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Transient Convection in Atmospheric Turbulent Boundary Layers: a Comparison between Flow over Smooth and Rough Surfaces

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Abstract. *In this study, transient convective turbulent boundary layers were experimentally simulated in a wind-tunnel environment. The behaviour of the thermal boundary layer subjected to a time-periodical step-wise heat flux imposed on the wall was investigated, and a comparison has been made between its response to a smooth and a rough surface. Using the classical boundary layer equations and the concept of the displacement height, new equations have been derived to account for the behaviour of the thermal transient boundary layer. Special attention has been given to the study of the behaviour of the displacement height and the Stanton number. The validity of the Reynolds analogy has also been analysed. The theoretical predictions are validated by the obtained experimental data and the results indicate that the displacement heights for the velocity and the temperature profiles assume very different values. It was observed that the Reynolds analogy is only approximately satisfied.*

Keyword: *Thermal boundary layer, turbulence, convection, roughness, experimental simulation.*

1. Introduction

Heterogeneity and non-steadiness are ubiquitous in nature. The physical processes involved in atmospheric flows are essentially non-uniform and transient. In fact, the consequences of the idealizations embodied in the assumptions of horizontal homogeneity and stationarity are paramount, since a handful of crucial effects which strongly influences the behaviour of the boundary layer is dismissed as unimportant. The differential heating of the ground by the varying position of the sun, the changes in surface and weather conditions, the earth’s rotation, all these factors are just a small illustration of the numerous effects that assure the actual dependence of the statistical properties of the boundary layer on time and space.

In particular, one of the main phenomena that clearly depicts this non-stationary characteristics of the atmospheric turbulent boundary layer is the process called by meteorologists as diurnal cycle. In response to the heating action of the sun, an unstable boundary layer is formed by the sunrise, as the warming of the ground by the sun rays indirectly heats the air parcels on its vicinity. This state is developed during the day, originating a convective boundary layer. The inverse process occurs as the sunset approaches, since the surface is progressively cooled, evolving into a stable boundary layer along the dawn. Consequently, during the time scale of one day, the atmospheric boundary layer assumes different characteristic states in response to the changing surface heating conditions.

The purpose of this work is to investigate the behaviour of the thermal boundary layer when a transient heating condition is imposed upon the surface. The response of the transient convective boundary layer to both a smooth and a rough surface has been separately studied. The focus of the present study is on assessing the behaviour of the displacement height for the logarithmic temperature profile, as well as on considering the temporal evolution of the Stanton number. Bearing in mind the physical process of the diurnal cycle, an attempt has been made to simulate in a controlled laboratory environment the temporal variations in surface heating and its influence on some of the characteristics of the resulting thermal boundary layer. To accomplish this task, it was used the atmospheric wind tunnel located at the Laboratory of Turbulence Mechanics (PEM/COPPE/UFRJ). A time-periodical step-wise heat flux was imposed upon the wall and the properties of the thermal boundary layer were experimentally determined by the graphical method of Perry and Joubert(1963). Furthermore, using the classical boundary layer equations and the concept of displacement height, new equations have been derived to account for the behaviour of the thermal transient boundary layer.

Herein, only the problem of forced convective heat transfer will be considered, that is, thermal boundary layers without coupling of the velocity field to the temperature field. The reason for doing so is clear, since we specifically want to study the response of the internal surface layer to a time-periodical heat flux imposed on the wall. Thus, keeping the velocity field unchanged, the friction coefficient will remain constant. However, it will probably not be the case for the Stanton number. Consequently, any change in Stanton number will be characterized as a break down in the Reynolds analogy hypothesis. This behaviour will be investigated here for flows over smooth and rough surface.

Despite its limitations, wind-tunnel modeling of the atmospheric flow field is widely recognised as the best alternative to the high-cost field experiments. However, there is only a limited number of experimental studies that deliver information on the behaviour of boundary layers developing over rough surfaces having non-uniform heat flux. In fact, scarcer are the field investigations conducted under these circumstances with a view to study the transient processes occurring in atmospheric flows. Indeed, the direct assessment of parameters such as the friction velocity, the roughness characteristic length and the error in origin for field atmospheric measurements cannot be easily attained. As a consequence, it is a common practice to evaluate them from at least five or six heights from a measured velocity profile, on the basis of the graphical method.

In the next section, we provided the reader with a brief overview of some of the main works which have been developed in the field.

2. A brief literature review

With regard to the surface properties variations, a more popular source of interest and investigation along the past fifty years has been the study of the roughness effects on the characteristics of the boundary layers. In a classical paper, Perry and Joubert(1963) reviewed many rough-wall boundary layer investigations in order to correlate resistance to a characteristic length of the flow, the distance below the top of the roughness elements, also known as the error in origin, **e**. The result was the development of a graphical method that has been used since then to determine the local boundary layer characteristics from each measured velocity distribution. The Perry and Joubert method was further improved and validated on two following publications, Perry et al.(1969) and Perry et al.(1987). In fact, the displacement height in the logarithmic velocity profile, according to Jackson(1981), is a concept which is commonly surrounded by a great deal of confusion. The main contribution of this work was to provide a better physical interpretation to the displacement height, showing that it could be regarded as the level at which the mean drag on the surface appears to act. Jackson also showed that the displacement height coincides with the displacement thickness for the shear stress. It then can be concluded that the error in origin, when considered from the bottom of the roughness elements is referred to as the displacement height, *d*, so that, therefore, a simple geometric relation can be established, $d = K - \varepsilon$, where *K* denotes the height of the roughness elements.

