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FLOW STABILITY AND TRANSITION

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Abstract. *The topic is self-explanatory and is a subject that remains of paramount importance in the study of fluid mechanics. This work will not offer any general solutions but is rather directed to an understanding and appreciation. For this purpose the salient history will be reviewed, the impact of the computer will be assessed, and the emergence of modern trends and usages of the theory, such as a means for flow control, will be given. Both the physics and the mathematics will be stressed and suggestions made for future exploration. And, because of modern understanding and significant results, the general problem has, in essence, been reformulated. Thus, not only will the possibility of flow control be shown to be possible but many other aspects for understanding the dynamics of shear flows will be found. Although there are two other essential prototypes in the general theory of stability theory (flows with curved streamlines and flows where there is no mean flow), emphasis in this presentation is on those shear flows that exist in the presence of one or no solid boundaries.*

1. Introduction and Historical Review

In many respects, the history of flow stability can be traced as far back as the fifteenth century when Leonardo da Vinci depicted the wake vortices (as we term them today). But, of course, da Vinci's talent is well known in many and quite diverse fields. The stability of fluid flow is one of the central fields in fluid mechanics. Practically all flows exhibit instabilities of various forms. A mixing layer will roll up into large eddies, a vortex street develops behind bluff bodies, convection rolls develop so as to maximize heat flux between two plates, vortical structures appear near concave walls, and smoke rings- even in still air - ultimately develop corrigations and then disperse once the scales become small enough. In general such instabilities are more often than not the precursor of a transition process where the flow evolves into a more complex form of fluid motion and eventually into turbulence. Laminar flow of any kind is the exception to fluid motion. Even without the prompting of the da Vinci picture, attempts to understand the process of transition have more than occupied research in this area for many decades. It remains elusive today.

As expressed by Betchov & Criminale (1967), stability can be defined as the ability of a dynamical system to be immune to small disturbances. Of course, disturbances need not necessarily be small but it is the result that the disturbances become amplified and a departure from any equilibrium is implicit. Helmholtz (1868) and Hagen (1855) were among the first to make contributiions to this field. Then, Reynolds (1883), Kelvin (1880,1887a,b) and Rayleigh (1879, 1880, 1887, 1892a,b,c, 1895, 1911, 1913, 1914, 1915, 1916a,b,) were extremely active. Other familiar names followed in due course. Orr (1907a,b), Sommerfeld (1908), Tollmien (1929), Heisenberg (1924), Prandtl (1921-1926, 1929, 1935), Taylor (1923), Schlichting (1932, 1933, 1934, 1935) and Lin (1944, 1945) are scientists who are well known for their contributions. And, yes, there are other names that have not been cited. However, save for the work of Taylor, the collective list had one major theme, namely incompressible shear flows that were parallel or almost parallel. Questions related to compressibility, magneto-hydrodynamics, gravitational or convective effects are also not included in in this listing. Moreover, in each case there is another important common thread, namely any solution that was obtained was done by analytical approximation for a linear system. The major objective was to be able to predict the breakdown of the particular laminar flow and the transition to turbulence. As such, it was not successful and this objective remains unfulfilled today.

The computer became a tool for solving the respective governing equations with the work of Thomas (1953), Brown (1959, 1961, 1962, 1965), Mack (1960, 1965), and Kaplan (1964). Nonlinear theory for these types of flow has not had as successful progress save, that is, for some ad hoc contributions. But, direct numerical computation for the full governing Navier-Stokes equations did follow and was pioneered by Fromm & Harlow (1963) with the wake pictures of da Vinci more than duplicated! And, of course the computer per se has progressed significantly since this time. But, to give a rough over view, the primitive computers of the 1950s required up to two weeks of computer time for solution or, if done by hand with a mechanical desk calculator, years of time (some estimates are 100 years!) would, by comparison, be needed. Results today can be available in a matter of minutes. But, alas, even with the computer, transition has yet to be predicted.

