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ANALYTICAL MODEL OF TWO-DIMENSIONAL TEMPERATURE DISTRIBUTION OVER A SINGLE ELECTRONIC CIRCUIT BOARD

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Abstract. *The thermal model of an electronic circuit board with installed heat dissipating components is presented as a two-dimension steady-state heat conduction problem with multiple sources distributed on a rectangular region. The corresponding energy equation includes a source term as well as a temperature-dependent term to account for linear heat transfer in z-direction. Boundary conditions are of first type with unique temperature along the perimeter. The integral-transform technique was applied to obtain closed-form integral solution. Assuming that all dissipated components have a rectangular contact area with the plate, multiple integrals for each dissipated sub-region are easily found. A temperature map over the plate is calculated from the closed expression with triple sums of series with respect to each coordinate and each source. The error was evaluated by the estimation of truncated terms values. A generalization of this solution is discussed. The solution was applied to find the temperature distribution over the electronic Driver Plate of the CEP block of the CIMEX Brazilian experiment planned to fly onboard a Space Shuttle mission as a Get Away Special payload.*

Keywords*. Temperature distribution, hot spot, integral transform, circuit board*

1. Introduction

The general trend in electronic devices development and production is minimization together with simultaneous increase of dissipated power density. It emphasizes the importance of an accurate thermal analysis throughout the design process. The circuit printed plate is a basic element of many electronic devices, and a correct prediction temperature map over it will provide evaluation of temperature of each component as well as its junction.

In equipment for Space application, these plates are often assembled in packages where these plates are fitted over the perimeter to a structural frame having a good thermal contact to a base plate. The later yields a thermo-mechanical interface to a surface whose temperature is maintained by onboard thermal control sub-system. The frame structure provides a thermal conductive path from dissipated components to the interface acting here as a heat sink. External surfaces of the package box are usually covered with multi-layer insulation to block the external radiative flux extremes at Space conditions. Before launch, as well as during assembling and testing phases, the electronic equipment is submitted to many functional tests. Under such tests, the convective portion of heat transfer contributes to the cooling of electronic plates and, finally, overall temperatures of the components usually are more favorable that under vacuum condition.

Extracted from this brief depiction, the problem under investigation is the following: prediction of steady-state temperature distribution over a plate with multiple spot-type heat dissipations under Dirichlet (1^{st} type) boundary conditions over the perimeter and with additional heat removal by convection in orthogonal direction.

A straightforward approach can be a numerical simulation with a fine grid capable to fit all multiple dissipated components of different dimensions. Finite differences, finite elements or boundary elements techniques can be applied. An alternative approach is the creation of an analytical model of temperature distribution. The analytical approach versus numerical one can give a closed-form expression, which is simple to use in engineering practice and is ready to imbedded in a higher-level analyzer. Beside that, the analytical solution can give more valuable information on peculiarities of the plate thermal design and can give the proper physical insight into interaction of different parameters.

Haque et al, in 1999 used a Fourier series method to obtaining steady-state temperature mapping over the power electronic building block processor. The method, in spite of being analytical, is realized by the TAMS-A software, developed by Ellison in 1990.

Green's Function technique (Beck, 1992) is very suitable for transient problems, whereas for steady-state problems the integral transformations seem more adequate. Pesare at al (2001) found an analytical model for 3-D package under several assumptions. A 2-D Fourier transform was applied to simplify the initial equation to the second-order ordinary differential equation. Temperature dependence of thermal conductivity was shifted to the boundary conditions by Kirchoff transformation and then the 1^{st} order Taylor expansion was applied to the inverse transform. Spatially distributed heat loads from components were approximated as a set of elementary point sources.

Culham et al in 2000 utilized the Fourier series technique for Laplace statement of heat equation under uniform boundary conditions to obtain a temperature distribution throughout a multi-layer electronic package with rectangular geometries. Package-board and layer-layer interfaces were considered.

Shukla in 2001 developed the 3-D thermal model of a circuit board with discrete surface heat sources by integral transform technique. The solution was compared to the numerical one obtained from the finite element method.

The present work continues the efforts to quest the analytical solutions for the temperature map over plates with multiple sources, staying within limits of a 2-D type model. An emphasis is made upon clarity of utilization, seeking of a simple procedure for evaluation of convection contribution and obtaining analytical expression for the upper error limit introduced by series truncation. The integral transform technique, developed and generalized by Özisik (1980), Mikhailov (1984) and Cotta (1993), was taken as a baseline for this study rather than Fourier series methods.