In fact, most of studies found in literature concerning changes in rough surfaces simplify the problem to the extent of considering changes to occur from one extensive uniform surface to another of a different surface. Bradley(1968) carried out a unique set of field measurements in the lower atmosphere, where the wind passed from one surface to another of different roughness. His measurements included both wind profiles and surface shear stress. His magnificent efforts where complemented by the wind-tunnel investigations of Antonia and Luxton(1971, 1972) and of Mulhearn(1978). The authors showed that the structure of the internal layer which grows downstream of step changes in roughness depends strongly on the type of the change, if rough-to-smooth or smooth-to-rough.

In analogy to the surface roughness problem, the surface heat flux is investigated using the spatial step change approach. The behaviour of the thermal boundary layer developing over surfaces having non-uniform heat flux or temperature distribution was investigated by many authors (Blom(1970), Antonia(1977)). However, most of the studies had been concerned to flows over smooth surfaces. Coleman(1976) and Ligrani and Moffat(1985) were the first to consider flow over rough surfaces. As remarked by the authors, the earth’s surface is rarely subjected to constant heat flux or temperature boundary conditions. In these works no particular consideration was given to the displacement height in the logarithmic profile since the temperature law of the wall was written in terms of the equivalent sand grain roughness for the considered surface.

An extension of the concept of displacement height to the temperature boundary layer was advanced by Avelino and Silva Freire (2002) for surfaces of types “K” and “D”, according to the classification established by Perry et al. (1969). The aim of the research was to investigate the behaviour of the temperature displacement height when velocity and temperature boundary layers with different states of development were considered. Under these conditions, it was not clear that a straight Reynolds analogy would work for the calculation of the friction coefficient and of the Stanton number. If that was to be the case, the values for the displacement height for the velocity and the temperature fields would have to have the same order of magnitude. The experiments were such that a flow that developed over a cold, smooth surface was made to pass over a heated, rough surface. As it turned out, surfaces of type “K” presented velocity and temperature displacement heights which were compatible whereas surfaces of type “D” presented displacement heights with very different values.

Although some important aspects of the behaviour of flow and turbulence on rough surfaces were enlightened by these studies, there are still a handful of open questions concerning the appropriate description of the flow field under more realistic conditions, for instance, theories that account for the effects of temporal dependence of the atmospheric flow phenomenon. As observed by Kalinin and Dreitsen (1985), “the problem of unsteady heat transfer is a conjugate one since the mathematical model for the description of heat transfer and hydrodynamics of a coolant is augmented with the equations of heat conduction in the material and with the conjugation conditions at boundaries”. Still, according to those authors, the current lack of experimental data on the turbulent flow structure would prevent us from obtaining a solvable closed system of equations. In our present analysis, this difficulty is overcome with a local analysis in the fully turbulent flow region.

3. Theoretical analysis

In the present section we firstly present a short revision of the classical steady-state problem by deriving the logarithmic law of the wall for the velocity and temperature fields. Secondly, the assumptions used are stated and the governing equations are derived in order to account for the temporal dependence of the thermal boundary layer. The obtained equations are then extended to the case of flow over rough surfaces, using the concept of the displacement height. At the end of the section we provide some comments on the Reynolds analogy and on the atmospheric stability conditions.

3.1 - Steady-state flow and thermal fields

The governing equations for a given incompressible fluid flowing over a smooth, heated surface under steady-state condition can be written as:

Continuity: $$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0, \quad (1)$$

x-Momentum: $$\rho u\frac{\partial u}{\partial x}+\rho w\frac{\partial u}{\partial z}=\mu\frac{\partial^2 u}{\partial z^2}-\rho\frac{\partial \overline{u'w'}}{\partial z}, \quad (2)$$

Energy: $$\rho c_p u\frac{\partial T}{\partial x}+\rho c_p w\frac{\partial T}{\partial z}=\kappa\frac{\partial^2 T}{\partial z^2}-\rho c_p\frac{\partial \overline{t'w'}}{\partial z}, \quad (3)$$

where the classical notation is used and the boundary layer hypothesis do apply.

Appropriate boundary conditions must be chosen so that these equations can be solved. As for the velocity field, the trivial conditions of no slip and impermeability can be used. For the temperature field, on the other hand, a number of different possible boundary conditions can be specified. Basically, the wall temperature, the wall heat flux or a combination of these two can be prescribed. Assuming now that the three-layered structure of the turbulent boundary layer is valid and furthermore considering that in one of these layers the turbulent effects are predominant, the governing equations can be reduced to:

x-Momentum: $$\frac{\partial \overline{u'w'}}{\partial z}=0, \quad (4)$$

Energy: $$\frac{\partial \overline{t'w'}}{\partial z}=0. \quad (5)$$

At this step, in order to solve the above equations, it is crucial to establish a relationship between the mean and turbulent quantities. A fairly simple way of doing this is to invoke the concepts of eddy diffusivities for momentum and heat together with the mixing length hypothesis. We further incorporate into our analysis two extra hypothesis: (i) von Karman's hypothesis that the mixing length can be considered proportional to the wall distance, i.e. $l=\kappa z$ and $l_t=\kappa_t z$, where κ and κ_t are constants; (ii) Prandtl's hypothesis that in the near wall region the total shear stress and the heat flux are constants. Thus, upon a simple integration, it results that in the fully turbulent region the solutions are given by:

$$u=\frac{u_\tau}{\kappa}\ln z+A, \quad (6)$$

$$T_w-T=\frac{t_\tau}{\kappa_t}\ln z+B, \quad (7)$$

where $u_\tau=\sqrt{\tau_w/\rho}$ and $t_\tau=q_w/(\rho\, c_p u_\tau)$.