On the other hand, experiments in this field were definitive. For the prime case of flow of the laminar boundary layer on a flat plate, Schubauer & Skramstad (1943) confirmed the essential features of the linear theory but, at the same time, found behavior not heretofore known. For example, disturbances evolved in the form of packets and, once nonlinearity was present, break down ensued but the flow again became laminar. Eventually complete breakdown did take place and the boundary layer became fully turbulent. Unlike the classical prototype of shear flow that is bounded from both above and below, such as the flow between two plates, the boundary layer has only one solid boundary in its description. This distinction will play a significant role in what is to come. At the same time, the cause of the instability in such flows has the same basis. This is known as a resistive type of instability with viscosity being the underlying cause. In other terms, this means there is a critical phasing between correlations of the perturbation velocity components that is needed in order to have the transfer of energy from the mean to the perturbation field. Contrary to this, even though it would seem to be of the same class of flow, viscous pipe flow has no solution for the perturbation problem. Modern computing has yet to resolve this enigma, regardless of additional input. But, surely everyone knows of the Reynolds pipe experiment and the origin of the Reynolds Number!

There is yet a stronger common element that is used in the linear perturbation analysis of viscous incompressible shear flows, namely solutions for the stability equations are obtained by use of discrete normal modes and hence the determination for stability became the calculations from a dispersion relation. Until recent times, if one positive eigenvalue could be found, then the flow was said to be unstable. Typical results are usually presented with the classical "hairpin" curve with the scalar wave plotted as a function of the Reynolds number. In spite of the fact that the problem is formulated as an initial-value, boundary-value problem, no consideration whatsoever was given to any particular initial-value specification. Had this step been taken at the outset, then more insight would have been earlier obtained.

As suggested, neither analytical nor computational means of modern day caliber were available for the early pioneers in this field. Even with linearity, the problem is complex. The major needs for the problem will be discussed but an excellent appreciation of the evolution of stability theory can be well reviewed with details provided by the major treatises that have been written over the years. Every aspect of the theory will not be given here but, for reference, works that do deal with all the nuances of stability of shear flows, see (1) Lin (1955); (2) Betchov & Criminale (1967); (3) Drazin & Reid (1984); (4) Schmid & Henningson (2001); (5) Criminale, Jackson & Joslin (2003). Each of these citations is a monograph devoted specifically to the subject of hydrodynamic stability and each has an extensive list of references that cover all aspects of the subject as of the time of the writing.

2. General Formulation

In a very general manner, it is linear perturbation analysis that is used for determining the stability of a mean flow that is parallel or almost parallel. For this problem the most general mean flow is defined as $\mathbf{U} = (U(y), 0, W(y))$ for the velocity and P is the mean pressure. This flow is then perturbed in a full three-dimensional manner and the instantaneous flow representation becomes $\mathbf{u} = (U(y) + u, v, W(y) + w)$; $P + p$. Substitution of these variables into the governing equations and, with the mean values subtracted from the balance, the governing linearized system is obtained. The choice of the axes is Cartesian and the mean flow is in the x-z plane and varies in the y-direction. In this way, the linearized equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

for incompressibility and,

$$\begin{aligned}
\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} + W\frac{\partial u}{\partial z} + U'v &= -\frac{\partial p}{\partial x} + \epsilon\nabla^2 u \\
\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + W\frac{\partial v}{\partial z} &= -\frac{\partial p}{\partial y} + \epsilon\nabla^2 v \\
\frac{\partial w}{\partial t} + U\frac{\partial w}{\partial x} + W\frac{\partial w}{\partial z} + W'v &= -\frac{\partial p}{\partial z} + \epsilon\nabla^2 w
\end{aligned}$$

for momenta where the primes on both U and W denote derivatives with respect to y. All quantities are nondimensional with respect to mean scales and are defined with respect to the mean density; ∇^2 is the three-dimensional Laplace operator, and $\epsilon = \Re^{-1}$, with $\Re$, the Reynolds Number. This formulation is a temporal initial-value, spatial boundary-value problem. Normally, a means for solution and the determination as to whether or not a particular flow is stable might be thought to be straight forward. However, there are many difficulties in the actual performance of this task-even though the problem is well posed. There are four equations with four unknowns and the conditions are clear.