2. Analytical solution in 2-D domain

The main assumptions made are the following. First, electronic components have a good thermal contact with the base plate. Second, thermal conductivity along the plate area is homogeneous. Third, the heat transfer coefficient for convective heat exchange in orthogonal direction is uniform over the whole area of the plate. Forth, temperatures on boundaries over perimeter of the plate are prescribed and uniform. Fifth, spot heat sources, representing electronic components, have rectangular shape.

Under these assumptions, the heat equation can be expressed in vector form in the following way

$$\begin{aligned} &\delta k \nabla^2 T(\mathbf{r}) + q(\mathbf{r}) - h_z (T(\mathbf{r}) - T_\infty) = 0 \quad \text{in region R} \\ &T(\mathbf{r}) = T_0 \;\; \text{on boundaries} \end{aligned} \tag{1}$$

Where q(r) is the distributed density of heat loads from components (W/m^2)

When rewriting the equations in rectangular coordinates with respect to a new variable

$$\Delta T(\mathbf{r}) = T(\mathbf{r}) - T_0 \,, \tag{2}$$

one can obtain

$$\begin{aligned} &\delta k \frac{\partial^2 \Delta T}{\partial x^2} + \delta k \frac{\partial^2 \Delta T}{\partial y^2} + q(x,y) - h_z (\Delta T - \Delta T_\infty) = 0 \quad \text{in region R} \\ &\Delta T = 0 \;\; \text{on boundaries} \end{aligned} \tag{3}$$

Where

$$\Delta T_\infty = T_\infty - T_0 \tag{4}$$

Note, that the last equation has homogeneous boundary conditions of 1st type.

Assuming the related multidimensional Sturm-Liouville problem is separable in the associated independent variables, i.e.

$$\psi(x,y) = X(x)Y(y); \tag{5}$$

the corresponding pair of auxiliary eigenvalue problems can be written as

$$\frac{\partial^2 X}{\partial x^2} + \beta^2 X = 0; \qquad \frac{\partial^2 Y}{\partial y^2} + \gamma^2 Y = 0; \tag{6}$$

under homogeneous Dirichlet conditions.

The integral-transform pair (Ozisik, 1980) is expressed as

$$\text{Integral transform:}\quad \Delta\bar{T}_{mn} = \Delta\bar{T}(\beta_m,\gamma_n) = \int_{x=0}^{a}\int_{y=0}^{b} X(\beta_m,x)Y(\gamma_n,y)\Delta T(x,y)dxdy$$
$$\text{Inversion formula:}\quad \Delta T(x,y) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{X(\beta_m,x)Y(\gamma_n,y)}{N(\beta_m)N(\gamma_n)}\Delta\bar{T}(\beta_m,\gamma_n) \tag{7}$$

The normalization integrals

$$N(\beta_m) = \int_{x=0}^{a}[X(\beta_m,x)]^2 dx; \qquad N(\gamma_n) = \int_{y=0}^{b}[Y(\gamma_n,y)]^2 dy. \tag{8}$$

For the homogeneous BC of 1st type, all eigen components are known (Ozisik, 1980)

$$X(\beta_m,x) = X_m = sin(\beta_m x) \quad N(\beta_m) = \frac{a}{2}; \quad \beta_m = \frac{m\pi}{a};$$
$$Y(\gamma_n,y) = Y_n = sin(\gamma_n y) \quad N(\gamma_n) = \frac{b}{2}; \quad \gamma_n = \frac{n\pi}{b}; \tag{9}$$

Where ***a*** and ***b*** are the dimensions of the plate in x and y directions correspondently; m and n – integer numbers.

Thus, the inversion

$$\Delta T(x,y) = \frac{4}{ab}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} sin\left(m\pi\frac{x}{a}\right)sin\left(n\pi\frac{y}{b}\right)\Delta\bar{T}(\beta_m,\gamma_n) \tag{10}$$

provides the base for closed analytical expression for the temperature distribution.

In spite of the alternative approach, suggested by Mikhailov et al in 1984 for multi-dimension problems, it could give only one infinite sum in this case, the straight approach, that involves double sums, was selected instead due to its symmetry, clarity and simplicity of integrating out the rectangular heat spot sub-domains.

