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MECHANISMS OF VOID FRACTION INSTABILITIES IN FLUIDIZED BEDS

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Abstract. *In this work the effect of various physical parameters on the stability of fluidized beds is evaluated by the means of a normal mode analysis. The effect of a inertial drag on the fluid particle interaction force is also explored. The present study is carried out for liquid-solid and gas-solid fluidized beds. It is verified that the behaviour of fluidized beds varies according to the type of fluidization, indicating that the specific mass ratio plays a key role in the stability of these systems. Different models are proposed for the particle pressure and the particle viscosity of the disperse phase.*

Palavras chave:*Fluidized Beds, Instability, Inertial Drag, Particle Pressure, Particle Viscosity.*

1. Introduction

Fluidized beds are suspensions of particles suspended by an upward flow. They have become a very frequent assemblage in general chemical industry as reactors of high mixture rates, since it brings a fluid into intimate contact with particulate solids. The efficiency of fluidized beds reactors is reduced when bubbles appear during their operation. This cause the by-passing of considerable amounts of fluid that are not brought into contact with the particles.

The fundamental equations for fluidized beds continue to be subject of critical discussions due to the difficulty or even impossibility of measuring the transport properties independently. In this paper, we propose a model that is systematically simplified but that retains the essencial physics present in fluidized beds.

Several works tried to describe the hydrodynamics of fluidized beds and the stability of these systems against bubble formation and void fraction. Anderson & Jackson (1967) were the first to introduce a continuum theory for examining such a system. Their approach has been the starting point to evaluate the stability of fluidized beds undergoing small disturbances (Anderson & Jackson, 1968; Garg & Pritchett, 1975; Batchelor, 1988). Most of these analysis have shown that a particle pressure accounts for the mechanism of momentum transfer on the disperse phase due to collisions between particles and to particle interactions. This mechanism plays a key role on the stability of the bed. This effect is discussed in Batchelor (1988) and in Sobral & Cunha (2001a,b). Different approaches were followed by Liu (1982) and Harris & Crighton (1995). The former has examined the stability of a bed in a wave hierarchy context and defined a formal stability criterion. The latter, on the other hand, explored the stability of concentration waves on the light of a solitary wave theory. The hydrodynamics of bubbles was recently discussed by Anderson et al. (1995), who found that bubbles are actually associated to voidage instability waves, as stated by Liu (1982).

The aim of this work is to analyse the behaviour a fluidized bed when small planar disturbances are imposed to the system. Differently from the matrix approach used in Sobral & Cunha (2001b) and Homsy et al. (1980), a hyperbolic equation in terms of particle concentration disturbances will be derived and will consist on the basis of a modal analysis. The effect of various parameters is investigated, as well as the effect of a inertial drag on the fluid particle interaction force. Finally, two different expressions are tested for the particle pressure and for the particle viscosity, in order to evaluate if the physical effects brought by these models are altered.

2. Governing equations

2.1. Balance equations

An isothermal fluidized bed system with no chemical reactions and with constant fluid and particles densities, is completely described by the continuity equations and the linear momentum equations for both fluid and particulate phase written in terms of local average equations. The continuity equations are, for incompressible fluid and particles, respectively (Anderson & Jackson, 1967):

$$\frac{\partial \epsilon}{\partial t} + \nabla \cdot \Big(\epsilon \mathbf{u}\Big) = 0 \quad \text{and} \quad \frac{\partial \phi}{\partial t} + \nabla \cdot \Big(\phi \mathbf{v}\Big) = 0. \tag{1}$$

