

# IN-FLIGHT THRUST DETERMINATION FOR TURBOFAN ENGINES AND THE ASSOCIATED UNCERTAINTY ANALYSIS

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**Abstract.** *In-flight thrust determination is important for the aircraft manufacturer because the aircraft drag must be precisely known. It depends on the drag the determination of the load carrying capacity of the aircraft. Since for level and steady flight drag equals thrust, measuring thrust is the same as measuring drag. Methods that might be considered for in-flight thrust determination (IFTD) and uncertainty analysis for turbofan engines are presented. Four IFTD methods are presented, three of which are standard in aircraft industry and are documented in SAE AIR 1703 as F/AP, F/mRootT, the Residual Error Method. The authors propose the fourth, named Specific Thrust Method. This paper includes also full uncertainty analysis assessment as per SAE AIR 1678. It is also intended to provide appropriate background information to gain a perspective of the major aspects and processes that might be used for the determination of in-flight thrust and its uncertainty. A case study is analyzed, reflecting a flight test program that illustrates the practices and results for determination of in-flight thrust for a modern turbofan engine.*

**Key words:** *IFTD, Residual Error, Performance, Uncertainty, Engine*

## 1. Introduction

The principal aim of aircraft performance flight-testing is the validation of mathematical models, which would have been established during design and development, to describe the aircraft and its propulsion system. If this is done, then any specific performance requirements can also be validated, though performance measured at relevant conditions, and appropriately corrected, may provide additional confirmation.

The engine performance assumed for a performance analysis model is normally specified by the engine manufacturer for engines that have been or will be qualification tested at the appropriate standard in ground test facilities. Confirmation of the drag polar of the aircraft performance model requires drag to be determined in flight, which in turn means that thrust must be determined in flight.

The term “in-flight thrust determination” is used synonymously with “in-flight thrust measurement” although in flight thrust is not directly measured but determined or calculated using mathematical modeling relationships between in-flight thrust and various direct measurements of physical quantities. The in-flight thrust determination process includes both Ground and Flight Testing. The mathematical modeling relationships between the in flight thrust and the measurements of the physical quantities are calibrated in Ground Tests. Error estimates for each item are required to calculate the uncertainty of the “in flight thrust measurement”.

## 2. Definitions and Basic Methodology

This section will present the definitions of thrust, thrust and drag accounting, and other basic terms for evaluating in flight thrust. Figure 1. presents a consistent station numbering system, according to Report SAE AIR 1703.

### 2.1. Net Thrust

In-flight thrust methodologies equate the propulsion system thrust and associated aircraft drag so they are equal in level, non-accelerating flight (Figure 2).

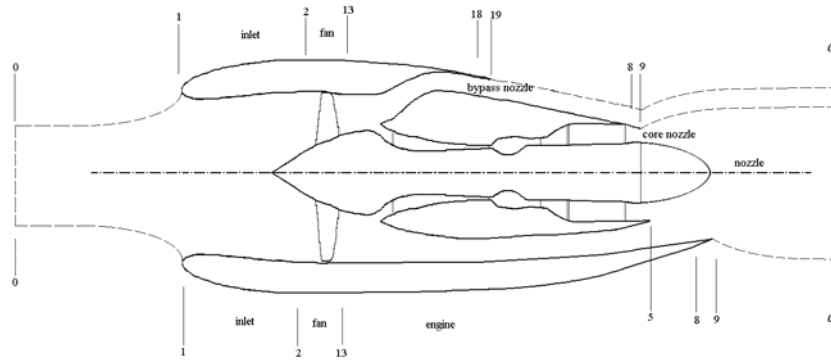


Figure 1. Engine Station Numbering System



Figure 2. Thrust and Drag at Steady Level Flight.

Several expressions for thrust can be adopted and used as models for in-flight thrust determination (SAE AIR 1703). In this work, for steady-state level flight, with undisturbed flow both far downstream and far upstream of the aircraft, the standard net thrust between station 0 and  $\infty$  for a dual stream propulsion system, illustrated in Figure 1, is represented by the Eq.(1), (Santos, 2001)

$$F_N = \dot{M}_{19} V_{19} + A_{19} (P_{s19} - P_{so}) + \dot{M}_9 V_9 + A_9 (P_{s9} - P_{so}) - \dot{M}_0 V_0 \quad (1)$$

where

$\dot{M}_0$ ,  $\dot{M}_9$  and  $\dot{M}_{19}$  are air mass-flow, respectively, at engine inlet, at station 9 and at station 19;

$V_0$ ,  $V_9$  and  $V_{19}$  are speed of air flow, respectively, at engine inlet, at station 9 and at station 19;

$P_{so}$ ,  $P_{s9}$  and  $P_{s19}$  are, respectively, ambient static pressure, static pressure at station 9 and static pressure at station 19.