The traditional method for solution involved the reduction of the number of equations and a Fourier decomposition in terms of the spatial variables x and z. This is possible because, in these terms, these variables define the infinite x-z plane and all coefficients are independent of x and z. In addition, the mean flow is assumed parallel. Then, the critical time behavior was expressed in terms of normal modes. After this steps was taken, the final result is a system of ordinary differential equations in terms of the final spatial variable y. Specificially, the two equations are

$$(\alpha U + \gamma W - \omega)\nabla^2 \hat{v} - \alpha U'' \hat{v} - \gamma W'' \hat{v} = -i\epsilon \nabla^2 \nabla^2 \hat{v}$$

and

$$(\alpha U + \gamma W - \omega)\hat{\omega}_y + i\epsilon \nabla^2 \hat{\omega}_y = -\gamma U' \hat{v} + \alpha W' \hat{v}$$

where all dependent variables are proportional to $e^{i\alpha x + i\gamma z} e^{-i\omega t}$ with $\hat{v}$ and $\hat{\omega}_y$ the Fourier amplitudes. Of course, this representation implies an infinite number of such modes in order to be the complete solution.

The first equation, when the W component of the mean velocity is omitted, is the Orr-Sommerfeld equation, so called even though there was no known collaboration between these two workers. It is fortuitous that it is fourth order and homogeneous in terms of the velocity component $\hat{v}$. then, when written in terms of the perturbation $\hat{w}$ component of the velocity in lieu of the vorticity $\hat{\omega}_y$ as the dependent variable, the other equation has become known as the Squire equation but, unlike the Orr- Sommerfeld equation, it is only second order but inhomogeneous. Strictly speaking, only the Orr-Sommerfeld equation needed to be solved in order to determine instability so long, that is, if one is only interested in the most unstable mode. Historically, the work of Orr and Sommerfeld was limited to two-dimensional perturbations and thus the Squire equation did not exist. Any ramifications due to three-dimensionality were left to be determined. In passing, it should be noted that neither equation is self adjoint and, although the step to remedy this can be made, there are no known eigenfunctions as are common for most boundary value problems. Moreover, both of these equations are singular and this makes the computing as well as the analytical task quite invloved. Again, it should be emphazied, no attention whatsoever was given to any specific initial-value input and, instead, it was tacitly assumed that any input would result in a fate determined by this approach.

Both Orr and Sommerfeld were interested in plane channel flows such as plane Couette (Orr: no mean pressure gradient and a linear variation of the mean velocity $U(y)$) and Poiseuille (Sommerfeld: constant mean pressure gradient and a parabolic variation for the mean velocity). Regardless, even for these cases, it was some twenty-two years after the derivation of the stability equation before any solutions whatsoever were found by the work due to Tollmien!

The problems, as posed by Orr and Sommerfeld, had finite boundaries in terms of the y variable. It is well known that this means that, when the initial-value problem is to be considered, there will be an infinite number of discrete normal modes by which any distribution can be expanded. But, as has been seen, only one mode was actually determined. For plane Couette flow, this is a moot point since this flow is not unstable. Nevertheless, damped modes should exist. Indeed, Grohne (1954) found numerous other modes for both of these flows. Save for the one positive eigenvalue for Poiseuille flow, all other modes were shown to be negative.

The question of three-dimensional effects was probed by Squire (1933) and the result was the Squire theorem. If an exact two-dimensional parallel flow admits an unstable three-dimensional disturbance for a certain value of the Reynolds number, it also admits a two-dimensional disturbance at a lower value of the Reynolds number. In short, the most unstable eigenvalue is for two-dimensional distrubances and these correspond to waves propagating in the direction of the mean flow. At the same time, as can be seen by the Squire equation, waves that propagate in a direction normal to the mean flow direction, may result in a time dependence that is also increasing but not exponentially. In the same vein, in this equation, a resonance is possible - even if the eigenmodes are damped - and this also results in secular behavior. So far as is known, such resonance is only possible for plane channel flows. In some respects, the fact that there could be secular time behavior was not thought to be of importance in that any exponentially increasing time behavior would be dominant.

With reference to the mathematical bases, the results cited above are not surprising. In addition to a resonance possibility, it should be noted that the eigen functions, even though they are not known explicitly, are not orthogonal. Then, it is only the viscous channel flows that have a complete set of eigenfunctions. A completeness proof has been provided by DiPrima & Habetler (1969). Inviscidly, there are no discrete eigenvalues or related eigenfunctions for these two flows. Instead, there is a continous spectrum. This remarkable result can best be seen by the use of the Laplace transform in time to solve the initial-value problem. Once the boundary value problem is complete, an inversion of the transform to give the time dependence will reveal the existence of branch cuts and this leads to algebraic behavior in time. It is the poles in the complex plane that correspond to exponential behavior.