Let multiply the Eq. (3) by $X_m Y_n$ and integrate over the area of the plate A:

$$\delta k\int_A X_m Y_n \frac{\partial^2\Delta T}{\partial x^2}dxdy + \delta k\int_A X_m Y_n \frac{\partial^2\Delta T}{\partial y^2}dxdy + \int_A X_m Y_n q(x,y)dxdy -$$
$$h_z\int_A X_m Y_n \Delta T dxdy + \Delta T_\infty h_z\int_A X_m Y_n dxdy = 0 \tag{11}$$

The first two terms can be evaluated by making use of the Green’s theorem in its particular case of homogeneous boundary conditions

$$\delta k\int_A X_m Y_n \frac{\partial^2\Delta T}{\partial x^2}dxdy + \delta k\int_A X_m Y_n \frac{\partial^2\Delta T}{\partial y^2}dxdy = \delta k\int_A \Delta T Y_n \frac{\partial^2 X_m}{\partial x^2}dxdy + \delta k\int_A \Delta T X_m \frac{\partial^2 Y_n}{\partial y^2}dxdy \tag{12}$$

Let multiply (Eq.6), expressed for each eigenvalue, by ΔT, Y then X and integrate over the same region:

$$\int_A \Delta T Y_n \frac{\partial^2 X_m}{\partial x^2}dxdy + \beta^2\int_A \Delta T X_m Y_n dxdy = 0; \qquad \int_A \Delta T X\frac{\partial^2 Y_n}{\partial y^2}dxdy + \gamma^2\int_A \Delta T X_m Y_n dxdy = 0; \tag{13}$$

Combining, one obtains

$$\int_A X_m Y_n \frac{\partial^2 \Delta T}{\partial x^2} dxdy + \int_A X_m Y_n \frac{\partial^2 \Delta T}{\partial y^2} dxdy = -(\beta_m^2 + \gamma_m^2) \int_A \Delta T X_m Y_n dxdy = -(\beta_m^2 + \gamma_m^2) \Delta \overline{T}_{mn} \tag{14}$$

Now, the original heat equation can be re-written for the transformed temperature difference

$$-\delta k(\beta_m^2 + \gamma_m^2) \Delta \overline{T}_{mn} + \int_A X_m Y_n q(x,y) dxdy - h_z \Delta \overline{T}_{mn} + h_z \Delta T_\infty \int_A X_m Y_n dxdy = 0 \tag{15}$$

It is a simple algebraic linear equation with regard to the transformed variable.

Let perform the integration of rest source terms. The integral of eigenfunction is

$$\int_A X_m Y_n dxdy = \int_{x=0}^{a} \int_{y=0}^{b} sin\left(m\pi \frac{x}{a} \right) sin\left(n\pi \frac{y}{b} \right) dydx = ab \frac{(1-\cos(m\pi))(1-\cos(n\pi))}{mn\pi^2} = \frac{ab}{\pi^2} \omega_{mn}$$

$$\omega_{mn} = \frac{1-(-1)^m - (-1)^n + (-1)^{m+n}}{mn} \tag{16}$$

Density distribution function is assumed to be discrete and homogeneous over its rectangular region. For each j-th heat load with coordinates (ξ_{1j}, ζ_{1j}) for its left bottom corner and (ξ_{2j}, ζ_{2j}) for its right upper corner, the density is

$$q_j(x,y) = \begin{cases} q_j & \text{for } x \in [\xi_{1j}, \xi_{2j}] \cap y \in [\zeta_{1j}, \zeta_{2j}] \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

and

$$q(x,y) = \sum_{j=1}^{J} q_j(x,y) \tag{18}$$

The integral over J heat sources is

$$\int_A X_m Y_n q(x,y) dxdy = \sum_{j=1}^{J} q_j \int_{x=\xi_{1j}}^{\xi_{2j}} \int_{y=\zeta_{1j}}^{\zeta_{2j}} sin\left(m\pi \frac{x}{a} \right) sin\left(n\pi \frac{y}{b} \right) dydx = \sum_{j=1}^{J} \frac{ab}{\pi^2} q_j \Omega_{mnj}$$

$$\Omega_{mnj} = \frac{1}{mn} \left[\cos\left(m\pi \frac{\xi_{1j}}{a} \right) - \cos\left(m\pi \frac{\xi_{2j}}{a} \right) \right] \left[\cos\left(n\pi \frac{\zeta_{1j}}{b} \right) - \cos\left(m\pi \frac{\zeta_{2j}}{b} \right) \right] \tag{19}$$