A common convention (SAE AIR 1703; Agardograph 237; SAE AIR 1678) is to define net thrust as the vector sum of a ram drag term and an appropriate gross thrust term. Ram drag,  $F_R$  or  $F_{RAM}$ , is the free-stream momentum,  $\dot{M}_0 V_0$ . The gross thrust term is the sum of the remaining terms on the right-hand side of the net thrust Eq.(1). Thus, for net thrust:

$$F_N = F_{G19} + F_{G9} + F_{RAM} \quad (2)$$

where  $F_{G19} = \dot{M}_{19} V_{19} + A_{19} (P_{s19} - P_{so})$ ,  $F_{G9} = \dot{M}_9 V_9 + A_9 (P_{s9} - P_{so})$ , and  $F_{RAM} = \dot{M}_0 V_0$

## 2.1. Ideal Thrust and Ideal Nozzle

Since the gross thrust,  $F_G$ , applies at the nozzle exit plane while practical considerations dictate that nozzle conditions be measured at the nozzle entry plane, a convention with several unique definitions has been developed to provide in-flight evaluations of thrust and mass-flow. The general procedure is to relate real nozzle performance to that of an ideal nozzle through the use of empirically established coefficients.

For the ideal nozzle definitions, one-dimensional, isentropic flow is considered to exist downstream of the nozzle entry plane. Real gas properties are used herein (Bathe, 1995).

Ideal-nozzle total temperature, total pressure and mass-flow are assumed to remain constant and equal to the entry values between the nozzle entry and exit stations. Static temperature, velocity and area (or static pressure) are calculated based on the one-dimensional isentropic-expansion assumption. The minimum area (throat) is designated station 8 or 18 and will be coincident with the exit plane (station 9 or 19) for the convergent nozzle.

## 2.2 Nozzle Coefficients

It is essential that a consistent set of coefficients be defined to relate actual to ideal nozzle performance in a manner that is practical for in-flight thrust determination. Nozzle coefficients are determined usually by nozzle model tests, but can also be established from full-scale tests.

### 2.2.1. Flow Coefficient ( $C_D$ )

The flow (or discharge) coefficient,  $C_D$ , is defined as the ratio of actual to ideal mass-flow for a given nozzle geometry and pressure ratio. It may equally be defined as the ratio of effective to actual (or geometric) throat area required to pass the actual mass-flow. Hence,

$$C_D = \frac{\dot{M}_{act}}{\dot{M}_{id}} = \frac{A_{eff}}{A_{act}} \quad (3)$$

### 2.2.2. Specific Thrust Coefficient ( $C_V$ and $C_X$ )

The specific thrust coefficient,  $C_V$ , is defined as the ratio of actual specific thrust to ideal specific thrust obtainable from an ideal flexible convergent-divergent nozzle at a given pressure ratio. This coefficient can be expressed as the ratio of an effective discharge velocity to the ideal velocity obtainable with an ideal flexible convergent-divergent nozzle. Thus,

$$C_V = \frac{\left(\frac{F_G}{\dot{M}}\right)_{act}}{\left(\frac{F_G}{\dot{M}}\right)_{id,con-di}} = \frac{V_{eff}}{V_{id,con-di}} \quad (4)$$

The resulting general thrust equation is:

$$F_{G,act} = C_V \cdot \dot{M}_{act} \cdot V_{id,con-di} \quad (5)$$

Another thrust coefficient,  $C_X$ , is defined similarly to  $C_V$  except that the ideal nozzle parameters are based upon an ideal convergent nozzle. Thus,

$$C_X = \frac{\left(\frac{F_G}{\dot{M}}\right)_{act}}{\left(\frac{F_G}{\dot{M}}\right)_{id,con}} \quad (6)$$

### 2.2.3. Gross Thrust Coefficient ( $C_G$ )

The gross thrust coefficient,  $C_G$ , is defined as the ratio of actual thrust to ideal thrust obtained from an ideal nozzle at a given pressure ratio. It may also be expressed as the product of the flow and specific thrust coefficients for a given nozzle geometry and pressure ratio, since, for instance,

$$C_G = \frac{(F_G)_{act}}{(F_G)_{id,con}} = \frac{\left(\frac{F_G}{\dot{M}}\right)_{act}}{\left(\frac{F_G}{\dot{M}}\right)_{id,con}} \cdot \frac{(\dot{M})_{act}}{(\dot{M})_{id,con}} = C_V \cdot C_D \quad (7)$$

Therefore

$$C_G = C_D \cdot C_V \text{ or } C_G = C_D \cdot C_X \quad (8)$$

### 3. Mathematical Formulation

The formulation used for thrust and mass-flow determination, for the IFTD methodologies analyzed, is discussed in this section.

The basic equation used is Eq. (2),  $F_N = F_{G19} + F_{G9} - F_{RAM}$ . A nozzle is said to operate “fully expanded” when the static pressure at the exit station,  $P_{s19}$  or  $P_{s9}$ , is at the ambient static pressure,  $P_{s0}$ , in the absence of base pressure effects acting in the local external environment. Then  $P_{s19}=P_{s0}$  and  $P_{s9}=P_{s0}$ , and the gauge pressure terms in the above equations become zero (Propesa, 1997). Following are indications of how to determine each term of Eq. (2):

- $P_{s0}$ ,  $P_{s9}$  and  $P_{s19}$  are determined directly by instrumentation installed on the engine;
- $A_9$  and  $A_{19}$ , geometric areas, are measured directly on the engine;
- $V_0$  is aircraft speed, from aircraft instrumentation.
- $V_{19}$  and  $V_9$  are determined considering thermally-perfect nozzle flow (Propesa, 1997), that is, at any station in a one-dimensional, uniform thermal flow of a gas, the steady energy equation may, from thermodynamic considerations, be written as

$$h_T = h_s + \frac{V^2}{2} \quad (9)$$

where  $h_T$  is the specific total (stagnation) enthalpy,  $h_T = \frac{H_T}{M}$ ;  $h_s$  is the specific static enthalpy  $h_s = \frac{H_s}{M}$ ;  $\frac{V^2}{2}$  is the specific kinetic energy.