Consequently, provided κ and κ_t are known, the skin friction coefficient and the heat transfer coefficient can be respective evaluated from the inclination of semi-logarithmic graphs of distance from the wall versus velocity and distance from the wall versus temperature.

One point of particular concern to the authors during the present work was the most appropriate value to be adopted for the constants κ and κ_t . A recently careful consideration of Zanoun et. al.(2003) of previous results on the behaviour of κ and *A* has shown that typical values of κ are observed to lie in the range [0.334, 0.436]. Here we have decided to use $\kappa = 0.4$, a value suggested by Coles(1956) and much preferred by several authors which undisputedly assured the existence of the flow logarithmic region. With respect to κ_t, its determination is obtained from the definition of the turbulent Prandtl number, which is written as: $\mathrm{Pr}_t = \nu_t / \alpha_t = \kappa / \kappa_t$. It is a common sense in literature that the turbulent Prandtl number varies across the boundary layer in a way that depends on both molecular properties of the fluid and the flow field. In the logarithmic region, however, many authors (e.g. Blackwell(1972)) have shown that the turbulent Prandtl number is approximately 0.9 which results in a value of 0.44 for κ_t.

3.2 - Transient thermal boundary layers on smooth surfaces

Let us now focus on the problem of a given incompressible fluid flowing steadily over a smooth surface of prescribed heat flux. Considering this hypothesis, the velocity field remains unaltered and consequently the solution for the fully turbulent region can still be approximated by the logarithmic equation.

On the contrary, the thermal field suffers considerable modifications since the surface boundary conditions have to change to accommodate a time varying imposed heat flux. Thus, the energy governing equation reduces to

$$\frac{\partial T}{\partial t} = -\frac{\partial \overline{t'w'}}{\partial z}. \tag{8}$$

Considering the relations between the mean and turbulent quantities presented in the previous section, the above equation can be rewritten as:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(u_t \kappa_t z \frac{\partial T}{\partial z} \right). \tag{9}$$

In order to find a solution, let us now take into account the temporal and spatial dependence of the temperature field:

$$T(z,t) = F(t)G(z). \tag{10}$$

Then, upon substituting Eq. (10) in Eq (9) it follows that

$$\frac{F'(t)}{F(t)} = u_t \kappa_t \left[\frac{G'(z) + zG''(z)}{G(z)} \right]. \tag{11}$$

So that a solution is sough from equations

$$\frac{F'(t)}{F(t)} = -\sigma, \tag{12}$$

$$G'(z) + zG''(z) + \frac{\sigma}{u_\tau \kappa_t} G(z) = 0, \tag{13}$$

where the sign of was **s** chosen so as to ensure that the temperature will decay in time. The solution of Eq. (12) is

$$F(t) = Je^{-\sigma t}. \tag{14}$$

To solve Eq. (13) consider the decaying time to be long enough so that $\varepsilon = (\sigma / \kappa u_\tau)$ can be considered a small parameter. Then we will search for a solution of the form

$$G(z) = G_0(z) + \varepsilon G_1(z). \tag{15}$$

The substitution of Eq. (15) onto Eq. (13) and the collection of the terms of the same order yields:

$$G_0'(z) + zG_0''(z) = 0, \tag{16}$$

$$G_1'(z) + zG_1''(z) + G_0(z) = 0, \tag{17}$$

whose solutions are:

$$G_0(z) = C \ln z + D\,, \tag{18}$$

$$G_1(z) = E \ln z + Rz \ln z + Sz + Q\,, \tag{19}$$

with $R = C$ and $2C + D - S = 0$. Thus, the fully turbulent approximate solution is given by

$$T(z,t) = Je^{-\sigma t}\left[(C \ln z + D) + (\sigma/(\kappa_t u_\tau))(E \ln z + Rz \ln z + Sz + Q)\right], \tag{20}$$

where all constants must be determined experimentally.

3.3 Transient thermal boundary layers on rough surfaces

It is known that for flows over K and D-type surface roughness, the viscous region is completely destroyed by the protuberances of the wall. Under this condition, the fully turbulent region just described above has to suffer some adjustments so as to yield a good description of the velocity and temperature fields. A universal expression can be written for the wall region provided that the origin of the velocity profile is set at some distance below the crests of the roughness elements. This displacement in origin, as said before, is generally referred to in literature as the error in origin, **e**. Then, for any kind of surface, it is possible to write:

$$\frac{u}{u_\tau} = \frac{1}{\kappa} \ln\left[\frac{(z_T + \varepsilon)u_\tau}{\nu}\right] + A - \frac{\Delta u}{u_\tau}\,, \tag{21}$$

$$\frac{\Delta u}{u_\tau} = \frac{1}{\kappa} \ln\left[\frac{\varepsilon\, u_\tau}{\nu}\right] + C_i\,, \tag{22}$$

where $\kappa = 0.44$, $A = 5.0$, C_i, $i = K, D$, is a characteristic parameter of the roughness.

Although of a universal character, Eqs. (21) and (22) have the inconvenience of needing two unknown parameters for their definition, the skin friction velocity and the error in origin, For an experimentalist, however, these equations are extremely useful since they provide a graphical method for the determination of the skin friction coefficient.

