeled as a two-dimensional diffusion problem, was accomplished in order to obtain the ablative thickness and speed. The Generalized Integral Transformed Technique (GITT) was used to solve the resultant equations. This technique transforms the original partial differential equation system into an ordinary differential equation system. A computer program was implemented using the Fortran Language to solve this equation system and IMSL Libraries routines. The variables of interest such as the temperature distribution, the ablative thickness and speed were obtained for heat fluxes different in the boundary.*

Keywords*. Ablation, phase change, GITT, moving boundary.*

1. Introduction

The transient heat transfer in solids with thermal protection are the great technological importance and in numerous engineering applications, Hsiao & Chung (1984). An example of theses applications includes the aerodynamic heating caused by the high speed with that the space vehicles reach as in the release as in atmospheric reentry.

A variety of systems of thermal protection has been proposed (Hatori & Pessoa-Filho, 1998; Sutton, 1982; Steg & Lew, 1962). Among these systems the more used are the ones with ablative materials.

Heat transfer with ablation in a two-dimensional region subjected to time variant heat fluxes at boundary were studied by Hsiao & Chung (1984). Due to non-linear features of this problem, exact analytical solutions are practical non-existent. Although many approximate analytical and numerical solutions to the ablation problem have been published in the literature, (Zien, 1981; Chung *et al.*, 1983; Kurokawa *et al.*, 2003), they are only restricted to the case of one-dimensional heat transfer process. Pantaleão (2003) studied the two-dimensional ablation problem in which the Galerkin finite element method was used for the space discretization, together with a totally implicit time iteration scheme.

The ablation phenomenon is complex involving heat and mass transfer, physical evaporation or pyrolysis, chemical reactions among another (Lacaze, 1967; Kreith, 1973). Due the complexity of the phenomenon, a convenient proposal to model the problem is to use a model that involves phase change with moving boundary with partial or whole loss of mass.

The process of heat transfer with ablation is inherently non-linear due to moving boundary, initially unknown (Chung & Hsiao, 1985; Zien, 1978; Chung *et al.*, 1983; Zien, 1981).

A technique that has been used to obtain exact solutions of complex problems is the Generalized Integral Transform Technique (GITT) (Cotta & Özisik, 1987; Cotta, 1993; Diniz, 1996; Diniz *et al.*, 1993). This technique is an analytical/numerical hybrid tool.

In this work the two-dimensional ablation problem is studied, considering a heat transfer with phase change and moving boundary. The GITT will be used in the analytical development, where for the problem solution a coupled system of ordinary differential equations should be solved. This system will be numerically solved by implementation of an algorithm in Fortran language using the IMSL library (IMSL, 1979).

2. Modeling of the problem

In the problem formulation it is considered a two-dimensional heat transfer in the stagnation point of a revolution body, which was approached to the long rectangular prism geometry. Also, it is assumed that the longitudinal length is much greater than the other dimensions, hence the end effects can be neglected. Fig. 1 depicts the problem geometry under consideration, where q_1 and q_2 are the uniform transient heat input and heat loss, respectively.

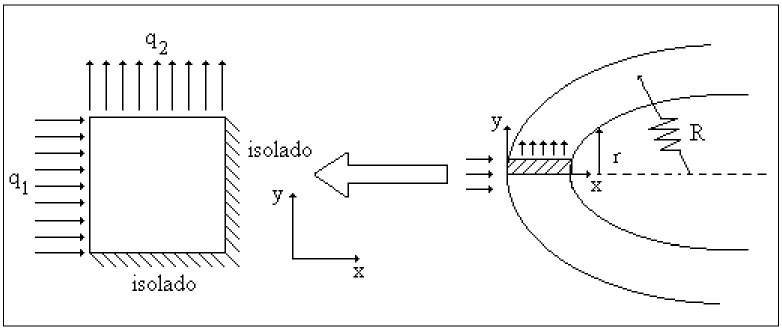


Figure 1. Schematic representation of ablation process model in the stagnation point region.

This problem is better described whether we divide the process in two periods: one named preablative, where the plate becomes warm due to the incident heat flux until the temperature surface reaches the fusion temperature of the material. In the other one, named ablative, where occurs material fusion that is dragged off to the environment, Fig. 2.