Thus Eq. (9) may be written as

$$V = \sqrt{2(h_T - h_s)} \quad (10)$$

Applying Eq. (10) to stations 9 and 19 of the engine, the velocity of the jet,  $V_{19}$  and  $V_9$  are

$$V_{19} = \sqrt{2(h_{T19} - h_{s19})} \text{ and } V_9 = \sqrt{2(h_{T9} - h_{s9})} \quad (11)$$

Here,  $h_{s19}$  and  $h_{s9}$  are the specific static enthalpies of the jet expanded isentropically to the static pressures  $P_{SB19}$  and  $P_{SB9}$ , respectively, both has been taken as ambient pressure,  $P_{s0}$ .

Applying Eq. (11) to an ideal nozzle, the fully-expanded velocity of the jet,  $V_E$  (or  $V_\infty$ ) is

$$V_E = \sqrt{2(h_T - h_{s\infty})} \quad (12)$$

Here,  $h_{s\infty}$  is the specific static enthalpy of the jet expanded isentropically to the ambient static pressure,  $P_{s0}$ .

In engines in which the nozzle gas comprises a mixture of air and the products of combustion of a defined fuel, the gas properties depend solely on fuel/air ratio ( $f/a$  = fuel mass-flow / air mass-flow) and the temperature of the gas (Bathe, 1995).

Thus, the value of the ideal velocity depends on the value of  $h_T$ , which corresponds to the total temperature,  $T_T$ , and nozzle pressure ratio.

The procedure is: Given  $T_T$ , calculate  $h_T$  and  $pr_T$  from polynomials (Bathe, 1995), then calculate  $pr=pr_T/NPR$ , then calculate  $h_{s19}$  and  $T_{s19}$  from polynomials, then calculate  $V_{19}$  from Eq. (11).

Therefore,  $V_{19}$  and  $V_9$  are calculated from total temperature ( $T_{T19}$  and  $T_{T9}$ ) measurements, total nozzle pressures ( $P_{T19}$  and  $P_{T9}$ ) and static base pressures ( $P_{SB19}$  and  $P_{SB9}$ ), here assumed to be  $P_{s0}$ .

- The mass flow at engine inlet is determined by

$$\dot{M}_0 = \dot{M}_{19} + \dot{M}_9 - \dot{M}_F \quad (13)$$

$\dot{M}_F$  is from aircraft instrumentation.

$\dot{M}_9$  is calculated assuming that high pressure turbine stators (station 4) operate always in choked conditions (Agardograph 237; Santos, Lewis and Barbosa, 2000). In this case the high pressure turbine stators flow function (HPFF) is constant and it is a characteristic of a given engine.

The flow function for choked flow at station 4, illustrated here for calorically-perfect flow, is:

$$\frac{\dot{M}_9 \sqrt{T_{T4}}}{P_{T4}} = \text{HPFF} = \frac{A}{\sqrt{R}} \sqrt{\gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} \quad (14)$$

where R is the gas constant,  $\gamma$  is the ratio between specific heat at constant pressure and at constant volume. Thus,

$$\dot{M}_9 = \frac{\text{HPFF} * P_{T4}}{\sqrt{T_{T4}}} \quad (15)$$

$\dot{M}_{19}$  is calculated from total temperature ( $T_{T19}$ ) measurements, total nozzle pressures ( $P_{T19}$ ) and, if the nozzle operates unchoked or sub-critically, static base pressures ( $P_{SB19}$ ). They can be written in “non-dimensional form” as Eq. (16), for calorically perfect flow when  $\gamma$  is constant:

$$\dot{M}_{19} = P_{T19} A_{19} \sqrt{\frac{\gamma}{T_{T19} R}} M_0 \left[ 1 + \frac{(\gamma-1)}{2} M_0^2 \right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (16)$$

- Thrust is then calculated:

$$F_{G19} = \dot{M}_{19} V_{19} + A_{19} (P_{s19} - P_{so})$$

$$F_{G9} = \dot{M}_9 V_9 + A_9 (P_{s9} - P_{so}), \quad F_{RAM} = \dot{M} V_0 \quad \text{and} \quad F_N = F_{G19} + F_{G9} + F_{RAM}$$

In this work, a mixed flow, single exhausted turbofan engine was used, as shown in Figure 3.