Now, the integral transform for temperature difference can be written

$$\Delta \overline{T}(\beta_m, \gamma_n) = \frac{a^3 b^3}{\pi^2 (k\delta\pi^2 (m^2 b^2 + n^2 a^2) + a^2 b^2 h_z)} \left[h_z \Delta T_z \omega_{nm} + \sum_{j=1}^{J} q_j \Omega_{jmn} \right] \tag{20}$$

Finally, the 2-D solution is the following

$$\Delta T(x,y) = \frac{4a^2 b^2}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{sin\left(m\pi \frac{x}{a} \right) sin\left(n\pi \frac{y}{b} \right)}{\left[k\delta\pi^2 \left(m^2 b^2 + n^2 a^2 \right) + a^2 b^2 h_z \right]} \left(h_z \Delta T_z \omega_{nm} + \sum_{j=1}^{J} q_j \Omega_{jmn} \right) \tag{21}$$

It yields the temperature map over a plate as a closed analytical function of all design parameters.

3. Convection and conduction contributions

The contribution of convection and conduction portions of heat transfer can be acquired from the obtained solution for each electronic component as well as for the entire plate. The convection portion can be obtained by integration over the surface of interest. The conduction portion can be defined through integration of thermal gradients along a closed contour. These calculations provide additional means for estimation of truncation errors in term of heat flux values through verification of overall balance

By selecting this contour fitted to the bounds of j-th heat source, the conductive portion (in W) can be defined as a sum of fluxes for each of the four sides:

$$Q_{kj} = k\delta\left(\int_{\zeta_{1j}}^{\zeta_{2j}} \left.\frac{\partial \Delta T}{\partial x}\right|_{x=\xi_{1j}} dy - \int_{\zeta_{1j}}^{\zeta_{2j}} \left.\frac{\partial \Delta T}{\partial x}\right|_{x=\xi_{2j}} dy + \int_{\xi_{1j}}^{\xi_{2j}} \left.\frac{\partial \Delta T}{\partial y}\right|_{y=\zeta_{1j}} dx - \int_{\xi_{1j}}^{\xi_{2j}} \left.\frac{\partial \Delta T}{\partial y}\right|_{y=\zeta_{2j}} dx \right) \quad (22)$$

The convection portion is obtained by integration over the appropriate surface:

$$Q_{hj} = h_z \int_{\xi_{1j}}^{\xi_{2j}} \int_{\zeta_{1j}}^{\zeta_{2j}} \Delta T dy dx - h_z A_j (T_\infty - T_0) \quad (23)$$

The relative error in terms of heat flux for each electronic component is defined as

$$\delta\overline{Q}_j = \left| 1 - \frac{Q_{kj}}{Q_j} - \frac{Q_{hj}}{Q_j} \right|$$

The solution re-written in the compact form is:

$$\Delta T(x,y) = \frac{4}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{mn} sin\left(m\pi \frac{x}{a} \right) sin\left(n\pi \frac{y}{b} \right) \quad (24)$$

where

$$\Phi_{\text{mn}} = \frac{a^2 b^2}{\left[k\delta\pi^2 \left(m^2 b^2 + n^2 a^2 \right) + a^2 b^2 h_z \right]} \left(h_z \Delta T_z \omega_{nm} + \sum_{j=1}^{J} q_j \Omega_{jmn} \right) \quad (25)$$

Using this function and performing differentiation-integration by Eq. (22), one can obtaine

$$Q_{kj} = \frac{4k\delta}{\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \Phi_{mn} \frac{(a^2 n^2 + b^2 m^2)}{abmn} \cdot \left[\cos\left(m\pi \frac{\xi_{1j}}{a} \right) - \cos\left(m\pi \frac{\xi_{2j}}{a} \right) \right] \left[\cos\left(n\pi \frac{\zeta_{1j}}{b} \right) - \cos\left(m\pi \frac{\zeta_{2j}}{b} \right) \right] \quad (26)$$

