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LAMINAR NATURAL CONVECTION IN CONCENTRIC AND ECCENTRIC ANNULI

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Abstract. *Steady laminar natural convection in two-dimensional concentric and eccentric horizontal annuli cavity isothermally heated from the inner cylinder and cooled from the outer cylinder is numerically analyzed using the finite volume method. Benchmark results for laminar flow are compared with similar numerical solutions by Cho et al, (1982)(JHT,104), Kenjeres & Hanjalic, (1995)(IJHFF,16) and the experimental data of Kuehn & Goldstein, (1978)(JHT,100). Governing equations are written in terms of primitive variables and are recast into a general form. The isotherms and streamlines for annuli of the same eccentricity, but located at different angular positions are presented for* $Ra_L=10^4$ *and* $R_i/R_o=0.3846$, *where* R_o *and* R_i *are the cylinder radii and* Ra_L *is the Rayleigh number based on a characteristic length* R_o - R_i.

Keywords: *Natural Convection, Horizontal Cylindrical Anulli, Generalized Coordinate*

1. Introduction

The analysis of buoyancy-driven flow between eccentric annulus cylinders is a problem which currently receives considerable attention from researchers in many fields of applications. The design of furnaces, in the operation of solar collectors, which contribute to energy losses minimization to increase collector efficiency, nuclear reactor insulation, the determination of the requirements for aircraft cabin insulation are some examples of applications.

Natural convection occurs as a result of gradients in density, which are in turn due to variations in temperature or mass concentration. Natural convection in a infinite horizontal layers of fluid heated from below has received extensive attention since beginning of 20th century, when Bérnard, (1901) observed hexagonal roll cells upon the onset of convection in molten spermaceti with a free upper surface.

The work of Rayleigh, (1926) was the first to compute a critical value, Ra_c, for the onset of convection. The accepted theoretical value of this dimensionless group is 1708 for rigid upper and lower surfaces.

The study of natural convection in enclosures still attracts the attention of researchers and a significant number of experimental and theoretical works have been carried out mainly from the 70's.

The first basic study on natural convection in cylindrical annuli was carried out by Beckman, (1931) and extended by Kraussold, (1934).

A very comprehensive analysis has been made on the concentric annuli by Kuehn & Goldstein, (1978). They have conducted both numerical simulation using finite elements technique and experimental study using Mach-Zehnder interferometer. Application of other type of finite differences method with ADI numerical solution has also reported by Charrier-Mojtabi et al, (1979) in solving the laminar horizontal concentric annuli problem formulated in cylindrical polar coordinates.

Small eccentric annuli has performed in the work of Yao, (1980), using an expansion in terms of the double series of eccentricity and Rayleigh number for small values of Ra. The work of Cho et al, (1982) extended the knowledge on the natural convection heat transfer in the horizontal cylindrical annuli, numerical analysis has been made using finite difference method in a bipolar coordinate system based on successive over-relaxation iteration.

Recent work of Kenjeres & Hanjalic, (1995), reports on the modeling and computational study of the natural convection in concentric and eccentric annuli by means of several variants of the algebraic model, based on the expression for turbulent heat flux $\overline{\theta u_i}$ obtained by truncation of the second-moment transport equation for this correlation. Various levels of closure included the low-Re number form of the k-ε model, but also a version in which the differential transport equations are solved for the temperature variance $\overline{\theta^2}$ and its decay rate ε_θ.

Previous simulations of buoyant heat transfer in clear vertical and horizontal square cavities (Braga & de Lemos, (2002a)) as well as in annuli filled with porous material have been presented (Braga & de Lemos, (2002b)). This work extends previous analyses on buoyant flows considering now the case of eccentric annuli. Results are compared with numerical and experimental data available in the literature.

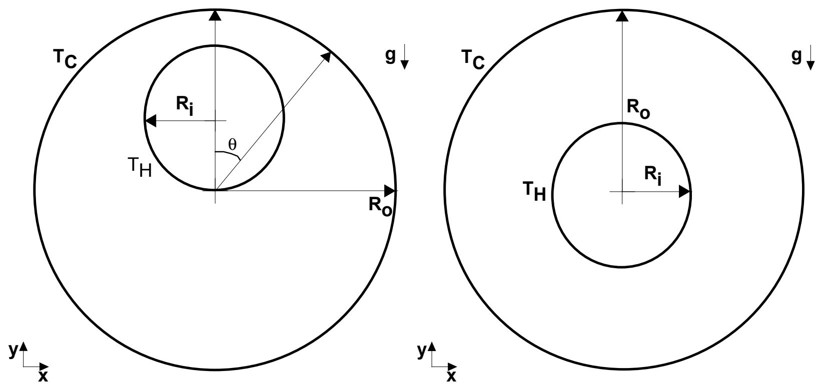


Figure 1. The eccentric and concentric annuli under consideration.