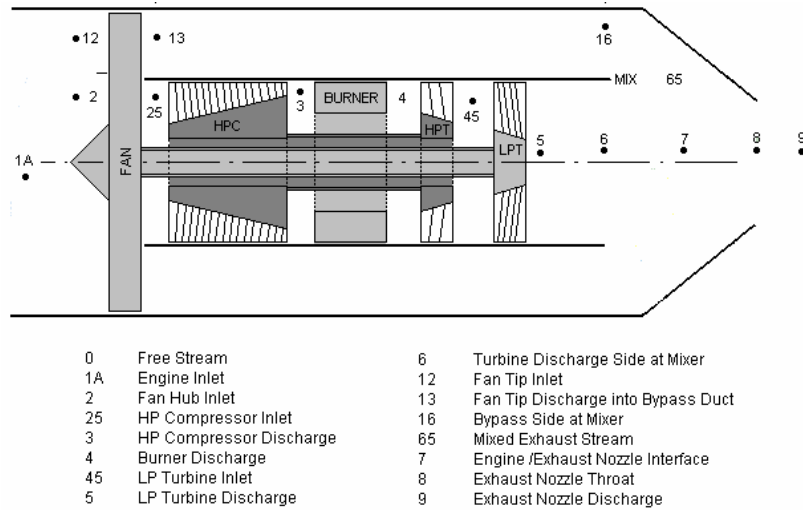


Figure 3. Single Exhaust Turbofan

The method used calculates a single ideal gross thrust from a nozzle-entry plane downstream of the annular or forced mixer. To use this method, average total pressure and average total specific enthalpy values downstream of the mixer,  $P_{T7}$  and  $h_{T7}$ , must be used together with the total nozzle flow, which equals the sum of the by-pass and core flows, and mixer efficiency.

The average total enthalpy,  $h_{T7}$ , is determined uniquely by energy conservation and mass-flow conservation:

$$H_{T7} = H_{T6} + H_{T16} \quad (17)$$

or

$$\dot{M}_7 h_{T7} = \dot{M}_6 h_{T6} + \dot{M}_{16} h_{T16} \quad (18)$$

given that  $\dot{M}_7 = \dot{M}_6 + \dot{M}_{16}$  hence,

$$h_{T7} = \frac{h_{T6} + \text{BPR} \cdot h_{T16}}{1 + \text{BPR}} \quad (19)$$

where, here,  $\text{BPR} = \frac{\dot{M}_{16}}{\dot{M}_6}$ . Note that, as the by-pass ratio, BPR, increases, the contribution made by the core flow to the mixed specific enthalpy decreases and the value of  $h_{T7}$  tends to  $h_{T16}$ , that is, BPR becomes less influential.

The determination of the average total pressure,  $P_{T7}$ , at station 7, is done using mass-flow weighted average total pressure value:

$$P_{T7} = \frac{\dot{M}_6 P_{T6} + \dot{M}_{16} P_{T16}}{\dot{M}_6 + \dot{M}_{16}} \quad (20)$$

The ideal thrust is given by

$$F_{G9,\text{ideal}} = (\dot{M}_6 + \dot{M}_{16}) \sqrt{2(h_{T7} - h_{s\infty})} \quad (21)$$

#### 4. Normalized Groups

In ideal one-dimensional flow, the ideal thrust expressions are functions of nozzle pressure ratio,  $P_T/P_{s0}$ , and specific heat ratio,  $\gamma$ , as summarized in Table 1.

Table 1. Ideal Nozzle Performance Groups (SAE AIR 1703).

Ideal Perform. Group	Nozzle condition	Nozzle Type	
		Convergent	Convergent-Divergent
$\frac{F_G}{AP_{s0}}$	Unchoked	$\frac{2\gamma}{\gamma-1} \left[ \left( \frac{P_T}{P_{s0}} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$	Same as convergent
	Choked	$\left[ 2 \left( \frac{2}{\gamma-1} \right)^{\frac{1}{\gamma-1}} \frac{P_T}{P_{s0}} \right] - 1$	$\frac{2\gamma}{\sqrt{\gamma^2-1}} \left( \frac{2}{\gamma+1} \right)^{\frac{1}{\gamma-1}} \frac{P_T}{P_{s0}} \sqrt{1 - \left( \frac{P_{s0}}{P_T} \right)^{\frac{\gamma-1}{\gamma}}}$
$\frac{F_G}{\dot{M} \sqrt{RT_T}}$	Unchoked	$\sqrt{\frac{2\gamma}{\gamma-1} \left[ 1 - \left( \frac{P_{s0}}{P_T} \right)^{\frac{\gamma-1}{\gamma}} \right]}$	Same as convergent
	Choked	$\sqrt{\frac{2(\gamma+1)}{\gamma} - \frac{P_{s0}}{P_T} \sqrt{\frac{1}{\gamma} \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma+1}{\gamma-1}}}}$	Same as convergent

Nozzle is considered unchoked, when  $\frac{P_T}{P_{s0}} < \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}}$ . On the other hand, for a choked nozzle:  $\frac{P_T}{P_{s0}} \geq \left( \frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}}$ .