In order to extend the transient thermal description for smooth surfaces, Eq. (20), to flow over rough surfaces, we will draw a direct analogy with Eq. (21). In addition, the similarity for turbulent flows suggests that:

$$T(z,t) = Je^{-\sigma t}\left[(C \ln z^+ + D) + (\sigma/(\kappa_t u_\tau))(E \ln z^+ + Rz^+ \ln z^+ + Sz^+ + Q)\right], \tag{23}$$

where $z^+ = (z_T + \varepsilon_t)u_\tau / \nu$ and the parameters to be determined may now be a function of the roughness.

In principle, the error in origin for the temperature field is expected to be time-dependent. Indeed, Eq. (23) provides a good means to measure the heat flux at the wall. Since we can evaluate the error in origin through one of the classical techniques, the slope of the temperature profile plotted in a semi-log graph will furnish the friction temperature and thus the heat transfer coefficient.

3.4 - The Reynolds analogy and atmospheric stability considerations

One of the main objectives of the present study is to analyse the validity of the Reynolds analogy hypothesis, $1/2C_f = S_t$, for thermal boundary layers subjected to transient heat flux.

Intuitively, a break down in the Reynolds analogy could be an expected result. On the one hand, the velocity field is made to be constant during the whole set of experiments so that its statistical properties remain unchanged. Thus, at any vertical profile, for al the time instants, the wall shear stress has a constant value. On the other hand, the heat flux at the wall changes with time, and so do the friction temperature and the wall temperature. Thus, if the Stanton number was to remain constant, these effects would have to compensate each other. Moreover, the displacement heights for the velocity and the temperature fields would have to be necessarily different. In the beginning of the heating cycle the displacement height for the temperature profile would be zero, whereas, the displacement height for the velocity profile would have a finite value. As time would progress, the displacement height for the temperature profile would reach a steady, finite value, returning to zero in the end of the cycle. Under these circumstances, it is not clear how the Reynolds analogy (Schlichting (1979)) between heat transfer and skin friction would behave.

In fact, for an isothermal flow, the definition of Stanton number,

$$S_t = \frac{u}{U_\infty} \frac{t_\tau}{T_w - T_\infty}, \tag{24}$$

results in a mathematical indetermination. However, as soon as heat is applied to the wall, T_w increases above T_∞ resulting in a non-zero value that makes Eq.(24) determined. Considering the definition of the Reynolds analogy, it implies that $u_\tau / U_\infty = t_\tau / T_w - T_\infty$. Thus, if Stanton number is to be kept constant, t_τ and $T_w - T_\infty$ must keep a constant relation during all the heating and cooling cycles. The implication of a break down in the Reynolds analogy is that the wall heat flux will have to be evaluated directly from the measured mean temperature gradient in the fully turbulent logarithmic region of the flow.

Concerning the atmospheric stability conditions, a word seems now in order. The wind-tunnel simulation of boundary layers with temperature gradients of up to 25ºC in addition to the assumption of the uncoupling of the flow field and the thermal field could raise some questions about the actual effects of buoyancy forces on the velocity profile. In order to clarify these questions, the simulated flow stability conditions were investigated through the estimation of the bulk Richardson number in the logarithmic region. It is defined by:

$$Ri_b = \frac{gH}{\overline{T}} \frac{\left(T_H - T_w\right)}{U_H^2}, \tag{25}$$

where the subscripts *H* and *w* denote respectively quantities to be evaluated at height *H* (= 36mm) and at the wall; $\overline{T}$ denotes the average temperature of the evaluated profile. This non-dimensional number is a parameter widely used by meteorologists to classify the levels of stability of the atmosphere. The results obtained are shown in the next section.

4. Experimental results

The experimental investigation was conducted at the Laboratory of Turbulence Mechanics, located at the Mechanical Engineering Department of COPPE/UFRJ. The atmospheric wind-tunnel, extensively described in Cataldi et. al.(2001), was used to simulate the transient thermal boundary layer.

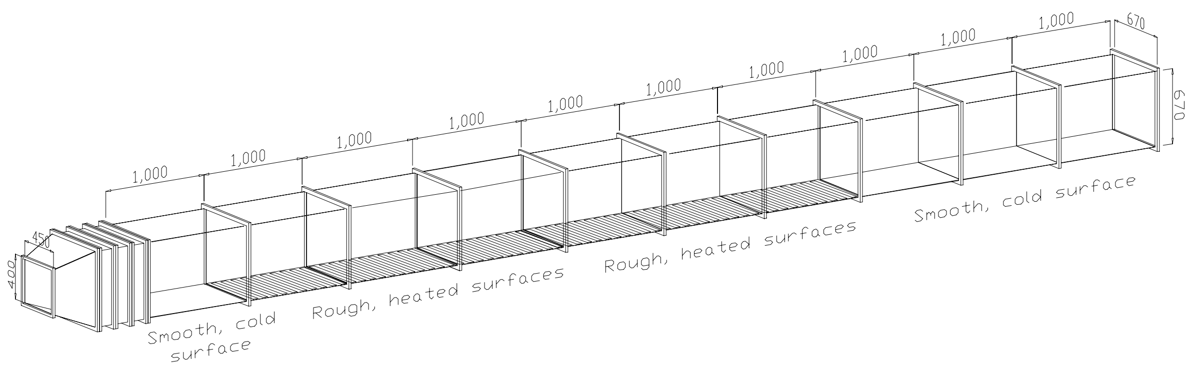

Figure 1: General view of the wind tunnel.