2. The Problem Considered

The problem considered is shown schematically in Fig. (1), and refers to the two-dimensional flow of a Boussinesq fluid of Prandtl number 0.71 in concentric and eccentric horizontal annulus of outer radius R_o and inner radius R_i with a eccentricity *e*=0.623 and R_i/R_o=0.3846 for both eccentric and concentric case. The cavity is assumed to be infinite depth along z-axis and is isothermally heated from the inner cylinder and cooled from outer cylinder

3. Governing Equations

For Steady flow, the equations for continuity, momentum and temperature take the form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\nabla^2 u \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\nabla^2 v+g\beta(T-T_o) \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\nabla^2 T \tag{4}$$

where u and v are the velocity components in x and y directions respectively and ρ is the density of the fluid. P is the total pressure and ν is the kinematic viscosity of the fluid. The gravity acceleration is defined by g and β is the thermal expansion coefficient. T and T_o is the temperature and the reference temperature and α is the thermal diffusivity. Finally L is a characteristic length. The no-slip condition is applied on the velocity and the resulting flow is treated as steady and, depending on the Rayleigh number defined as $\frac{g\beta L^3\Delta T}{\nu\alpha}$ with L equal to (R_o-R_i) and the eccentricity *e* defined as the ratio between the distance of the centers of the circular cylinders and the radius difference of the two cylinders (R_o - R_i).

4. The Numerical Method and Solution Procedure

The numerical method employed for discretizing the governing equations is the control-volume approach with a collocated grid. A hybrid scheme, Upwind Differencing Scheme (UDS) and Central Differencing Scheme (CDS), is used for interpolating the convection fluxes. With the help of the Fig. (2), the following operators can be identified:

$$\Delta x_\eta^e=(x_{ne}-x_{se}),\ \Delta y_\eta^e=(y_{ne}-y_{se}) \tag{5}$$

$$\Delta x_\xi^n=(x_{ne}-x_{nw}),\ \Delta y_\xi^n=(y_{ne}-y_{nw}) \tag{6}$$

The vector form of the area of the control-volume at *east* and *north* faces, respectively, are given by

$$\vec{A}_e=\Delta y_\eta^e\vec{i}-\Delta x_\eta^e\vec{j}\ ,\ \vec{A}_n=-\Delta y_\xi^n\vec{i}+\Delta x_\xi^n\vec{j} \tag{7}$$

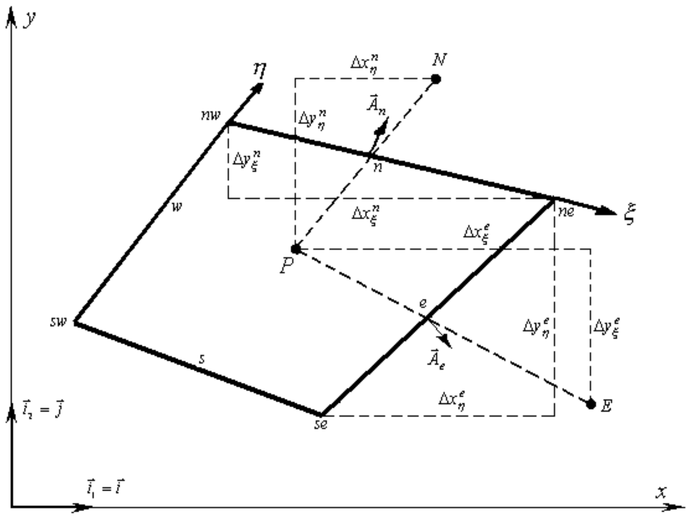
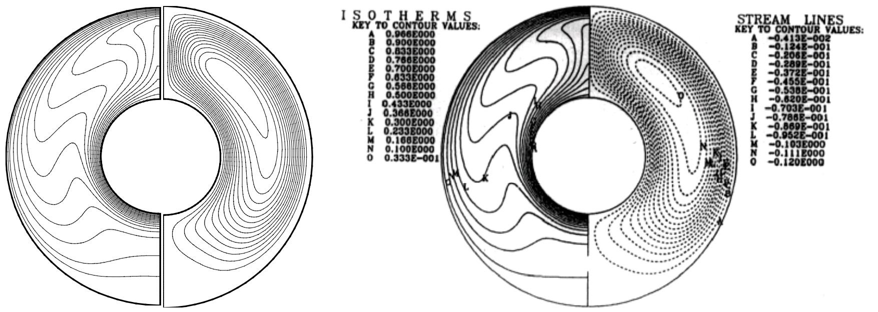


Figure 2. Control-volume and notation.

The well-established SIMPLE, Patankar & Spalding, (1972), is followed for the momentum equations which is solved by a line-by-line procedure namely SIP, Stone, (1968). The generalized coordinate transformation and the process of discretization is fully documented in the work of Pedras & de Lemos, 2001.