#### 5. IFTD Methods

Four forms of gas generator methods will be discussed. These utilize calibrated measurements of gas-generator-flow properties at various stations within the engine and at nozzle entry. Flow characteristics can be calculated and related to thrust through calibrations of the engine and nozzle in a ground test facility or test stand. The process of in-flight thrust measurement becomes one of relating measurements made in flight to similar measurements made in controlled conditions on a ground level test bed. The following sections depict the methods used for this study, more

detailed description of them may be found in the literature (Santos, Lewis and Barbosa, 2000; Santos, 2001; SAE AIR 1703; Agardograph 237).

### 5.1 Area Pressure Method ( $F/AP$ )

Thrust is calculated from the measured nozzle exit areas, the nozzle pressure ratios, and the nozzle gross thrust coefficients, as given by the Eq. (22):

$$F_G = C_G \cdot \left[ \frac{F_G}{AP_{s0}} \right]_{ideal} \cdot A \cdot P_{s0} \quad (22)$$

### 5.2. Mass-flow - Temperature Method ( $F/M\sqrt{T}$ )

Thrust is calculated from the measured nozzle exit areas, the nozzle pressure ratios, and the nozzle gross thrust coefficients, as given by the Eq. (23).

$$F_G = C_V \cdot C_D \cdot \left[ \frac{F_G}{\dot{M}\sqrt{RT_t}} \right]_{ideal} \left[ \frac{\dot{M}\sqrt{RT_t}}{AP_t} \right]_{ideal} \cdot A \cdot P_t \quad (23)$$

### 5.3. Residual Error Method (REM)

The procedure of the Residual Error Method (REM) is based on a full-scale sea level calibration method, using a specific calibration technique, the “Residual Error” approach, to determining the fan pressure correlation characteristic. Fan pressure correlation is a scalar which when multiplied to the measured fan duct pressure,  $P_{T13}$ , drives closure between the nozzle calculated thrust and flow and the test stand measurements. The method is applicable to both separate and mixed flow nacelles.

The REM assumes fan discharge pressure  $P_{T13}$  measurement error to be the primary cause for any disagreements between nozzle calculated versus facility measured thrust and flow.

The REM method drives these disagreements to a minimum using a fan discharge pressure scalar or fan pressure correlation. Fan discharge pressure error is an independent parameter, and has no impact on residual error, RERR, levels.

Errors only in fan discharge pressure would result in zero residual error, because a unique fan pressure correlation would simultaneously drive exact closure on the calculated to measured differences. On the other hand, non-zero RERR exists when a unique pressure correlation cannot simultaneously drive these differences to zero, thus leaving residual thrust and flow errors,  $(\Delta F_G, \Delta \dot{M}_2)$ . This method derives a single compromised pressure correlation that minimizes the root-sum square of these residuals, that is, minimizes  $\sqrt{(\Delta F_G)^2 + (\Delta \dot{M}_2)^2}$ .

Therefore the presence of measurement/modeling errors other than those related to fan discharge pressure level result in non-zero residual error. The method assumes that RERR is the result of unknown errors.

A unique pressure correlation characteristic as a function of power is derived for each podded system. On-wing, the personalized pressure correlation of each calibrated propulsion system is applied to the average fan duct radial rake measured pressure. The resulting fan duct “gas path average pressure” is then used in the determination of nozzle calculated in-flight thrust and airflow.

### 5.4. Specific Thrust Method

The procedure for this method is similar to the Residual Error Method (described in section 5.3), the difference being on the full-scale sea level calibration method, using a different calibration technique, the “Specific Thrust” approach to determining the fan pressure correlation characteristic.

The method, as well as the residual error method, also will assume fan discharge pressure measurement error,  $P_{T13}$  to be the primary cause for any disagreements between nozzle calculated versus facility measured thrust and flow. The intention of this method is to drive these disagreements to zero using a fan discharge pressure correlation. A unique fan pressure correlation will be derived, which will simultaneously drive exact closure on the calculated to measured

differences and zero  $\Delta \left( \frac{F_G}{\dot{M}_2} \right)$  will exist when a unique pressure correlation drives these differences to zero. In this

case, the method will derive a single compromised pressure correlation that zeroes the specific thrust, that is

$$\Delta\left(\frac{F_G}{\dot{M}_2}\right) = 0.$$

## 6. Large Sample Uncertainty of a Measured Variable

A single number, some combination of bias and precision, is needed to express a reasonable limit of error. This single number, called uncertainty,  $U$ , must have a simple interpretation and be useful. One may not define a single rigorous statistics because the bias is an upper limit involving judgment which has unknown characteristics. Any function of these two numbers must be a hybrid combination of an unknown quantity, bias ( $B$ ), and statistics, precision ( $S$ ).

For 95% confidence level,  $t_{95}=2$ . Then, (Santos, 2001; SAE AIR 1678)

$$U_{RSS} = \sqrt{B^2 + (2S)^2} \quad (25)$$

Therefore, the appropriate random uncertainty to use with a variable  $X$  that is determined with a reading is the standard deviation of the sample population times 2. The estimate of random uncertainty used in this work is:

$$P_{\bar{X}} = 2S_{\bar{X}} = 2 \left\{ \left[ \frac{1}{N_s(N_s - 1)} \sum_{i=1}^{N_s} (X_i - \bar{X})^2 \right]^{\frac{1}{2}} \right\} \quad (26)$$

where  $N_s$  is the number of samples in a set of data,  $P$  is the estimate of random uncertainty,  $S_{\bar{X}}$  is the standard deviation of the mean,  $\bar{X}$  is the mean of sample population.