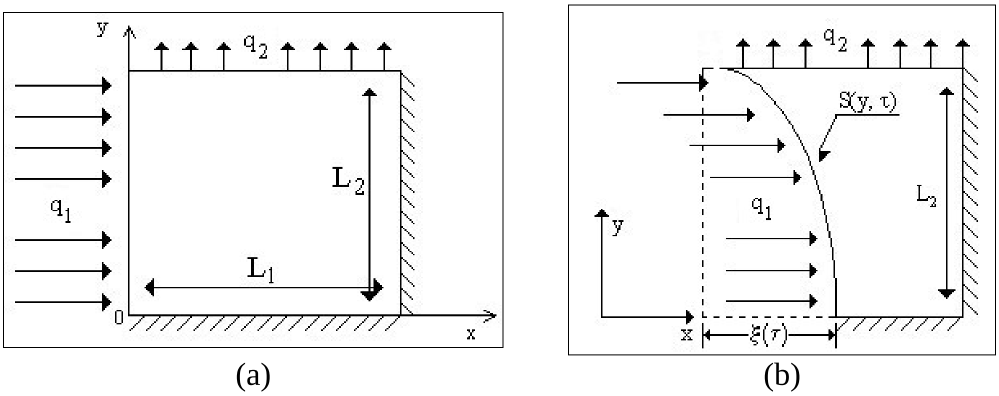


Figure 2. Description of ablation problem under consideration: (a) preablative and (b) ablative.

3. Formulation mathematical

The governing equations that model these two periods, in the dimensionless form, are given as:
Preablative period:

$$\frac{\partial\theta(x,y,\tau)}{\partial\tau} = \frac{\partial^2\theta(x,y,\tau)}{\partial x^2} + \frac{\partial^2\theta(x,y,\tau)}{\partial y^2} \qquad \tau > 0 \qquad \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le l \end{matrix} \tag{1}$$

with initial and boundary conditions:

$$\theta(x,y,\tau) = 0 \qquad \tau = 0 \qquad \begin{matrix} 0 \le x \le 1 \\ 0 \le y \le l \end{matrix} \tag{2}$$

$$\frac{\partial\theta(x,y,\tau)}{\partial x} = -Q_1 \qquad x = 0 \qquad 0 \le y \le l \tag{3}$$

$$\frac{\partial\theta(x,y,\tau)}{\partial x} = 0 \qquad x = 1 \qquad 0 \le y \le l \tag{4}$$

$$\frac{\partial \theta(x,y,\tau)}{\partial y} = 0 \qquad y=0 \qquad 0 \le x \le 1 \tag{5}$$

$$\frac{\partial \theta(x,y,\tau)}{\partial y} = -Q_2 \qquad y=l \qquad 0 \le x \le 1 \tag{6}$$

Ablative period:

$$\frac{\partial \theta(x,y,\tau)}{\partial \tau} = \frac{\partial^2 \theta(x,y,\tau)}{\partial x^2} + \frac{\partial^2 \theta(x,y,\tau)}{\partial y^2} \qquad \tau > \tau_f \qquad \begin{matrix} S \le x \le 1 \\ 0 \le y \le l \end{matrix} \tag{7}$$

with initial and boundary conditions:

$$\theta(x,y,\tau) = \theta_{in}(x,y,\tau) \qquad \tau = \tau_f \qquad \begin{matrix} S \le x \le 1 \\ 0 \le y \le l \end{matrix} \tag{8}$$

$$\frac{\partial \theta(x,y,\tau)}{\partial x} = 0 \qquad x=1 \qquad 0 \le y \le l \tag{9}$$

$$\theta(x,y,\tau) = 1 \qquad x = S(y,\tau) \qquad 0 \le y \le l \tag{10}$$

$$\frac{\partial \theta(x,y,\tau)}{\partial y} = 0 \qquad y=0 \qquad S \le x \le 1 \tag{11}$$

$$\frac{\partial \theta(x,y,\tau)}{\partial y} = -Q_2 \qquad y=l \qquad S \le x \le 1 \tag{12}$$

where: $\theta_{in}(x,y,\tau)$ is the temperature distribution obtained for the preablative period. The initial condition for the ablative period is the $\theta(x,y,\tau)$ value for the time $\tau = \tau_f$. Due to boundary moving by the phase change process, there is a boundary velocity equation, named restriction condition, that results from the energy balance in the interface, Hsiao & Chung (1984):

$$-\left[1 + \left(\frac{\partial S}{\partial y}\right)^2\right]\frac{\partial \theta}{\partial x} + \nu \frac{\partial S}{\partial \tau} = Q_1 \qquad x = S(y,\tau) \tag{13}$$

where $S(y,\tau)$ and ν are respectively the position of the boundary and the inverse of Stefan number.

4. Analytical solution

In this work the Generalized Integral Transform Technique (GITT) was utilized to obtain the solution of the two-dimensional ablation problem. Considering that temperature potential of the preablative phase can be defined by:

$$\theta(x,y,\tau) = \theta_1(x,y,\tau) + \theta_2(x,y,\tau) \tag{14}$$

Then, there is two problems: one for $\theta_1(x,y,\tau)$ and other for $\theta_2(x,y,\tau)$. Applying the GITT, the appropriate eigenvalue auxiliary problem is described in Kurokawa (2003) and the $\theta_1(x,y,\tau)$ function can be expresses by:

$$\theta_1(x,y,\tau) = Q_1\left(\frac{x^2}{2} - x\right) + A_1 B_1 \hat{\hat{\theta}}_{11}(\tau) + A_1 \sum_{m=2}^{\infty} B_m \cos(\lambda_m x)\hat{\hat{\theta}}_{1m}(\tau) \tag{15}$$

where:

$$A_i = \sqrt{\frac{2}{l(1+\delta_{i1})}}$$

$B_m = \sqrt{\dfrac{2}{(1+\delta_{m1})}}$, $\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$ is the Kronecker delta.