In order to validate the theory introduced above, the velocity and the temperature profiles were measured at one specific station, namely 6500mm downstream of the settling chamber. The experiments were conducted in accordance to the following procedure: the velocity field was evaluated and set to remain constant at a predetermined value. Then, for over 10,000 seconds a constant heat flux was applied at the wall. After this time, and for an extra 10,000 seconds, the heating was turned off. This cycle was repeated at least twice for every wind tunnel run. The laboratory controlled environment allowed the ambient temperature to be maintained within ≤ 0.5 ºC.

Figure (1) presents a general view of the wind tunnel. The heating system is consisted by a series of six independent one-meter long panels, and each one has a heating capacity of about 0.75 kW/m^2. The whole facility is capable of developing gradients of up 65 ^{o}C at uniform mean speeds in the range 1.5-3.5 m/s. The velocity measurements were made with the aid of hot-wire anemometry. A special probe support, illustrated in Figure (2), was designed in order to precisely distribute a series of eight micro-thermocouples along the thermal logarithm region of the simulated atmospheric boundary layer.

The simulated rough surface consisted of equally spaced transversal rectangular slats made of aluminium. Figure (3) illustrates the distribution and the dimensions of the roughness elements, where K denotes the height, S the length, W the gap and λ the pitch of the protuberances.

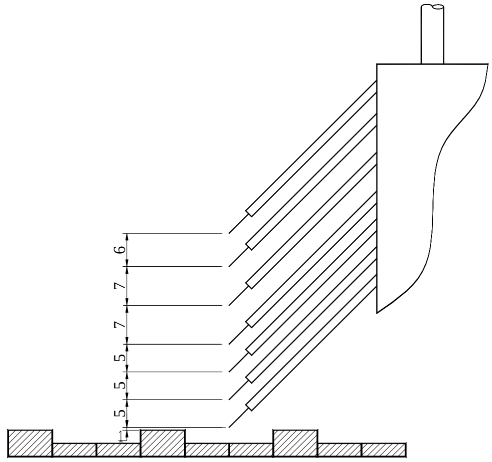

Figure 2: Illustration of the probe support designed to hold the thermocouples.

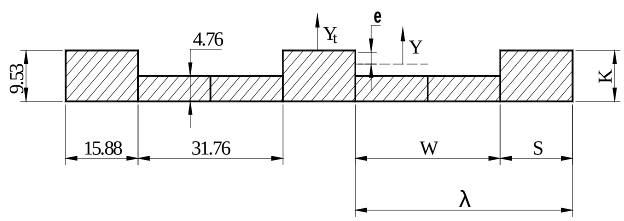

Figure 3: Description of the roughness elements. Dimension in milimetres.

Figure (4) presents the longitudinal mean velocity and the turbulent intensities profiles for the cases of flow over a rough surface and flow over a smooth surface. The simulated flow conditions were classified through global and local parameters, which are shown in Table (1).

Table 1 – Boundary layer parameters.

Surface	U (m/s)	u_τ (m/s)	δ^* (mm)	θ (mm)	G	q_w (kW/m^2)
Smooth	3.0	0.125	13.25	9.94	6.12	0.75
Rough	3.0	0.161	28.79	19.15	8.53	0.75

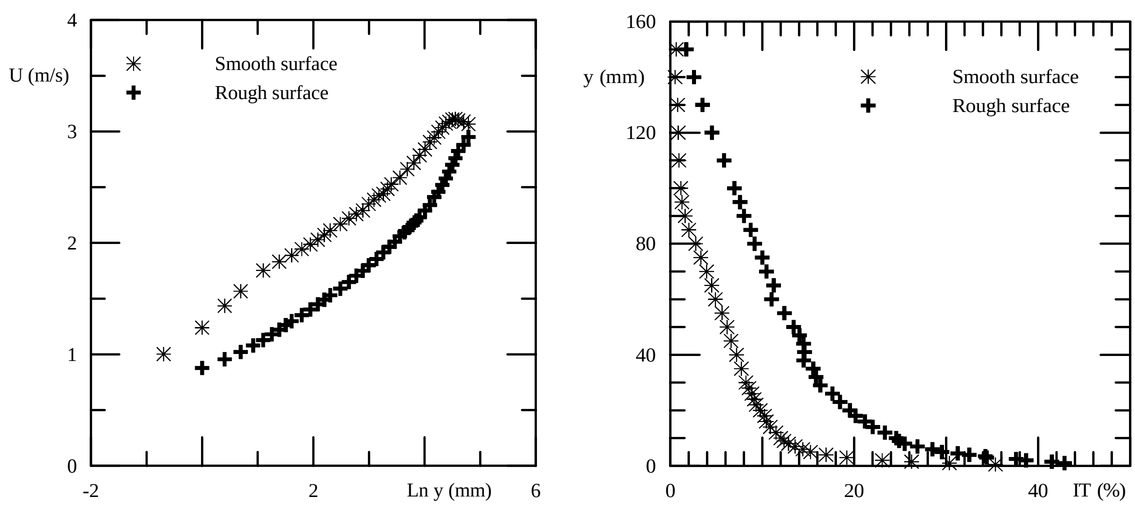

Figure 4: Mean velocity profile and longitudinal turbulent intensities. Comparison between smooth and rough surfaces.

Prior to the evaluation of the friction velocity for the boundary layer over rough surface, it is necessary to determine first the velocity error in origin, ε. Using the graphical method of Perry and Joubert(1963), the velocity profiles were plotted in semi-log graphs in dimensional coordinates. Then, the normal distance from the top of the roughness elements were incremented in steps of 0.1mm and a straight fit was applied to the logarithmic region. The most appropriate curve was chosen by searching for the maximum coefficient of determination, R-squared. Others statistical parameters were also considered in this evaluation process. The determination of the error in origin for the velocity boundary layer is illustrated in Figure (5).