(a) (b)

Figure 3. (a): Isotherms and Streamlines for a laminar flow in a concentric annulus with Ra_L=5x10^4, R_i/R_o=0.3846, (b): Kenjeres & Hanjalic, (1995)

5. Results and Discussion

5.1 Laminar flow in concentric anulli

Numerical computations was performed for concentric annulus using a grid with 50x50 (CV), considering only half domain. Selected were those cases investigated experimentally by Kuehn & Goldstein, (1978), in this case at Ra_L=5x10^4. The results show good agreement with the experiments both qualitatively and quantitatively. The qualitatively agreement is observed when flow patterns are compared with the interferometer results, (Fig. 2 in the paper of Kuehn & Goldstein, (1978)) and the numerical result of Kenjeres & Hanjalic, (1995) like shown in Fig. (3), exhibiting identical behavior. The quantitatively agreement is shown in Fig. (4) in the distribution of the temperature.

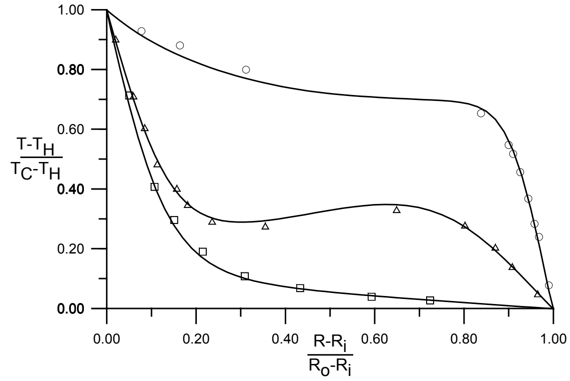


Figure 4. Temperature behavior for laminar regime in a concentric annulus, R_i/R_o=0.3846, Ra_L=5x10^4, Symbols: experiments of Kuehn & Goldstein, (1978), O $\Rightarrow$ θ=0^o, $\Delta \Rightarrow \theta$=90^o, □ $\Rightarrow$ θ=180^o

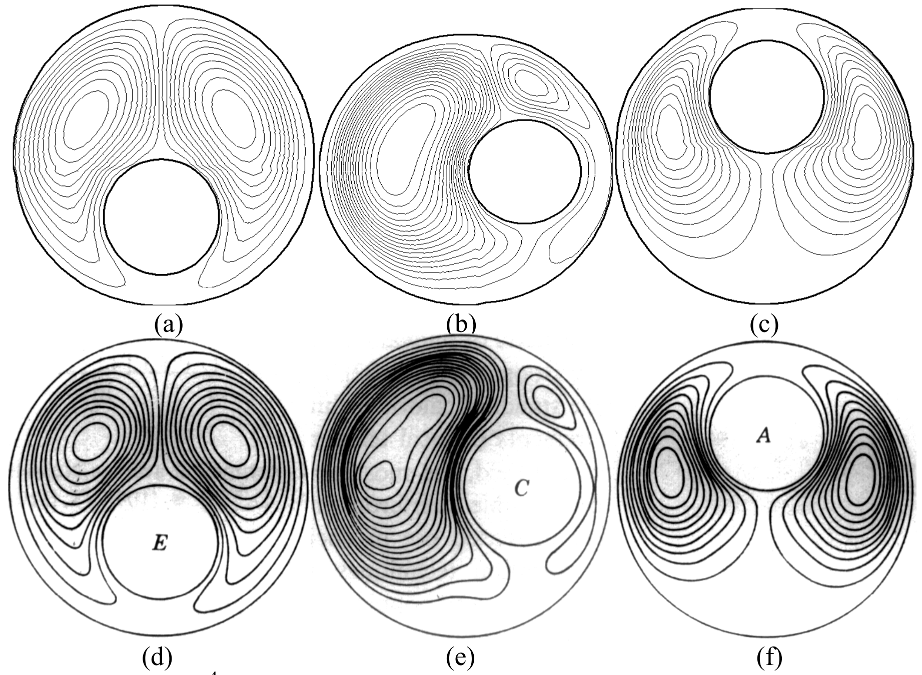


Figure 5. Streamlines for ($Ra_L=10^4$, $R_i/R_o=0.3846$, $e=0.623$), Present Results, (a), (b) and (c), Cho et al, (1982), (d), (e) and (f)

5.2. Laminar flow in eccentric annulus

Calculations were performed using a grid with 40x60 control volumes (CV). Figure (5) shows the streamlines of a eccentric annuli heated from the inner cylinder and cooled from outer cylinder for $Ra_L=10^4$, $e=0.623$ and for three diferent angles, 180°, 90°, 0°, respectively. The figures show a good agreement with the work of Cho et al, (1982) and reproduce the basic features of the flow. The use of generalized coordinate instead of bipolar coordinates leads to some distortions in the streamlines as can be seen clearly in the Fig. (5a). In the narrowest part of the gap, the conduction is dominant has seen in the isotherm plots. Also, in the streamlines contours, more fluids is mobilized in the convection currents with increase θ to deliver thermal energy from the inner heated cylinder to the cooled outer cylinder. The positional influence on the heat transfer is felt more strongly from the isotherm plots than from the streamlines, since