To obtain the values $\hat{\tilde{\theta}}_{im}(\tau)$, the ordinary differential equation below must be solved:

$$\frac{\partial \hat{\tilde{\theta}}_{im}(\tau)}{\partial \tau} + (\mu_i^2 + \lambda_m^2)\hat{\tilde{\theta}}_{im}(\tau) = \hat{\tilde{P}}_{im}(\tau) \tag{16}$$

with initial condition:

$$\hat{\tilde{\theta}}_{im}(0) = \hat{\tilde{f}}_{im} \tag{17}$$

where:

$$\hat{\tilde{P}}_{im}(\tau) = \int_0^1 \tilde{P}_i(x,\tau)\phi_m(x,\tau)dx = \int_0^1\int_0^l \psi_i(x,y,\tau)\phi_m(x,\tau)P_1(x,y,\tau)dydx$$

$$\hat{\tilde{f}}_{im}(\tau) = \int_0^1 \tilde{f}_i(x,\tau)\phi_m(x,\tau)dx = \int_0^1\int_0^l \psi_i(x,y,\tau)\phi_m(x,\tau)\, f_1(x,y)dydx$$

The solution obtained for Eq. (16) is:

$$\hat{\tilde{\theta}}_{11}(\tau) = \exp[-(\mu_1^2 + \lambda_1^2)\tau]\left\{\frac{\sqrt{l}}{3}Q_1(0) + \sqrt{l}\left[\int_0^\tau Q_1(\tau)\exp[(\mu_1^2 + \lambda_1^2)\tau]d\tau + \frac{1}{3}\int_0^\tau \dot{Q}_1(\tau)\exp[(\mu_1^2 + \lambda_1^2)\tau]d\tau\right]\right\} \tag{18}$$

$$\hat{\tilde{\theta}}_{1m}(\tau) = \exp[-(\mu_1^2 + \lambda_m^2)\tau]\left[-\sqrt{2l}Q_1(0)\frac{1}{\lambda_m^2} - \frac{\sqrt{2l}}{\lambda_m^2}\int_0^\tau \dot{Q}_1(\tau)\exp[(\mu_1^2 + \lambda_m^2)\tau]d\tau\right] \tag{19}$$

For $\theta_2(x,y,\tau)$, a new eigenvalue auxiliary problem was defined in the Kurokawa's work (2003), where the solution of the potential problem is given by:

$$\theta_2(x,y,\tau) = Q_2\frac{y^2}{2l} + A_1B_1\hat{\tilde{Z}}_{11}(\tau) + B_1\sum_{i=2}^{\infty} A_i\cos(\mu_i y)\hat{\tilde{Z}}_{i1}(\tau) \tag{20}$$

where the values of $\hat{\tilde{Z}}_{im}(\tau)$ were obtained solving the following ordinary differential equation:

$$\frac{\partial \hat{\tilde{Z}}_{im}(\tau)}{\partial \tau} + (\mu_i^2 + \lambda_m^2)\hat{\tilde{Z}}_{im}(\tau) = \hat{\tilde{S}}_{im}(\tau) \tag{21}$$

with initial condition:

$$\hat{\tilde{Z}}_{im}(0) = \hat{\tilde{h}}_{im}(\tau) \tag{22}$$

where:

$$\hat{\tilde{S}}_{im}(\tau) = \int_0^1 \tilde{S}_i(x,\tau)\phi_m(x,\tau)dx = \int_0^1\int_0^l P_2(x,y,\tau)\psi_i(x,y,\tau)\phi_m(x,\tau)dydx$$

$$\hat{\hat{h}}_{im}(\tau)=\int_0^1 \tilde{h}_i(x,\tau)\phi_m(x,\tau)dx=\int_0^1\int_0^l f_2(x,y)\psi_i(x,y,\tau)\phi_m(x,\tau)dydx$$

Then the solution of the Eq. (21) is:

$$\hat{\hat{Z}}_{11}(\tau)=\exp[-(\mu_1{}^2+\lambda_1{}^2)\tau]\left\{\sqrt{l}Q_2(0)\frac{l}{6}+\sqrt{l}\left[-\frac{1}{l}\int_0^\tau Q_2(\tau)\exp[(\mu_1{}^2+\lambda_1{}^2)\tau]d\tau+\frac{l}{6}\int_0^\tau \dot{Q}_2(\tau)\exp[(\mu_1{}^2+\lambda_1{}^2)\tau]d\tau\right]\right\} \quad (23)$$

$$\hat{\hat{Z}}_{i1}(\tau)=\exp[-(\mu_i{}^2+\lambda_1{}^2)\tau]\left[\sqrt{2l}Q_2(0)\frac{l}{\mu_i{}^2}\cos(\mu_i^*)+\sqrt{2l}\,\frac{l}{\mu_i{}^2}\cos(\mu_i^*)\int_0^\tau \dot{Q}_2(\tau)\exp[(\mu_i{}^2+\lambda_1{}^2)\tau]d\tau\right] \quad (24)$$