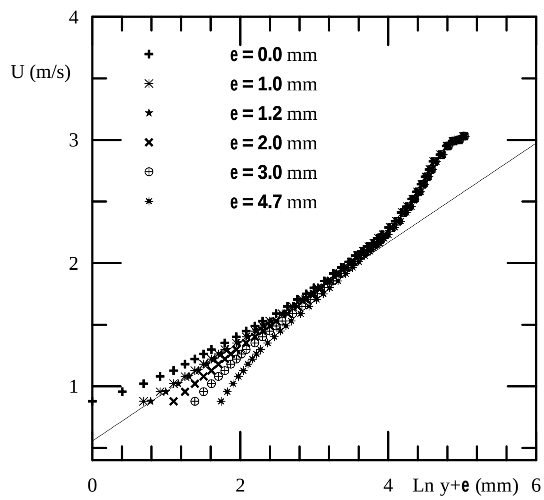

Figure 5: Illustration of the graphical method for determination of the error in origin.

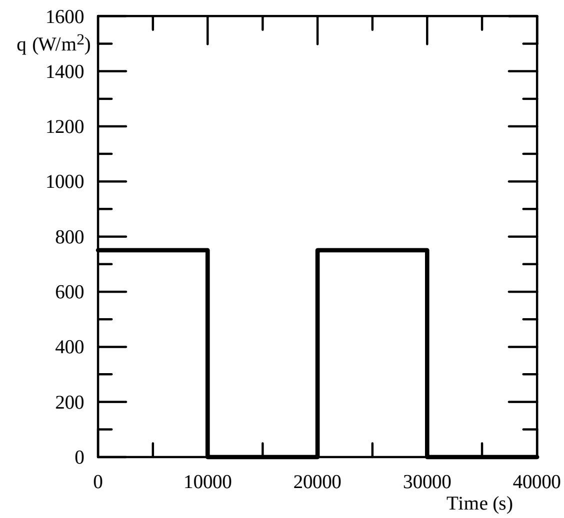

Figure 6: Temporal evolution of the step-wise heat flux imposed on the wall.

Now turning to the thermal problem, the temporal evolution of the heat flux imposed on the wall is shown in Figure (6). The resulting time-dependent behaviour of the thermal boundary layer is presented in Figure (7) for both smooth and rough surface.

For the transient thermal boundary layer over rough surface, the error in origin was also determined through the graphical method, described above for the velocity profile. Consequently, the gradient of the temperature log-law was used to determine the friction temperature, and, further, the wall heat flux. It was concluded that, from the time instants considered in this work, the temperature error in origin always assumed the same constant value, 4.7mm. Two explanations are now possible: first, that the transient response of the temperature error in origin is very short, so that it quickly adjusts to its maximum possible value, the height of the protuberances. The second possibility is that the error in origin is a parameter which do not has a temporal dependence for the thermal case. In fact, this is an interesting point that deserves deeper investigations, so that a final and solid conclusion can be reached. The determination of the thermal error in origin is presented in Figure (8).

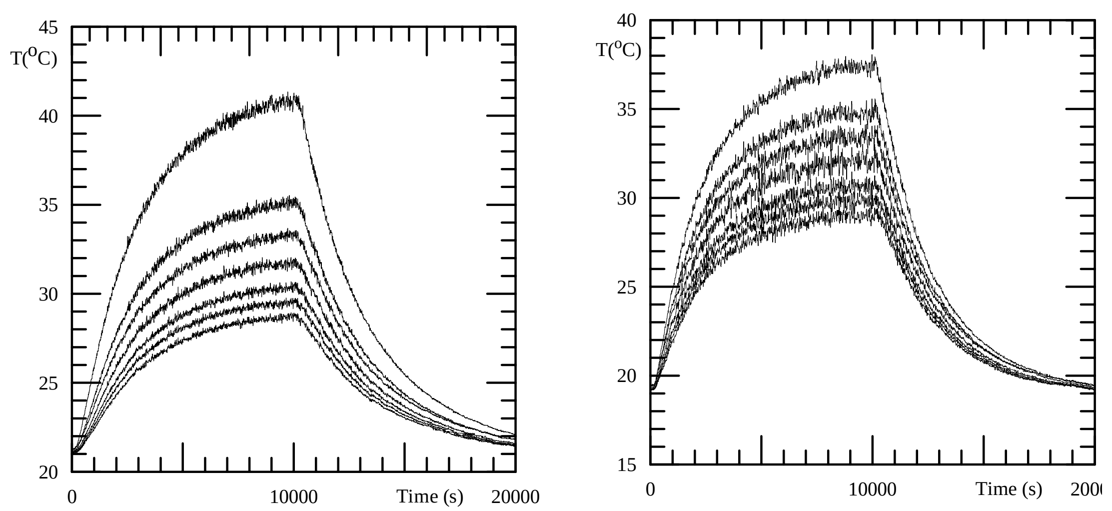

Figure 7: Time-dependent behaviour of the thermal boundary layer: smooth surface (left) and rough surface (right). Curves from top to bottom: y = 0, 5, 10, 15, 22, 29, 35mm. y denotes the distance from top of roughness elements.