Therefore, for 95% confidence, (Coleman and Steele, 1998)

$$U_{RSS} = \sqrt{B^2 + P_{\bar{X}}^2} \quad (27)$$

### 6.1. Propagation of Errors into Experimental Results

Equation (28) is the data reduction equation used for determining  $r$  from the measured values of the variables  $X_i$ . Then the uncertainty in the result is given by (Coleman and Steele, 1998):

$$U_r^2 = \left( \frac{\partial r}{\partial X_1} \right)^2 U_{X_1}^2 + \left( \frac{\partial r}{\partial X_2} \right)^2 U_{X_2}^2 + \dots + \left( \frac{\partial r}{\partial X_J} \right)^2 U_{X_J}^2 \quad (28)$$

## 7. Case Study - Mixed exhaust/Single Stream Turbofan Engine

Figure 3 presents a sketch of the single stream turbofan engine used in this work. The propulsion thrust included all the forces and losses inside the propulsion stream tube (inlet, fan and primary flows), while forces outside this stream tube were considered as part of airplane drag. Scale model static tests were conducted to determine  $C_{V9}$  (static, model) and  $C_{D9}$  (static, model). ATF (Altitude Test Facility) tests were conducted, with a production-equivalent engine, to obtain  $C_{D9}$  (ATF),  $C_{V9}$  (ATF). GLTB (Ground Level Test Bed) tests were conducted for flight test engines to determine  $\Delta P_{T13}$ .

### 7.1. Nozzle coefficients results

Nozzle flow and thrust coefficients were obtained by testing nozzles over the nozzle operating pressure ratio range, at several different mass-flows, nozzle-entry total pressure and nozzle-entry total temperature.

Coefficient results from sub-scale model nozzle tests and full-scale tests are represented in Figures 4 and 5.



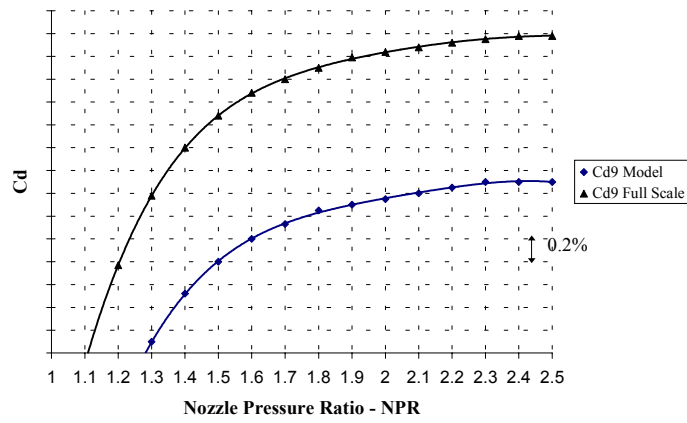


Figure 4. Mass-Flow Coefficient,  $C_{d9}$

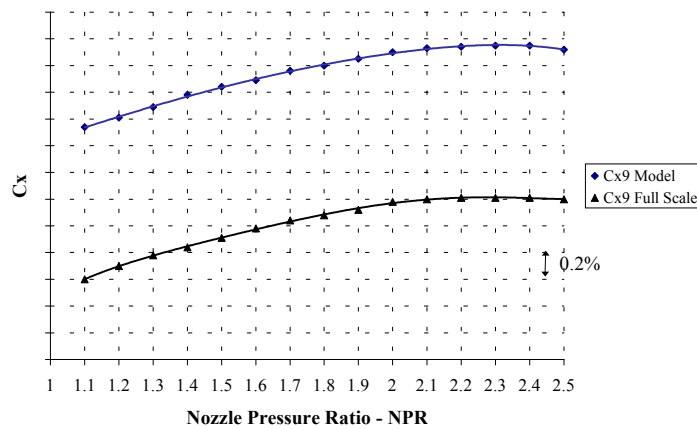


Figure 5. Specific Thrust Coefficient,  $C_{x9}$

**7.2. Instrumentation**

Figure 6 indicates the instrumentation installed in the engine and used for the data acquisition referred to in the following sections.

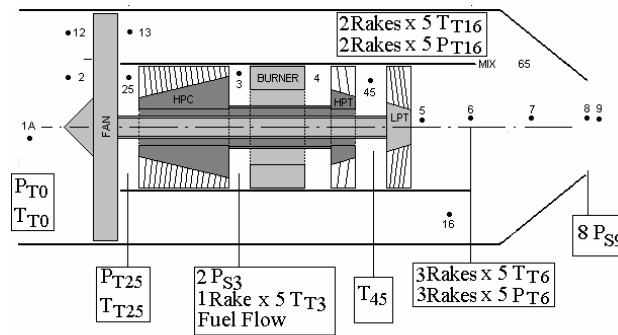


Figure 6. Measured Data

Information on static ambient pressure ( $P_{s0}$ ), static ambient temperature (SAT), total ambient temperature (TAT or  $T_{T0}$ ), air speed ( $V_0$ ), and aircraft configuration were information provided by the aircraft manufacturer.