Or, using the previously defined function Ω, Eq.(19), the final expression is

$$Q_{kj} = \frac{4k\delta}{ab\pi^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} (a^2 n^2 + b^2 m^2) \Phi_{mn} \Omega_{mnj} \quad (27)$$

The overall balance of conduction portion over the whole plate gives a similar result

$$Q_k = \frac{4k\delta}{ab\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(a^2n^2 + b^2m^2)\Phi_{mn}\omega_{mn} \tag{28}$$

The corresponding convection portions are the following:

$$Q_{hj} = \frac{4abh_z}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Phi_{mn}\Omega_{mnj} - h_z A_j (T_\infty - T_0) \tag{29}$$

$$Q_h = \frac{4abh_z}{\pi^4}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\Phi_{mn}\omega_{mn} - h_z A(T_\infty - T_0) \tag{30}$$

The relative error in terms of heat flux for the whole plate is defined as

$$\delta\overline{Q} = \left| 1 - \frac{Q_k + Q_h}{\sum_{j=1}^{J} Q_j} \right| \tag{31}$$

A separate evaluation of the contribution of different mechanisms of heat transfer to the overall heat balance gives the designer an important information about how to evaluate the efficiency of adopted means of thermal control.

4. Error supremum

Now we will estimate the residue of the series to obtain (conservative) upper limit of the error due to series truncation. Expression for the error of truncation is the following

$$\delta^*(\Delta T(x,y)) = \frac{4}{\pi^2}\sum_{m=M}^{\infty}\sum_{n=N}^{\infty}\tilde{\Phi}_{mn} \tag{32}$$

where

$$\tilde{\Phi}_{mn} = \Phi_{mn} sin\left(m\pi\frac{x}{a}\right) sin\left(n\pi\frac{y}{b}\right) \tag{33}$$

and (M-1) and (N-1) – number of terms in corresponding finite series.

For relatively large M and N the estimation of the supreme of Φ can be accomplished by setting terms of nominator to their maximum and the terms of denominator to their minimum:

$$\sup(\omega_{mn}) = \frac{2}{mn}; \quad \sup(\Omega_{mnj}) = \frac{4}{mn}; \tag{34}$$

Taking into account that max(Sin())=1, it is possible to obtain

$$\sup(\tilde{\Phi}_{mn}) = \frac{\left(2h_z\Delta T_z + 4\sum_{j=1}^{J} q_j\right)}{k\delta\pi^2 nm\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)} \tag{35}$$

Now the double sum can be evaluated

$$\sum_{m=M}^{\infty}\sum_{n=N}^{\infty}\tilde{\Phi}_{mn} < \sum_{m=M}^{\infty}\sum_{n=N}^{\infty}\sup(\tilde{\Phi}_{mn}) = \frac{a^2}{k\delta\pi^2}\left(2h_z\Delta T_z + 4\sum_{j=1}^{J}q_j\right)\Psi$$
$$\Psi = \sum_{m=M}^{\infty}\sum_{n=N}^{\infty}\frac{1}{nm\left(m^2+r^2n^2\right)} \qquad (36)$$

where a – largest board side size and r=a/b.

As soon as r≥1, a conservative estimation can be obtained, using the geometric inequality

$$\frac{1}{nm\left(m^2+r^2n^2\right)} < \frac{1}{nm\left(m^2+n^2\right)} \le \frac{1}{2m^2n^2} \qquad (37)$$

Now the sum of double infinite series can be expressed via Euler Gamma functions

$$\Psi \le \sum_{m=M}^{\infty}\sum_{n=N}^{\infty}\frac{1}{m^2n^2} = \gamma(M,N)$$
$$\gamma(M,N) = \frac{\left(\Gamma'(M)^2 - \Gamma(M)\Gamma''(M)\right)\left(\Gamma'(N)^2 - \Gamma(N)\Gamma''(N)\right)}{\Gamma(M)^2\,\Gamma(N)^2} \qquad (38)$$

where

$$\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt; \qquad \Gamma'(z) = (z-1)\int_0^{\infty} t^{z-2}e^{-t}dt; \qquad \Gamma''(z) = (z^2-3z+2)\int_0^{\infty} t^{z-3}e^{-t}dt; \qquad (39)$$

The presented Euler Gamma functions are incorporated in the Mathematica package, Wolfram, 1991, or can be easily coded in accordance with Press, 1992.