There are other prototypical flows in this family besides the enclosed channel flows. Boundary layers have only one solid boundary in the definition for the flow. Boundary layers can also be those on a swept surface and W has a finite value and distribution. Free shear flows, such as the wake, jet or mixing layer, have no solid boundaries. As a result, because of the conditions that must be met for the boundary conditions at infinite distances, these flows must definitively have a continuous spectrum for the perturbation field. The discrete spectrum, however, is marginal with only a few values (depending on the value of the Reynolds number, Mack (1976) found only seven discrete eigen modes, for the boundary layer, for example). It is clear that no general initial-value problem could be explored without this data. Grosch & Salwen (1978) and Salwen & Grosch (1981) have lucidly provided the deails needed for these conclusions. A continuous spectrum is tantamount to algebraic temporal behavior. Because of this fact, the interest shifted to the transient state of the dynamics since the continuous spectrum makes for algebraic time behavior. Even for an instability that grows exponentially, it may well be that the early transient period has already increased to a level where there is breakdown or at least to a point where the assumption of linearity is no longer valid. Indeed, if this is the case, the existence of an unstable eigenvalue predicts the asymptotic fate (and instability) and nothing more. In principal, if the problem is viscous, any early growth will eventually be dissipated but there is need to assess how influential the initial period may be in the dynamics. To do this, an initial-value investigation for any shear flow is crucial. A more general nonmodal approach for solutions has led to a better understanding of the transition process.

The early state of flow instabilities becomes even more important in that it provides a basis for developing a means for flow control and, as a consequence, a delaying of the transition process. For this purpose, control theory is joined with hydrodynamic stability in order to achieve this goal.

3. The Basic Perturbation Physics

The two equations that were obtained from the full linear system are those for vorticity. Specificially, taking the curl of the momentum equations will eliminate the pressure and the curl of the velocity field is the vorticity. Then, from kinematics, it can be shown that

$$\nabla^2 v = \frac{\partial \omega_z}{\partial x} - \frac{\partial \omega_x}{\partial z}$$

where ω_x and ω_z are the perturbation vorticity components in the x and z directions respectively. By combining this relation with the three vorticity equations, then

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + W\frac{\partial}{\partial z}\right)\nabla^2 v - U''\frac{\partial v}{\partial x} - W''\frac{\partial v}{\partial z} = \epsilon \nabla^2\nabla^2 v$$

results and is the more general Orr-Sommerfeld equation in partial differential equation form. The equation for the y-component of the perturbation vorticity is

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x} + W\frac{\partial}{\partial z}\right)\omega_y - \epsilon\nabla^2\omega_y = -U'\frac{\partial v}{\partial z} + W'\frac{\partial v}{\partial x}$$

and, in like manner, is the more eneral Squire equation. Solutions to these two master equations first makes use of Fourier decomposition in the x, z plane or, in terms of standard integral definitions,

$$\hat{v} = \iint_{-\infty}^{+\infty} v e^{i\alpha x + i\gamma z} dx dz$$

and

$$\hat{\omega}_y = \iint_{-\infty}^{+\infty} \omega_y e^{i\alpha x + i\gamma z} dx dz$$

In wave space the above two equations become

$$\left(\frac{\partial}{\partial t} + i\alpha U + i\gamma W\right)\nabla^2\hat{v} - i\alpha U''\hat{v} - i\gamma W''\hat{v} = \epsilon\nabla^2\nabla^2\hat{v}$$

and

$$\left(\frac{\partial}{\partial t} + i\alpha U + i\gamma W\right)\hat{\omega}_y - \epsilon\nabla^2\hat{\omega}_y = -i\gamma U'\hat{v} + i\alpha W'\hat{v}$$

In this form, the Squire results regarding three-dimensionality become clear. Polar wave variables, $\tilde{\alpha}$ and φ are defined as $\tilde{\alpha}^2 = \alpha^2 + \gamma^2$, and $\gamma/\alpha = \tan\varphi$. In like fashion,

$$\tilde{\alpha}\tilde{u} = \alpha\hat{u} + \gamma\hat{w}$$
$$\tilde{\alpha}\tilde{w} = -\gamma\hat{u} + \alpha\hat{w}$$

are the planar velocity components when expressed in polar terms. By expressing the system in terms of $\tilde{u}$ and v, then an equivalent two-dimensional perturbation system evolves. But note that the Reynolds number in these terms is proportional to $\cos\varphi$. Consequently, it is for $\varphi = 0$ that yields the most unstable eigenvalue.