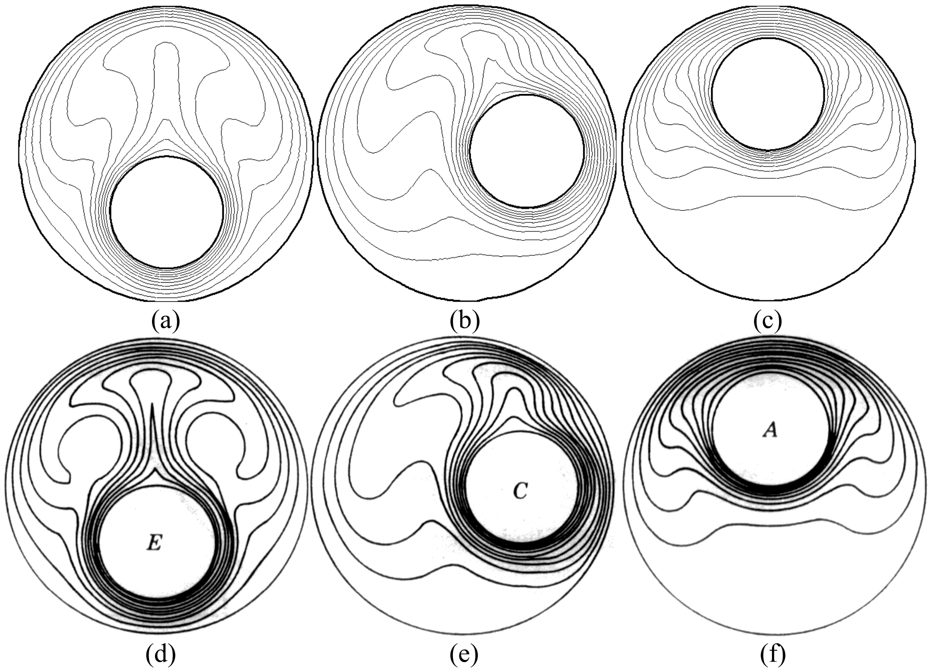


Figure 6. Isotherms for $Ra_L=10^4$, $R_i/R_o=0.3846$, $e=0.623$. Present Results, (a), (b) and (c); Cho et al, (1982), (d), (e) and (f).

the temperature inversion phenomenon becomes very distinguished as θ is increased.

Figure (5a) depicts lines of constant stream function for the case of $\theta=180^o$. The convective cell is both larger and stronger and the effect of the more favorable geometry is clearly evident. Figure (5b) shows a case with $\theta=90^o$ and the convection is more pronounced in the left side where the enclosure is not obstructed. Finally, Fig. (5c) shows a picture with $\theta=0^o$. In this case the convective cell center has moved towards and the cell is also less powerful than others ones. Clearly, this geometry inhibits convective motion in the fluid. In all cases, the motion of fluid in the right hemisphere is clockwise. Plots of the temperature fields are shown in Fig. (6). Figure (6a) consider a $\theta=180^o$ and shows that the temperature inversion and the thermal plume are more pronounced than in concentric cases. The two cylinders are very close to each other and the effects of convection are diminished, and the conduction is dominant in that area for all configurations. Figure (6c) shows a $\theta=0^o$ case. The less favorable geometry between the two cylinders inhibits convection. Accordingly Yao, (1980), the isotherms show smaller temperature inversions when compared to the concentric case. The thermal plume is absent and conduction is the dominant heat transfer mechanism at the top. The bottom of the annulus exhibits an enlarged stagnant region of fluid. Consequently, the isotherms are not as uniformly spaced as they would be if conduction were the dominant heat transfer mechanism in this region.

Finally, as a last comment one should say that, unfortunately, experimental temperature distributions for horizontal eccentric annuli are not easily found in the literature. For that, detailed comparison with numerical simulations is still a challenge task for this particular geometry.

6. Conclusion

This paper presented computations for simulation of laminar natural convection in concentric and eccentric annuli heated on the inner cylinder and cooled at the outer surface. The present results showed grid independency with the grid used in the calculations. The results yielded generally satisfactory agreement with the experimental data and also in the distribution of the temperature at several cross sections. The computed flow pattern generally agrees with that detected by optical and others experimental methods. For all case considered, there is a stagnant region below the inner cylinder. Finally, the study showed that such model is able to yield satisfactory predictions of major flow details relevant to industrial computations.

9. Acknowledgments

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