The final expression for the truncation error is the following:

$$\sup\delta^* = \frac{2a^2}{k\delta\pi^4}\left(2h_z\Delta T_z + 4\sum_{j=1}^{J}q_j\right)\gamma(M,N) \qquad (40)$$

This is a very important result, clearly showing how design parameters can have an influence the introducing error because of series truncation.

Some values of the function γ are presented in the table

Table 1. Values of the function γ

M	5	10	20 (10)	20	30	50	100
N	5	10	10 (20)	20	30	50	100
γ	0.024	0.0055	0.0027	0.0013	0.00057	0.00020	0.000051

For typical values k=5 W/m/K, δ=0.001 m, h_z=5 W/m^2/K, ΔT_z=10 K, J=10, Sum(q_j)=20000 W/m^2, a=0.2 m:

$\sup\delta^* \approx 13800^*\gamma$, it means that for N=M=100 we have $\delta^*<0.7$ K.

Note, that it is a very conservative estimation. If we suppose for example, that electronic components cover less than 25% of whole electronic board area (i.e. sup(Ω_{mn})=1 instead 4), the estimation of error for this case will be $\delta^*<0.18$ K.

5. Results

The model was used for temperature mapping of the electronic board of the DC/DC converter of the CIMEX experiment CEP block. The board has 6 dissipated elements with good thermal contact to base surface. Figure 1 shows contour lines for natural ambient condition of ground testing with heat exchange in z-direction, and for vacuum condition. Maximal temperature differences above base frame temperature T_0 for first case was obtained 33.9 K, whereas for second case was 37.7 K

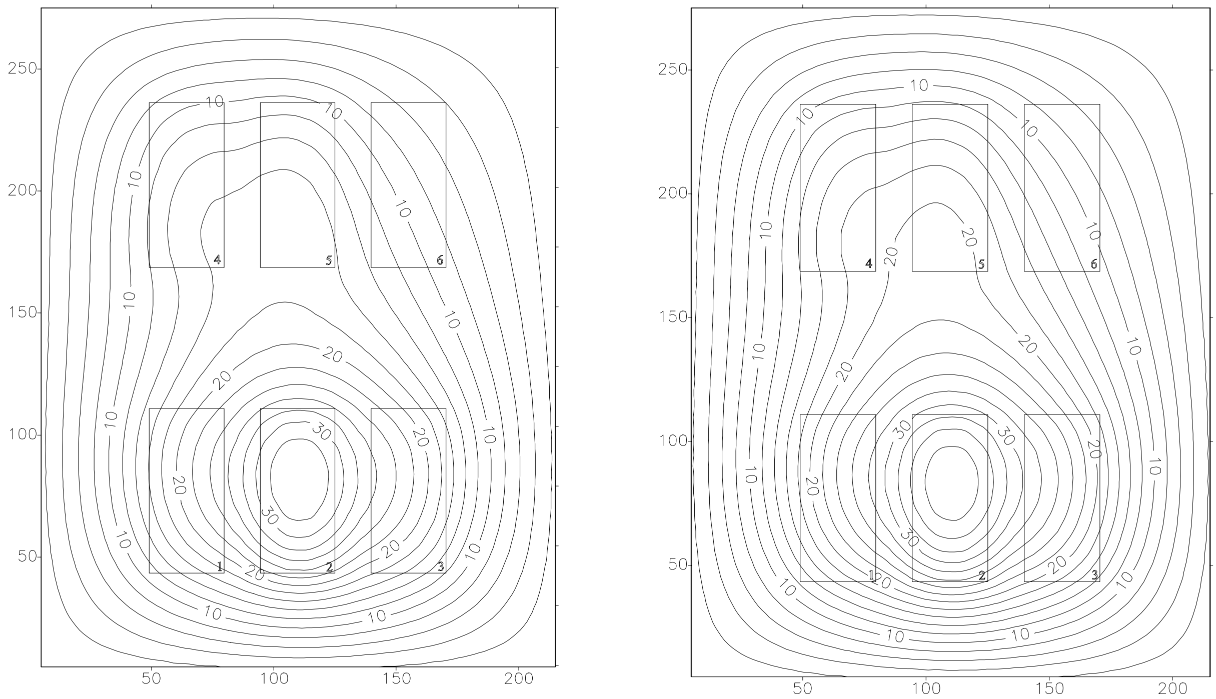

Figure 1. Maximal dissipation: With (left) and without (right) convection cooling.