As suggesed, the two principal governing equations are those governing the perturbation vorticity. The normal component equation is explicit. The other is better seen by recognizing that, in wave space,

$$\nabla^2\hat{v} = i\alpha\hat{w}_z - i\gamma\hat{w}_x$$

is the vorticity component in the φ-direction. Of course there is a component in the $\tilde{\alpha}$-direction as well but does not appear in any of the governing equations. Each equation has the expected transport by the mean velocity and diffusion by viscosity. The inhomogeneous terms are due to the interaction of the fluctuating strain and the mean vorticity, $\Omega_x = W'$, $\Omega_z = -U'$. Parenthetically, it can be observed from here why two-dimensional plane Couette flow is stable. For, even if there is initial vorticity, it can only be advected and diffused. There is no mechanism for production.

The classical means of modal solutions in time reduced the problem even further to ordinary differential equations. Then, by a combination of analytical approximations and numerial integration, the ordinry differential equations are solved and a dispersion relation is obtained, namely $\omega = F(\tilde{\alpha}, \varphi, \epsilon)$. It is from this relation that leads to the "hairpin" curve (actually "hairpin" curves if all modes have been determined) of stability theory. As been suggested, this is not a profitable means for treating an initial-value problem. Even though the continuous spectrum can be obtained in this way, solutions are just as involved and tedious. In addition, all discrete modes, including those that may be damped, must be determined.

An alternative is to directly integrate the partial differential governing equations. For example, a convenient online method for this purpose is the method of lines. In this way, arbitrary input can be made and the time dependence naturally evolves. This route also allows for initial expansion to be interms of alternative eigen functions and ones that meet the far field boundary conditions. These steps have been taken with excellent results even for much larger values of the Reynolds number than was done before; Criminale et al. (1997), e.g. But, no matter how good a numerical scheme might be, it is imperative to have some means for an analytical representation by which comparisons can be made as well as properly formulate the initial-value problem. A recent approach that has met this challenge is one that uses the techniques of multiple spatial and multiple time scales. In addition to providing an analytical representation, this method explicitly encompasses the full range of the dynamics, namely the transients as well as the asymptotic fate. And, as an added bonus, this method is one in terms of a regular rather than a singular perturbation expansion. Hence, any effect due to viscosity is explicitly assessed and shear flows with or without the presence of solid boundaries can be so analyzed. Numerically, though, the problem remains one that is characterized as stiff but these difficulties have long been resolved.

4. Multiple Scale, Multiple Time Analysis

The two principal governing partial differential equations, once Fourier transformed from real to Fourier space, remain the bases for exploration. Now, it has already been noted that $\nabla^2\hat{v}$ is the vorticity component in the φ direction in wave space. Define this as $\hat{\Omega}$. With this relation, the governing system now has three principal equations. In much the same way, direct numerical computation of the Navier-Stokes equations uses this step in formlating integration schemes in order to remove the pressure from the numerical scheme. It should also be recognized that, for any case where $\hat{\Omega} = 0$, then there is no vorticity. Any velocity field can be decomposed into three basic components, namely solenoidal, rotational and harmonic. For an incompressible fluid there can be no solenoidal component. Hence, without vorticity the velocity would only be harmonic. Explicitly, the three equations are

$$\nabla^2\hat{v} = \hat{\Omega}$$
$$\frac{\partial\hat{\Omega}}{\partial t} - \epsilon\frac{\partial^2\hat{\Omega}}{\partial y^2} = -i(\tilde{\alpha}\cos\varphi U + \tilde{\alpha}\sin\varphi W)\hat{\Omega} + i(\tilde{\alpha}\cos\varphi U'' + \tilde{\alpha}\sin\varphi W'')\hat{v} - \epsilon\tilde{\alpha}^2\hat{\Omega}$$

and

$$\frac{\partial\hat{\omega}_y}{\partial t} - \epsilon\frac{\partial^2\hat{\omega}_y}{\partial y^2} = -i(\tilde{\alpha}\cos\varphi U + \tilde{\alpha}\sin\varphi W)\hat{\omega}_y + i(-\tilde{\alpha}\sin\varphi U' + \tilde{\alpha}\cos\varphi W')\hat{v} - \epsilon\tilde{\alpha}^2\hat{\omega}_y$$

where the relations, $\alpha = \tilde{\alpha}\cos\varphi$ and $\gamma = \tilde{\alpha}\sin\varphi$ have been inserted directly.