Therefore, the temperature distribution in the preablative phase is given by:

$$\begin{aligned}\theta(x,y,\tau)&=Q_1(\tau)\left(\frac{x^2}{2}-x\right)-Q_2(\tau)\frac{y^2}{2l}+\frac{1}{\sqrt{l}}\exp[-(\mu_1{}^2+\lambda_1{}^2)\tau]\left\{\frac{\sqrt{l}}{3}Q_1(0)+\sqrt{l}Q_2(0)\frac{l}{6}+\right.\\&+\sqrt{l}\left[\int_0^\tau Q_1(\tau)\exp[(\mu_1{}^2+\lambda_1{}^2)\tau]d\tau+\frac{1}{3}\int_0^\tau \dot{Q}_1(\tau)\exp[(\mu_1{}^2+\lambda_1{}^2)\tau]d\tau\right]+\\&\left.+\sqrt{l}\left[-\frac{1}{l}\int_0^\tau Q_2(\tau)\exp[(\mu_1{}^2+\lambda_1{}^2)\tau]d\tau+\frac{l}{6}\int_0^\tau \dot{Q}_2(\tau)e^{(\mu_1{}^2+\lambda_1{}^2)\tau}d\tau\right]\right\}+\\&-\frac{1}{\sqrt{l}}\sum_{m=2}^{\infty}\sqrt{2}\cos(\lambda_m x)\exp[-(\mu_1{}^2+\lambda_m{}^2)\tau]\frac{\sqrt{2l}}{\lambda_m{}^2}\left[Q_1(0)+\int_0^\tau \dot{Q}_1(\tau)\exp[(\mu_1{}^2+\lambda_m{}^2)\tau]d\tau\right]+\\&+\sqrt{\frac{2}{l}}\sum_{i=2}^{\infty}\cos(\mu_i y)\exp[-(\mu_i{}^2+\lambda_1{}^2)\tau]\frac{l\sqrt{2l}}{\mu_i{}^2}\cos(\mu_i^*)\left[Q_2(0)+\int_0^\tau \dot{Q}_2(\tau)\exp[(\mu_i{}^2+\lambda_1{}^2)\tau]d\tau\right]\end{aligned} \quad (25)$$

For $\tau>\tau_f$, it begins the phase change period, called ablative period. A variable transformation applied to Eqs. (7) and (13) turns this in a homogeneous problem, defined as:

$$\theta^*(x,y,\tau)=\theta(x,y,\tau)-1 \qquad \begin{array}{l}S(y,\tau)\le x\le 1\\ \tau>\tau_f\end{array} \quad (26)$$

and after a coordinate transformation, defined as $\eta=1-x$, the boundary position is denoted as:

$$S=S(y,\tau)=\xi\left(1-\frac{Q_2}{Q_1}\frac{y}{l}\right) \quad (27)$$

where ξ represents the ablative thickness.

Again, defining an appropriate eigenvalue auxiliary problem, Kurokawa (2003), the following ordinary differential equation is obtained applying the TTIG to the ablative phase:

$$\begin{aligned}&\frac{\partial\hat{\hat{\theta}}_{ik}{}^*(\tau)}{\partial\tau}+\mu_k{}^2\hat{\hat{\theta}}_{jm}{}^*(\tau)+\sum_{j=1}^{\infty}\sum_{m=1}^{\infty}\int_0^l A_{ij}M_k(y,\tau)M_m(y,\tau)dy\hat{\hat{\theta}}_{jm}{}^*(\tau)-\sum_{j=1}^{\infty}\sum_{m=1}^{\infty}\int_0^l B_{ij}M_k(y,\tau)\frac{\partial M_m(y,\tau)}{\partial y}dy\hat{\hat{\theta}}_{jm}{}^*(\tau)+\\&+\sum_{m=1}^{\infty}\int_0^l \lambda_i{}^2(y,\tau)M_k(y,\tau)M_m(y,\tau)dy\hat{\hat{\theta}}_{jm}{}^*(\tau)=-\frac{1}{2}\sum_{j=1}^{\infty}\sum_{m=1}^{\infty}M_k(l,\tau)M_m(l,\tau)B_{ij}(l,\tau)\hat{\hat{\theta}}_{jm}{}^*(\tau)+\\&+\frac{1}{2}\sum_{j=1}^{\infty}\sum_{m=1}^{\infty}B_{ij}(0,\tau)M_k(0,\tau)M_m(0,\tau)\hat{\hat{\theta}}_{jm}{}^*(\tau)+M_k(l,\tau)f_i(l,\tau)Q_2(\tau)\end{aligned} \quad (28)$$