**7.3. Fan Pressure Correlation Curves**

The curves of Figures 7 and 8 were generated by running the engine mathematical model at several fan speed ( $N_1$ ), at sea level and static conditions, simulating a ground test.

On the flight tests, the instrumentation were installed at station 16 of the engine rather than at station 13, and the fan pressure correlation curves reflect that.

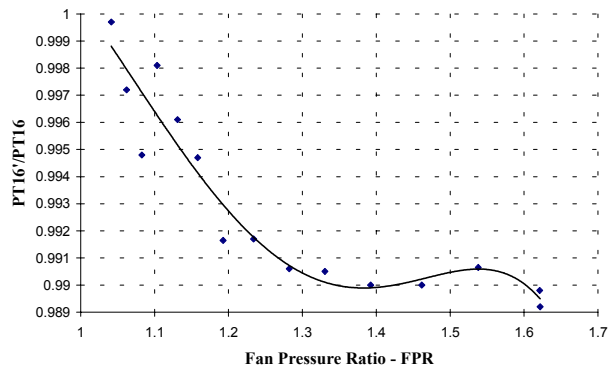


Figure 7. Fan Pressure Correlation Curve - Residual Error Method

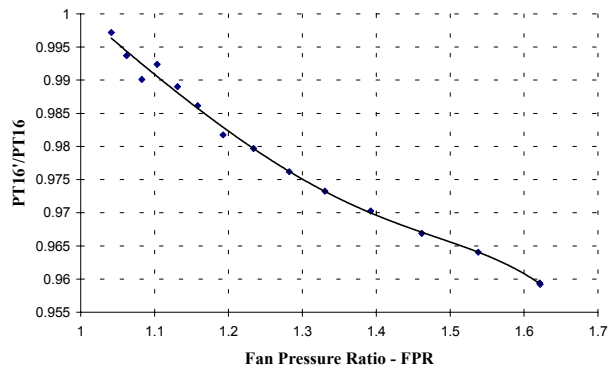


Figure 8. Fan Pressure Correlation Curve - Specific Thrust Method

### 7.4 In-Flight Thrust Results

This section will discuss actual in-flight results. As requested by the aircraft manufacturer, real values of parameters were normalized by reference values. Some parameters must be carefully selected as indicators of steady state flight conditions. In this work, altitude and speed were selected as indicators, and static air temperature and inter-stage turbine temperature (ITT or T45) were the engine parameters. Figure 9, 10 11 and 12 show results of the behavior of selected parameters during the flight.