In order to demonstrate the technique, consider the case of the Blasius boundary layer. Immediately all terms involving the W-component of the mean velocity are eliminated from the equations. Now define new dependent variables by the relations

$$\hat{\Omega} = e^{-\epsilon\tilde{\alpha}^2 t} e^{-i\tilde{\alpha}\cos\varphi t}\mathbf{\Gamma}$$
$$\hat{v} = e^{-\epsilon\tilde{\alpha}^2 t} - i\tilde{\alpha}\cos\varphi t\mathbf{V}$$
$$\hat{\omega}_y = e^{-\epsilon\tilde{\alpha}^2 t} e^{-i\tilde{\alpha}\cos\varphi t}\omega_y$$

The first factor is clear; the second indicates that the system is now in terms of moving coordinates and the shift is in terms of the value in the free stream external to the boundary layer where $U = 1$ in nondimensional terms. In this way, the three equations become

$$\nabla^2\hat{v} = \frac{\partial^2\mathbf{v}}{\partial y^2} - \tilde{\alpha}^2\mathbf{v} = \mathbf{\Gamma}$$

$$\frac{\partial\mathbf{\Gamma}}{\partial t} - \epsilon\frac{\partial^2\mathbf{\Gamma}}{\partial y^2} = -\, i\tilde{\alpha}\cos\varphi(U-1)\mathbf{\Gamma} - i\tilde{\alpha}\sin\varphi V\mathbf{\Gamma} + i\tilde{\alpha}\cos\varphi U'' v$$

and

$$\frac{\partial\omega_y}{\partial t} - \epsilon\frac{\partial^2\omega_y}{\partial y^2} = -\, i\tilde{\alpha}\cos\varphi(U-1)\omega_y - i\tilde{\alpha}\sin\varphi U' v$$

In this form, the equations have salient implications. In the far field outside the boundary layer the $y \to \infty$ condition is direct since $U \to 1$ and $U'' \to 0$ in this limit. Moreover, in terms of the physics, the two equations for the vorticity are simply those of classical heat diffusion and are readily solveable. In addition, the case where $\varphi = \pi/2$, all equations can be solved exactly.

The classical results for eigen modes all indicate that the scalar wave number, $\tilde{\alpha}$, is $O(1)$ or smaller for instability. In other words, instability occurs for large spatial scales. Small scales are heavily damped by viscosity. This identifies $\tilde{\alpha}$ as a small parameter. Further inspection will then show that this translates into spatial and time scale identification. Specifically, there are two spatial scales: y and $Y = \tilde{\alpha}y$. For time there are three scales: t, $\tau = \tilde{\alpha}t$ and $T = \tilde{\alpha}^2 t$. In this formulation, let

$$\mathbf{\Gamma} = \mathbf{\Gamma}(y, Y, t, \tau, T)$$

etc. for $\mathbf{v}$ and ω_y as well. An expansion procedure results and can be written as

$$\mathbf{\Gamma} = \tilde{\alpha}\mathbf{\Gamma}_1 + \tilde{\alpha}^2\mathbf{\Gamma}_2 + \ldots$$
$$\mathbf{v} = \tilde{\alpha}\mathbf{v}_1 + \tilde{\alpha}^2\mathbf{v}_2 + \ldots$$
$$\omega_y = \tilde{\alpha}\omega_{y_1} + \tilde{\alpha}^2\omega_{y_2} + \ldots$$

The series begins in this way in order that all terms are ordered properly as can be seen from the Squire relations and incompressibilty.