The transformed restriction equation for the coupling is:

$$\left[1+\left(\frac{\xi}{l}\frac{Q_2}{Q_1}\right)^2\right]\sum_{i=1}^{\infty}\frac{\sqrt{2}}{\sqrt{l}}\frac{\lambda_i^{*}}{\overline{\eta}_b^{\,3/2}}(-1)^i\,\hat{\hat{\theta}}_{i1}^{\,*}(\tau)-\nu\frac{\partial\overline{\eta}_b}{\partial\tau}=Q_1 \tag{29}$$

where:

$$\overline{\eta}_b=\int_0^l 1-\xi\left(1-\frac{Q_2}{Q_1}\frac{y}{l}\right)dy=1-\xi\left(1-\frac{1}{2}\frac{Q_2}{Q_1}\right)$$

With transformed initial condition:

$$\hat{\hat{\theta}}_{ik}^{\,*}(\tau)=\hat{\hat{\varphi}}_{ik}(\tau)-\hat{\hat{\gamma}}_{ik}(\tau) \tag{30}$$

where:

$$\hat{\hat{\varphi}}_{ik}(\tau)=\int_0^l M_k(y,\tau)\tilde{\varphi}_i(y,\tau)dy$$

$$\hat{\hat{\gamma}}_{ik}(\tau)=\int_0^l M_k(y,\tau)\tilde{\gamma}_i(y,\tau)dy$$

Therefore, the Eqs. (28) and (29) form an infinite system of coupled ordinary differential equations that is solution for the studied two-dimensional ablation problem, with the Eq. (30) being the initial condition.

The numerical results for the temperature solution were calculated through a Fortran language program. So, it was necessary to transform this infinite system in a suitable finite system of order N. Making N sufficiently great we obtain a good approximated solution for the infinite problem. The numerical solution was performed using available IMSL subroutines (1979).

5. Results

Results were obtained by analytical/numerical hybrid analysis through of application GITT for an ablation problem modeled by a diffusion equation. In the present study, it is assumed constant heat fluxes $Q_1 = 2$ and $Q_2 = 0$, heat input and heat loss, respectively, where Q_1 and Q_2 are dimensionless heat flux.

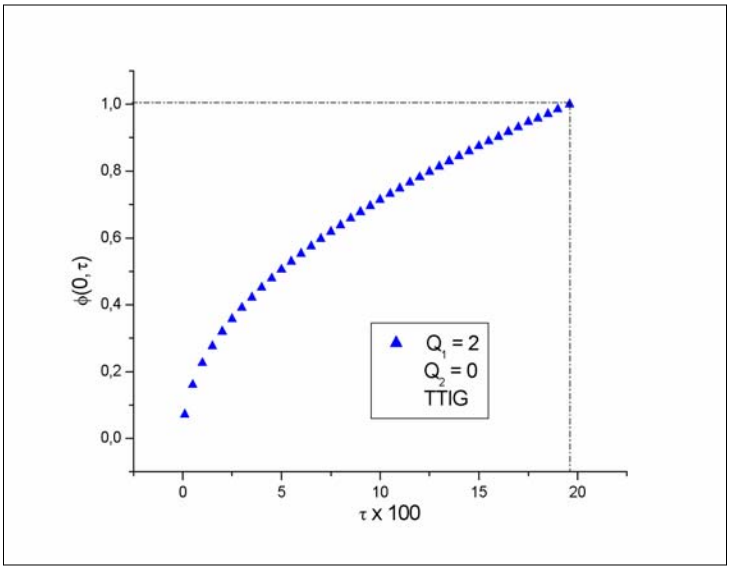


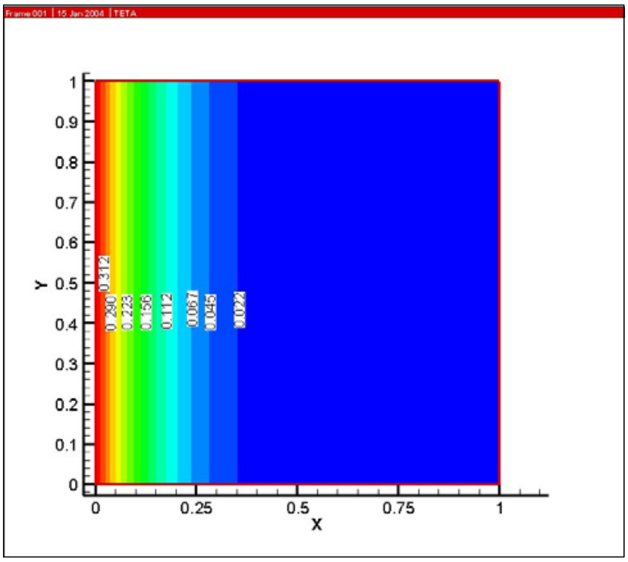
Figure 3. Determination of the time-interval for the beginning of the ablative phase.