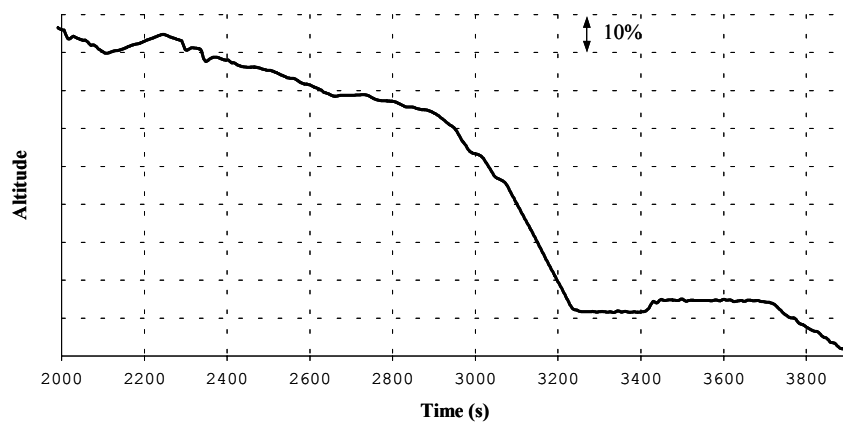


Figure 9. Analysis of Stabilization - Altitude

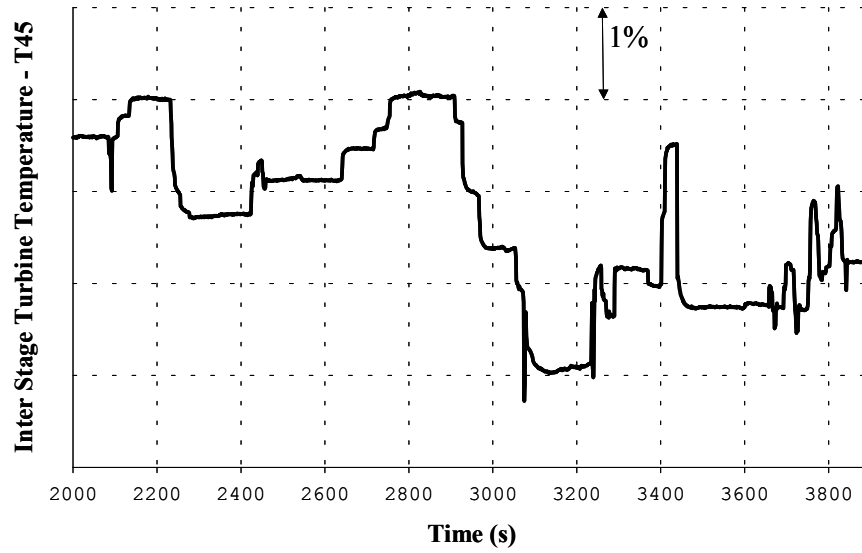


Figure 10. Analysis of Stabilization - Inter-stage Turbine Temperature

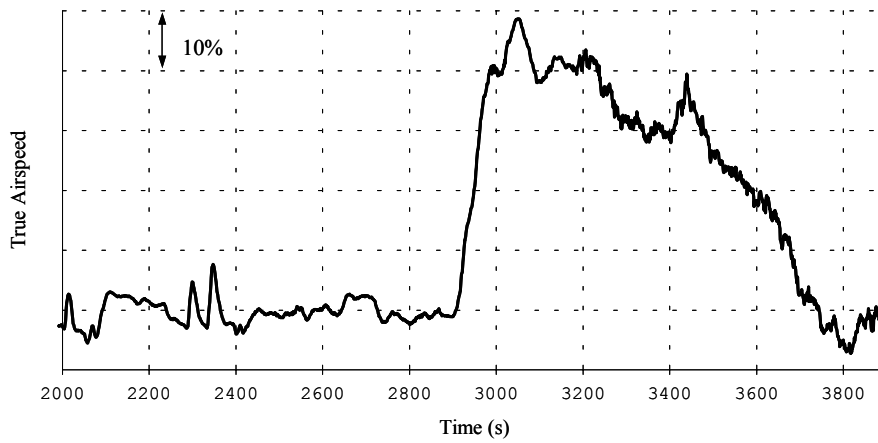


Figure 11. Analysis of Stabilization - True Air Speed

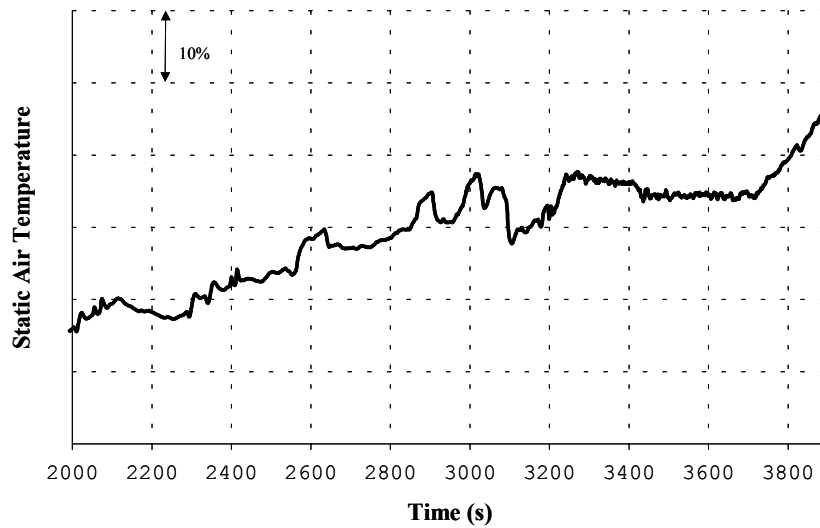


Figure 12. Analysis of Stabilization - SAT

For this flight, two windows were selected, the first one, called CASE 1, is between 2150 seconds and 2230 seconds, and the second one, called CASE 2, is between 3300 seconds and 3350 seconds. Due to limitation of number of pages, flight test results for other parameters are not presented in this report.

## 7.5 Net Thrust Results

Net thrust and uncertainty analysis results are provided by Table 2..

Table 2. Net Thrust and Uncertainty Results

Method	Case 1	Case 2
$F/A_P$	14234 ± 141 N	6428 ± 239 N
$F/M\sqrt{T}$	14049 ± 141 N	6120 ± 239 N
Residual Error	14195 ± 154 N	6167 ± 216 N
Specific Thrust	13972 ± 152 N	5893 ± 212 N

## 8. Conclusions

This work provided background theory involved in in-flight thrust determination and uncertainty analysis. An application to a specific program was developed and applied practically. Efforts were concentrated on a turbofan engine. Although the methods are very different from each other, the results in Table 2 show good agreement both on thrust and uncertainty limits, particularly at CASE 1 flight conditions.

The comparison between Specific Thrust and Residual Error methods exposed a systematic difference in thrust. The characteristics of the determination of the Fan Pressure Correlation curve makes that the Residual Error always calculates a higher by-pass pressure that calculated by the Specific Thrust Method. This causes the Residual Error method to output higher gross thrust than the Specific Thrust method. If these methods use the same mass-flow determination procedure, then one may expect the Residual Error method to provide net thrust always greater than the one provided by the Specific Thrust method.

The results show that significant net thrust difference over four different thrust determination methods, that is, shows method bias. Therefore, for every IFTD flight test campaign there should be always more than one IFTD method implemented. This method redundancy is important to increase confidence in aircraft drag, performance assessment, to expose bias between methods, and provide more realistic performance uncertainty limits to compare wind-tunnel aircraft-model performed.

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