The first two orders for this expansion will be at $O(\tilde{\alpha})$

$$\frac{\partial^2\mathbf{v}_1}{\partial y^2} = \mathbf{\Gamma}_1;$$

$$\frac{\partial\mathbf{\Gamma}_1}{\partial t} - \epsilon\frac{\partial^2\mathbf{\Gamma}_1}{\partial y^2} = 0;$$

$$\frac{\partial\omega_{y_1}}{\partial t} - \epsilon\frac{\partial^2\omega_{y_1}}{\partial y^2} = 0;$$

and, at order $O(\tilde{\alpha}^2)$,

$$\frac{\partial^2\mathbf{v}_2}{\partial y^2} = -2\frac{\partial^2\mathbf{v}_1}{\partial y\partial Y} + \mathbf{\Gamma}_2$$

$$\frac{\partial\mathbf{\Gamma}_2}{\partial t} - \epsilon\frac{\partial^2\mathbf{\Gamma}_2}{\partial y^2} = -\frac{\partial\mathbf{\Gamma}_2}{\partial\tau} + 2\epsilon\frac{\partial^2\mathbf{\Gamma}_1}{\partial y\partial Y} + i\cos\varphi(U-1)\mathbf{\Gamma}_1 + i\cos\varphi U''\mathbf{v}_1$$

$$\frac{\partial\omega_{y_2}}{\partial t} - \epsilon\frac{\partial^2\omega_{y_2}}{\partial y^2} = -\frac{\partial\omega_{y_1}}{\partial\tau} + 2\epsilon\frac{\partial^2\omega_{y_1}}{\partial y\partial Y} + i\cos\varphi(U-1)\omega_{y_1} - i\sin\varphi U'\mathbf{v}_1$$

Now, just as the boundary layer per se is the result of a vorticity input in that a constant mean flow is sheared at the leading edge and has vorticity down the plate, the perturbation problem can also be taken as one that has an initial input of vorticity. From this, fluctuating velocity will result. Boundary conditions, however, are in terms of the velocity: all components must vanish at the surface of the plate (y = 0) and be bounded as $y \to \infty$. The vorticity should be bounded as well. It is further noted that this method allows for the input of vorticity in the free stream, making for an assessment of the interaction or influence of such fluctuations within the boundary layer. Directly, because of the the relations, then $\mathbf{v}(0, 0, t, \tau, T) = 0$ and bounded as $y \to \infty$.

$\mathbf{\Gamma}_1(y, Y, 0, 0, 0) = \mathbf{\Gamma}_0(y)$; $\mathbf{\Gamma}_2 = \mathbf{\Gamma}_3 = \ldots = 0$; $\omega_{y_1}(y.Y, 0, 0, 0) = \omega_{y_0}(y)$ and all subsequent terms zero at initial time. In addition, $\omega_y(0, 0, t, \tau, T) = 0$ and bounded as $y \to \infty$. After the initial input, the equations are then a series of forced heat equations for the vorticity with the vertical velocity component forced at the outset.

This scheme can be applied to all shear flows whether visocus or not. The only difference is that, for the inviscidly defined mean flows, then there are only two time scales, t and in the perturbation problem. In fact, this point is prominent in the definitions of the transposed dependent variables where it is explicit that the ultimate decay by viscosity involves the factor $\epsilon\tilde{\alpha}^2 t$ or ϵT. It also follows that any instabiliity in these flows is not caused by viscosity.

5. General Remarks

There are, of course, other prototypical flows that have been examined for stability. Such flows can be catalogued under two main headings: (1) mean flows with curved stream lines and (2) flows with no mean velocity. Both of these cases have been thoroughly treated. In both cases, viscosity results only in dissipation, such as the problem of flow betwen two concentric cylinders. With no mean flow, the best example is that of advection caused by a thermal gradient. These problems do not require alternative analysis in that the instability that results is not one of wave propagation that is amplified. Instead, the flow evolves to another state where there are cells, for example. The neutral stability is one that is time independent. And, the reason for the instability is due to a simple non balance of the forces involved. And, there has been substantial and excellent experimental verification.

As suggested, the majority of stability analyses are based on an initial-value construction. In reality, a flow such as the boundary layer, is actually one of a spatial initial-value problem. Simply stated, an input is established at an upstream location and then moves downstream. The time dependence is merely that of oscillations. Theoretical work has been made for this means of formulation. And, although very much involved, direct numerical simmulation has also been done. To date, no alternative analytical method has been suggested for the problem when cast in this manner. At the same time, newer concepts, such as convective and absolute instabilities, must now be considered when the problem is approached from spatial stability.

An extended list of references on the subject is included below.

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