The preablative phase solution allows determining the time where the ablative phase begins. It's occur when averaged dimensionless temperature $\phi(x,\tau)$, defined by Eq. (31), in the position $x=0$ is equal to 1:

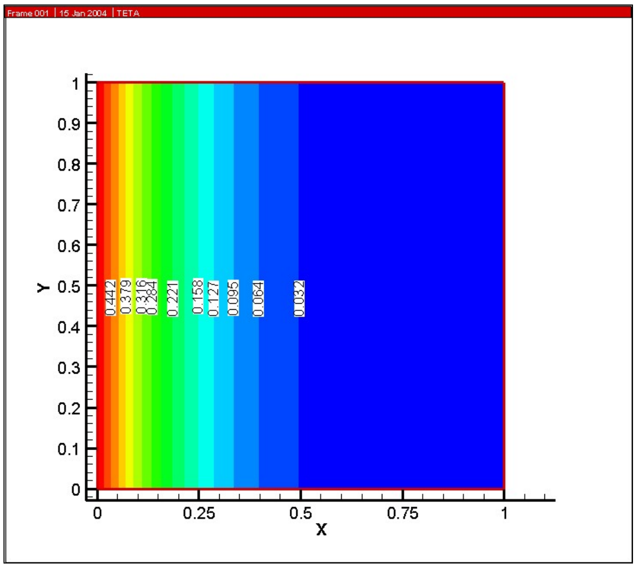
$$\phi(x,\tau) = \int_0^l \theta(x,y,\tau)dy \tag{31}$$

Figure 3 exhibits the results obtained to determine the dimensionless time value for the beginning of the ablative phase, τ_m. The temperature distribution for the preablative phase at τ_m is the initial condition for the ablative phase. Fig. 3 shows that the ablative phase starts at $\tau_m = 0.196$.

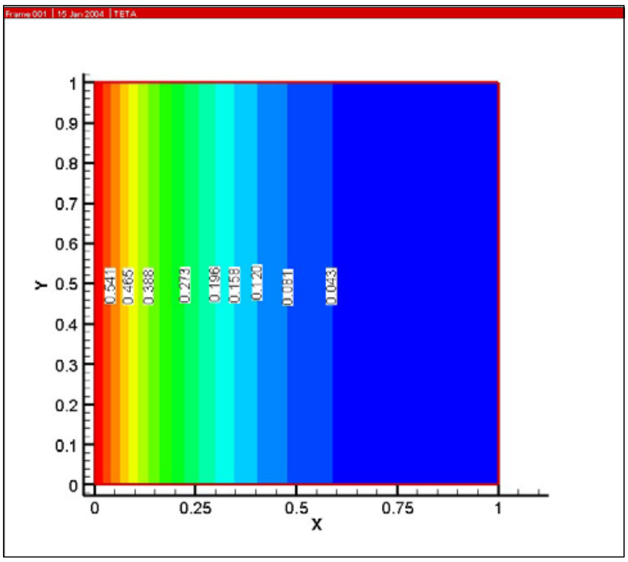
Fig. 4 presents the dimensionless temperature distribution in the solids, showing the temperature profiles for the following dimensionless time values: (a) $\tau = 0.025$, (b) $\tau = 0.05$, (c) $\tau = 0.075$, (d) $\tau = 0.1$, (e) $\tau = 0.125$, (f) $\tau = 0.15$, (g) $\tau = 0.175$ and (h) $\tau_m = \tau = 0.196$.



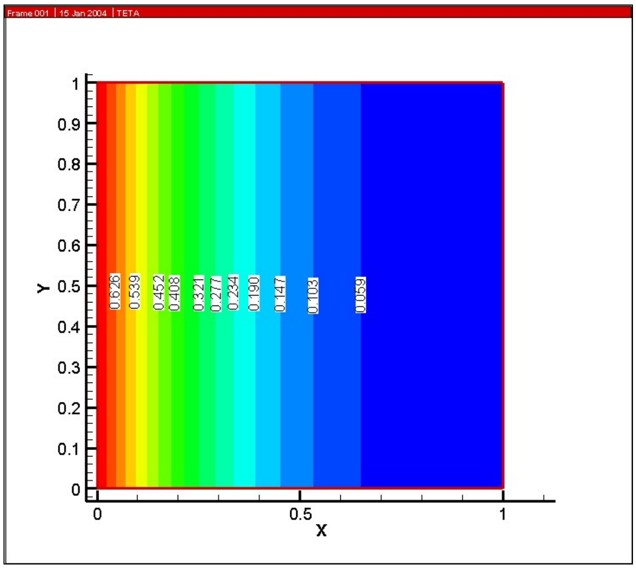
(a)



(b)



(c)