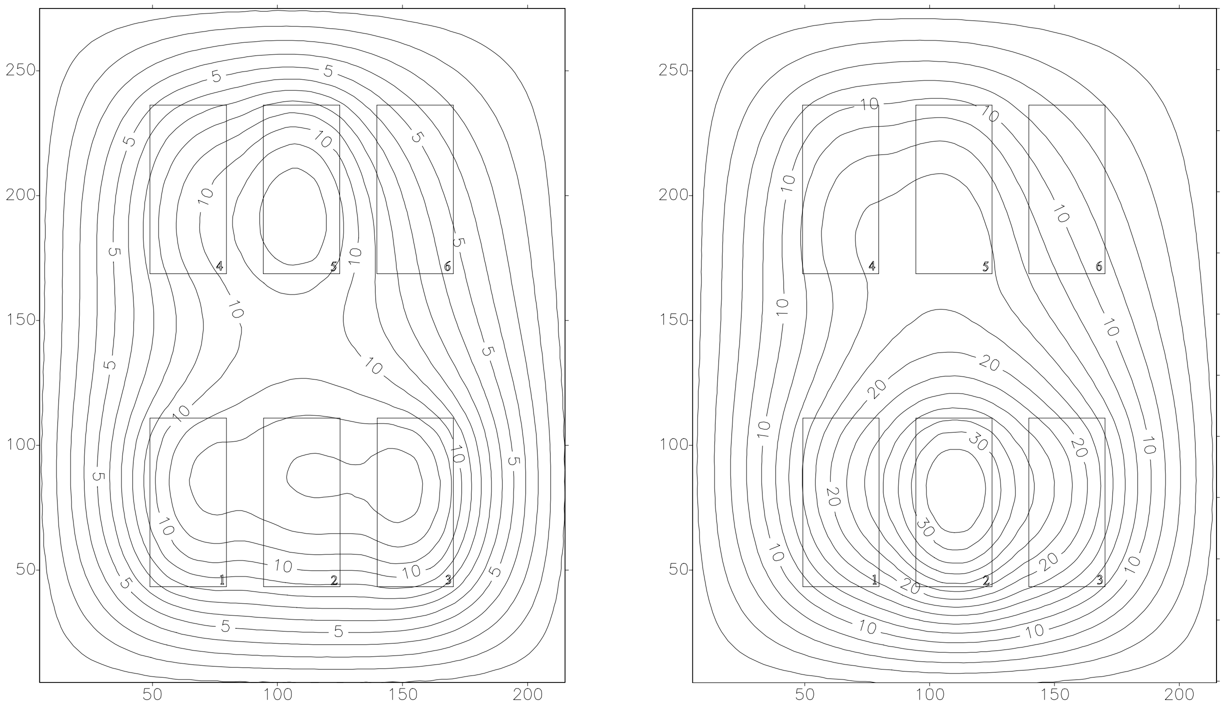

Figure 2. Combined conduction and convection cooling: Minimal dissipation (left) and maximal dissipation (right).

Figure 2 shows contours for two extreme cases of heat dissipation (cold and hot cases) under combined conduction and convection cooling.