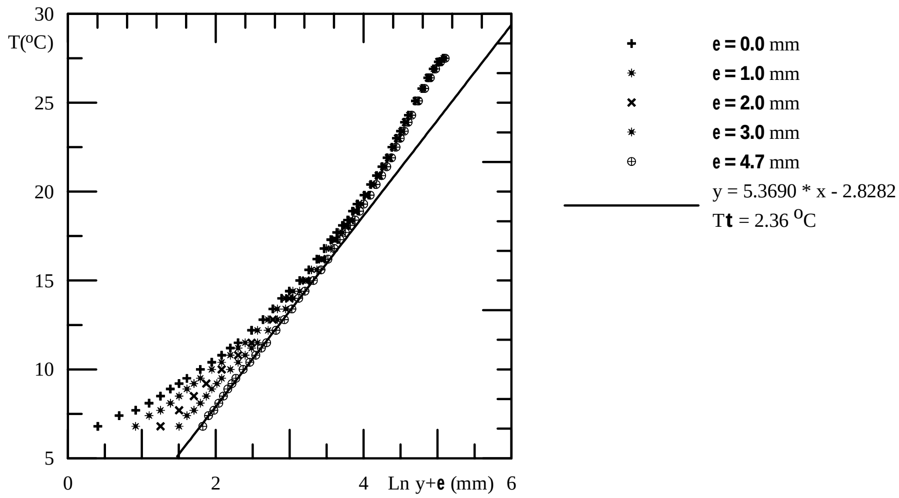

Figure 8: Graphical method for determination of the error in origin for the thermal boundary layer.

The behaviour of the friction temperature is illustrated in Figure (9). The friction temperature was evaluated directly from the inclination of the temperature law of the wall, for the smooth case, and for the rough case, it was evaluated from the law of the wall corrected by the error in origin, as explained before.

During the heating cycle, the following equation was used for the data reduction:

$$T_w - T = \frac{I - Je^{-\sigma t}}{\kappa_t} \ln(z + \varepsilon_t) + B' . \qquad (26)$$

In the cooling cycle, $(I - Je^{-\sigma t})$ was replaced by $(Je^{-\sigma t})$. A best-curve fitting of the data presented in Figure (9) is shown in Table (2).

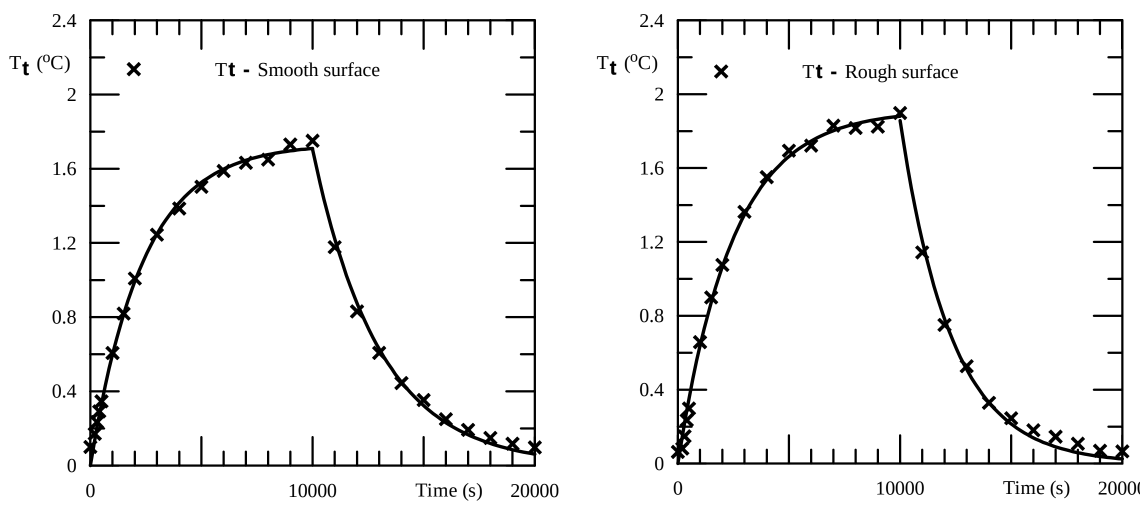

Figure 9: Temporal behaviour of the friction temperature: smooth surface (left) and rough surface (right).

Table 2 – Characterizing parameters for the behaviour of t_t.

Surface	Cycle	s	*I*	*J*
Smooth	Heating	-0.00042 ≤ 0.00002	1.73422 ≤ 0.02122	1.73422 ≤ 0.02122
Smooth	Cooling	-0.00033 ≤ 0.00001	-	47.46499 ≤ 5.42583
Rough	Heating	-0.00041 ≤ 0.00002	1.91345 ≤ 0.02645	1.91345 ≤ 0.02645
Rough	Cooling	-0.00043 ≤ 0.00002	-	132.29752 ≤ 24.06960

Now we turn to the discussion concerning the Stanton number. In order to investigate the validity of the Reynolds analogy hypothesis, remembering the discussion carried out in section (3.4), it was done a best-fit to the time-varying wall temperature profiles shown in Figure (7), according to the curve:

$$T_w(t) - T_\infty = M - Ne^{-\Sigma t}\,. \tag{27}$$

The results are presented in Table (3).

Table 3 – Best-curve fitting parameters for the behaviour of (T_w-T¶).