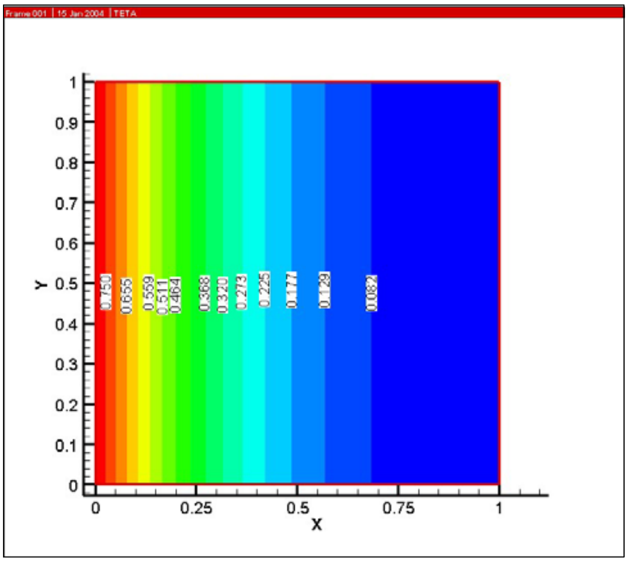


(d)

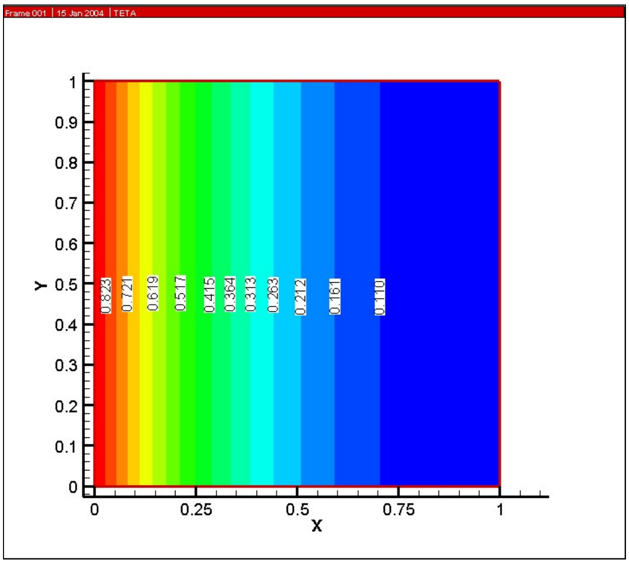
Figure 4. Temperature distribution of the preablative phase.

Note that the case in study is a particular case of the two-dimensional problem, once $Q_2 = 0$ represents a one-dimensional problem. A comparison between the time value for the beginning of the ablative phase obtained by the one-dimensional formulation ($\tau_m = 0.197$) provided by Kurokawa *et al.* (2003), with the present work value ($\tau_m = 0.196$) shows a good agreement, where both results applied the GITT.

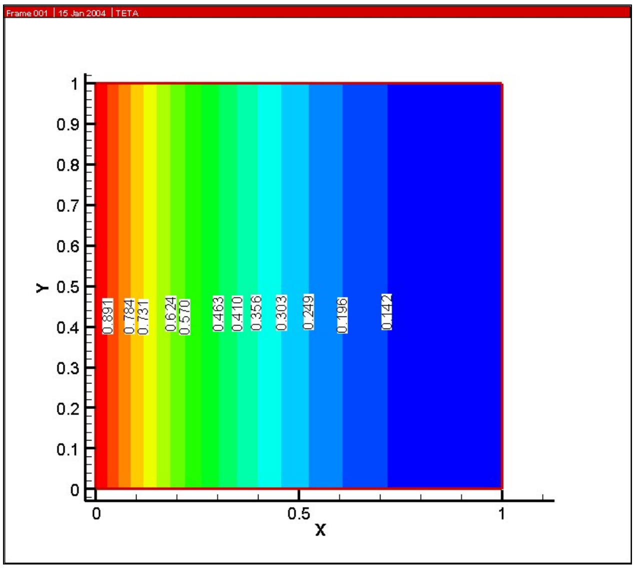
In this work a solution was presented for the heat transfer problem for a two-dimensional phase change ablation in solids. It is noticed that the temperature distribution, Fig. 4, exhibits a stratified profile, that also occurs in the one-dimension case.



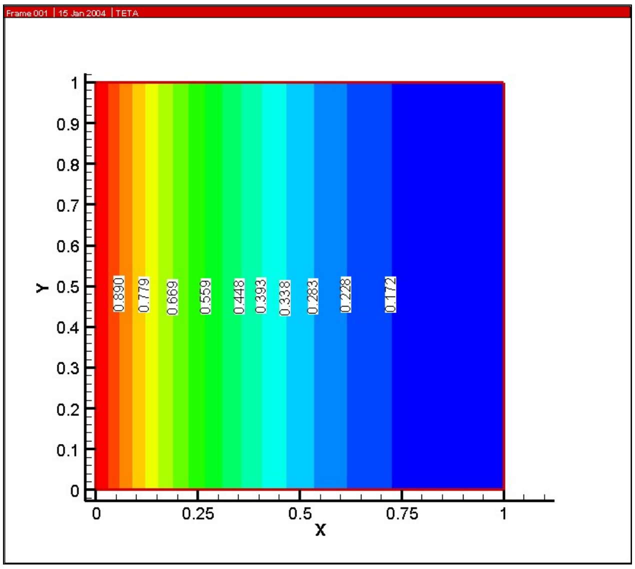
(e)



(f)



(g)



(h)

Figure 4. … temperature distribution of the preablative phase *(continuation)*.