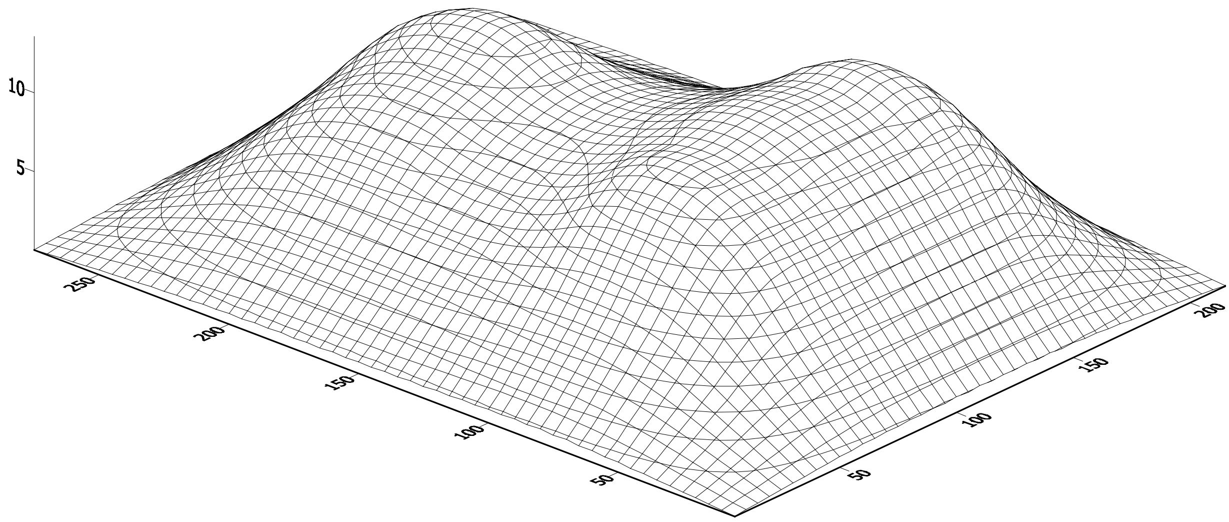

Figure 3. 3-D view for the temperature surface of the board for the case of minimal dissipations.

The last figure shows 3-D view in the case of minimum heat load. Contour lines are plotted in a 2 K interval. The dissipations and coordinates of electronic components are presented in the following table

Table 2. Input data for the dissipation components of the DC/DC board.

n/n	x1 [mm]	x2 [mm]	y1 [mm]	y2 [mm]	Qmin [W]	Qmax[W]
1	46.5	78.4	40	110.1	2.8	2.8
2	94	126	40	110.1	1.5	8.9
3	141.5	173.5	40	110.1	3.5	3.5
4	46.5	78.4	169.9	240	1.5	2.8
5	94	126	169.9	240	3.1	3.1
6	141.5	173.5	169.9	240	0	0

The general input data, used in all of these calculations, were the following: board size is 220x280 mm, k=60 W/m/K, δ=0.0016 m, h_z=0 or 5 W/m^2/K, ΔT_z=0 K

6. Conclusions

The developed analytical model provides a relatively simple and clear tool for 2D temperature mapping of circuit boards with electronic components having a good thermal contact to the base plate. Convection cooling is also taken into account. As soon as the exact analytical expression for the upper limit of truncation error is available, the obtained solution can be used as a benchmark for the evaluation of numeric solution of such problems.

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8. References

Beck, J.V., Cole, K.D., Haji-Sheikh, A., Litkouhi, B., 1992, “Heat conduction using Green’s functions”, Hemisphere publishing.

Cotta, R.M., 1993, “Integral transforms in Computational Heat and Fluid Flow”,CRC Press, USA.

Culham, J.R., Yovanovich, M.M., Lemczyk, T.F., 2000, “Thermal Characterization of Electronic Packaging Using a Three-Dimensional Fourier Series Solution”, Journal of Electronic Packaging, Vol.122, pp. 233-239.

Ellison G.N., 1990, “TAMS-A Thermal analyzer for multilayered structures”, Electrosoft, Vol.1, pp. 85-97.

Haque, S., Stinnet, W.A., Nelson, D.J., Lu, G-Q., 1999, “Thermal management of power electronics modules packaged by a stacked-plate technique”, Microelectronics Reliability, Vol.39, pp. 1343-1349

Mikhailov, M.D., Ozisik, M.N., 1984, “An Alternative General Solution of the Steady-State Heat Diffusion Equation”, John Willey, NY.

Ozisik, M.N., 1980, “Heat Conduction”, John Wiley, NY.

Pesare, M., Giorgio, A., Perri, A.G., 2001, “An analytical method for the thermal laypout optimization of multilayer structure solid-state devices”, Solid-State Electronics, Vol.45, pp. 511-517.

Press, W., Teukolsky, S.A., Vetterling, W.T., Flannery, B.P., 1992, “Numerical Receipts in C”, Cambridge University press, 2nd edition.

Shukla, K.N., 2001, “Thermal Math Modeling and Analysis of an Electronic Assembly”, Journal of Electronic Packaging, Vol.123, pp. 372-378.

Wolfram, S., 1991, “Mathematica”, Addison Wesley.