Surface	Cycle	S	*M*	*N*
Smooth	Heating	-0.00041 ≤ 0.00001	47.07727 ≤ 0.38668	47.07227 ≤ 0.38668
Smooth	Cooling	-0.00036 ≤ 0.00001	-	1649.2136 ≤ 166.7986
Rough	Heating	-0.00042 ≤ 0.00002	42.1186 ≤ 0.72038	42.11186 ≤ 0.72038
Rough	Cooling	-0.00042 ≤ 0.00001	-	2700.56288 ≤ 209.5625

The behaviour of the Stanton number can now be evaluated for the limit cases of time tending to zero and to infinity by substituting the fitted curves into Eq. (24), that is,

$$S_t(0) = \lim_{t\to 0}\frac{u_\tau}{U_\infty}\frac{J\left(1-e^{-\sigma t}\right)}{N\left(1-e^{-\Sigma t}\right)} = \frac{u_\tau}{U_\infty}\frac{J\sigma}{N\Sigma}\,, \qquad S_t(\infty) = \lim_{t\to\infty}\frac{u_\tau}{U_\infty}\frac{Je^{-\sigma t}}{Ne^{-\Sigma t}} = \frac{u_\tau}{U_\infty}\frac{J}{N}e^{(-\sigma+\Sigma)t}\,. \tag{28}$$

Therefore, if the Reynolds analogy is to be satisfied at all the time, the following relations must hold for both the heating and the cooling cycles.:

$$\frac{1}{2}\frac{u_\tau}{U_\infty} = \frac{J}{N}\,, \quad \sigma = \Sigma\,, \tag{29}$$

The temporal behaviour of the Stanton number for the cases of smooth and rough surface are compared in Fig (10). It can be observed that for most of the heating cycle S_t remains constant but assumes values 12% below *(1/2)*C_f. In the cooling cycle, however, S_t seems to increase linearly. The conclusion is that t_t goes to zero at a slower rate than (T_w-T¶). Hence, an apparent result from Figure (10) is that the Reynolds analogy is only approximately satisfied for the present problem.

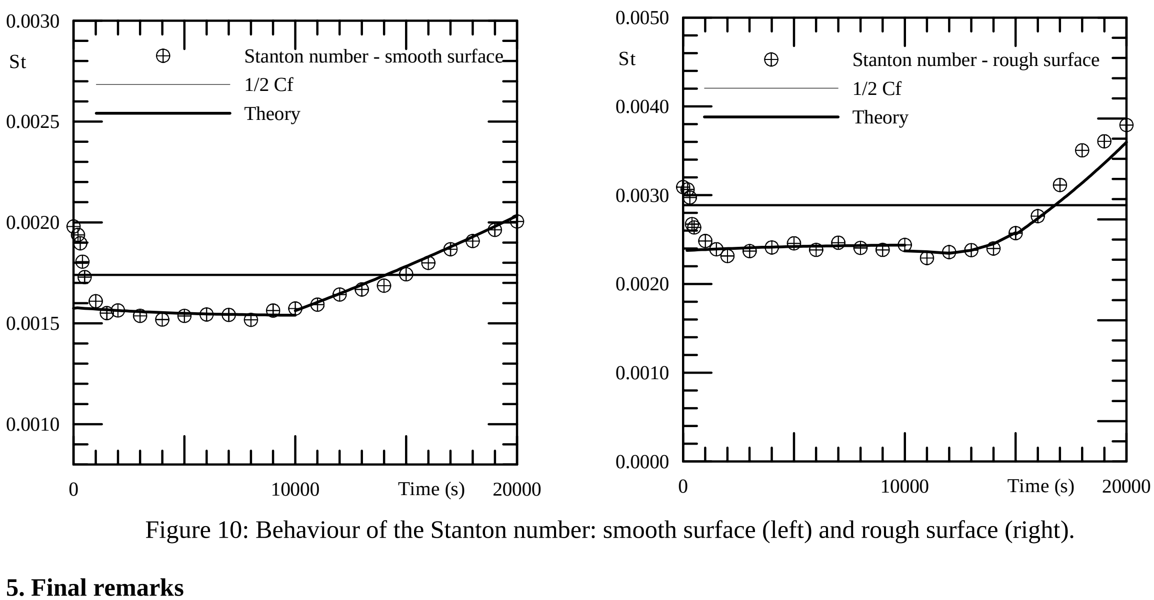

Figure 10: Behaviour of the Stanton number: smooth surface (left) and rough surface (right).

5. Final remarks

The present work has studied the behaviour of the thermal boundary layer subjected to a time-periodical step-wise heat flux imposed on the wall was investigated, and a comparison has been made between its response to a smooth and a rough surface. In particular, we have noticed that the temperature displacement height reaches a constant value in a relatively very short time. This constant value is observed to be very different from the velocity displacement height, taking on the height of the roughness protuberances. The consequence is that the wall heat flux can be evaluated directly from the inclination of a corrected temperature profile in the fully turbulent region of the flow through Eq. (26).

Indeed, the simple theory developed here was validated by the obtained experimental data. This theory provides indication that under transient condition a logarithmic region can be identified with a constant temperature displacement height but with a time dependent inclination.

As far as the Reynolds analogy is concerned, the present indication is that it holds in the transient regime. The results of Table (2) and (3) imply that the variations in friction temperature and in wall temperature do present the same decaying rate and when divided by each other produce a constant value not much different from half of the non-dimensional friction velocity. The present results furnish further indication that departures from the Reynolds analogy are of the order of 10%. This could be due to our choices for the constants κ and κ_t. A 10% disagreement in the Reynolds analogy expression is within the expected error provided by the Perry and Joubert (1963) graphical method so that it appears that for the present experimental conditions the Reynolds analogy is satisfied.

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