As previously mentioned, a simplification of phenomenon was considering, since the ablation phenomena is treated merely as a phase change process with boundary moving, neglecting other physical effects.

6. Final Coments

The principal objective this work was to validate the results obtained for a two-dimensional ablation problem by the application of the GITT. Once considering the heat loss null, the modeling mathematical of the two-dimension problem transform in a one-dimension ablation problem. Therefore, the GITT should be a useful tool for complex problem solution involving non-linear heat transfer.

7. Acknowledgments

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8. References

Chung, B. T. F., Chang, T. Y., Hsiao, J. S. and Chang, C. I., 1983, "Heat Transfer with Ablation in a Half-Space Subjected to Time-Variant Heat Fluxes", Trans. ASME – Journal of Heat Transfer, vol. 105, pp. 200-203.

Chung, B. T.F. and Hsiao, J. S., 1985, "Heat Transfer with Ablation in a Finite Slab Subjected to Time-Variant Heat Fluxes", AIAA Journal, vol. 23, pp. 145-150.

Cotta, R. M., 1993, "Integral Transforms in Computational Heat and Fluid Flow", CRC Press, Baca Raton, Florida, USA.

Cotta, R. M. and Özisik, M. N., 1987, "Diffusion Problems with General Time-Dependent Coefficients", Mechanical Sciences RBCM, vol. 9, n. 4, pp. 269-292.

Diniz, A. J., 1996, "Proteção Térmica por Ablação em Corpos de Várias Geometrias Aquecidos Cineticamente", Tese (Doutorado em Engenharia Mecânica-Aeronáutica), Instituto Tecnológico de Aeronáutica (ITA), São José dos Campos – SP.

Diniz, A. J., Aparecido, J. B. e Zaparoli, E. L., 1993, "Solução de Problemas Térmicos com Acoplamentos Não Lineares", XII COBEM, Brasília.

Hatori, M. E. e Pessoa-Filho, J. B., 1998, "Soluções Similares em Escoamentos Supersônicos", VII Encontro Nacional de Ciências Térmicas, ENCIT 98, pp. 257-262.

Hsiao, J. S. and Chung, B. T. F., 1984, "A Heat Balance Integral Approach for Two-Dimensional Heat Transfer in Solids with Ablation", AIAA 22nd Aerospace Sciences Meeting, AIAA – 84 – 0394, Reno, Nevada.

IMSL Library, 1979, Edition 7, GNB Building Blvd, Houston, Texas, 77036.

Kreith, F., 1973, "Princípios da Transmissão de Calor", Cap. 12, Ed. Edgard Blucher Ltda.

Kurokawa, F. Y., 2003, "Estudo Híbrido Analítico/Numérico da Equação de Difusão Bidimensional em Sólidos com Proteção Térmica Ablativa", Dissertação (Mestrado em Engenharia Mecânica), Universidade Estadual Paulista (UNESP), Ilha Solteira – SP.

Kurokawa, F. Y., Diniz, A. J. and Campos-Silva, J. B., 2003, "Analytical/Numerical Hybrid Solution for One-Dimensional Ablation Problem", Proceedings of HT 2003, Summer Heat Transfer Conference, ASME, Las Vegas, Nevada.

Lacaze, H., 1967, "La Protection Thermique Par Ablation", Doc-Air-Espace, n. 105.

Pantaleão, A. V., 2003, "Análise Bidimensional de Proteção Térmica por Ablação", Tese (Doutorado em Engenharia Mecânica-Aeronáutica), Instituto Tecnológico de Aeronáutica (ITA), São José dos Campos – SP.

Steg, L. and Lew, H., 1962, "Hypersonic Ablation", In Agard Meeting on High Temperature – Aspects of Hypersonic Fluid Dynamics, v. 68, part 6, pp. 629-680.

Sutton, G. W., 1982, "The Initial Development of Ablation Heat Protection, an Historical Perspective", AIAA 82 4038, Journal Spacecraft, v. 19, n. 1, pp. 3-11.

Zien, T. F., 1978, "Integral Solution of Ablation Problems with Time Dependent Heat Flux", AIAA Journal, v. 16, pp. 1287-1295.

Zien, T. F., 1981, "Approximate Solutions of Transient Heat Conduction in a Finite Slab", Heat Transfer and Thermal Control, Progress in Astronautics and Aeronautics, v. 78, edited by A. Crosbie, AIAA, New York, pp. 229-248.

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