The linear momentum equations for the fluid and the particulate phases are, respectively:

$$\frac{\partial}{\partial t}(\epsilon \rho_f \mathbf{u}) + \nabla \cdot \Big(\epsilon \rho_f \mathbf{u}\mathbf{u}\Big) = \rho_f \epsilon \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = \epsilon \rho_f \mathbf{g} + \nabla \cdot (\epsilon \mathbf{T}_f) - \mathbf{f}, \tag{2}$$

$$\frac{\partial}{\partial t}(\phi \rho_p \mathbf{v}) + \nabla \cdot \Big(\phi \rho_p \mathbf{v}\mathbf{v}\Big) = \rho_p \phi \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v}\right) = \phi \rho_p \mathbf{g} + \nabla \cdot (\phi \mathbf{T}_p) + \mathbf{f}. \tag{3}$$

On the above equations, $\mathbf{u}$ denotes the averaged velocity of the fluid phase, $\mathbf{v}$ denotes the average particulate phase velocity, ϕ represents the particle concentration and ϵ is the void fraction, $\phi + \epsilon = 1$. The sub-index $_f$ and $_p$ are associated to fluid and particulate properties, respectively, when written on the specific mass ρ and on the stress tensor $\mathbf{T}$. The term $\mathbf{f}$ denotes the fluid particle interaction force and $\mathbf{g}$ the gravitational acceleration. A complete derivation of these equations can be found in Sobral & Cunha (2001 a,b) and in Anderson & Jackson (1967).

2.2. Constitutive equations for the stress tensors

The stress tensors are modelled considering stokesian newtonian phases, namely:

$$\epsilon \mathbf{T}_f = -p_f \mathbf{I} + \mu_f \left(\nabla \mathbf{u} + \nabla^T \mathbf{u} - \frac{2}{3}(\nabla \cdot \mathbf{u})\mathbf{I}\right), \tag{4}$$

$$\phi \mathbf{T}_p = -p_p \mathbf{I} + \mu_p \left(\nabla \mathbf{v} + \nabla^T \mathbf{v} - \frac{2}{3}(\nabla \cdot \mathbf{v})\mathbf{I}\right). \tag{5}$$

In Eq. (4) and (5), p denotes the pressure and μ the viscosity of the correspondent phases, the upper-index ^{T} indicates the transpose of the tensor and $\mathbf{I}$ denotes the identity tensor.

The particle pressure, represented by p_p, is a key term in the proposed model. It takes into account the effect of a linear momentum transport due to particle collisions and hydrodynamic interactions and it is considered to be a stabilizing term (Garg & Pritchett, 1975; Sobral & Cunha, 2001b). Besides, a particle viscosity μ_p represents the mechanism of viscous dissipation in the particulate phase. Expressions to p_p and μ_p are proposed and are considered to be functions of ϕ. Two expressions will be examined for the particle pressure. First, an ad-hoc expression proposed by Hernández & Jiménez (1991) and second, the one proposed by Harris & Crighton (1994) based on a monotonic functional dependence of the particle pressure on the particle concentration. When $\phi \to \phi_c$ p_p tends to infinity, and $\phi \to 0$, p_p should vanish. Writing $p_p = \sigma G(\phi)$, where σ is the particle pressure coefficient, we define:

$$G_1(\phi) = \phi^3 \exp\left(\frac{r\phi}{\phi_c - \phi}\right) \quad \text{or} \quad G_2(\phi) = \frac{\phi}{\phi_c - \phi}. \tag{6}$$

$G_1(\phi)$ is the form proposed by Hernández & Jiménez (1991) and $G_2(\phi)$ by Harris & Crighton (1994). In these equations, ϕ_c denotes the close packing of particles, assumed to be $\phi_c = 0.65$ and r is a constant subjected to calibration. Anderson et al. (1995) proposed $r = 0.3$ to fit their experiments, and this value is retained on the present work. For the particle viscosity, the asymptotic expression obtained by Frankel & Acrivos (1995) and, in analogy with the particle pressure, the functional proposed by Harris & Crighton (1995) are considered. Introducing the same notation as for the particle pressure, we have $\mu_p = Mh(\phi)$, where:

$$h_1(\phi) = \frac{\phi}{1 - (\phi/\phi_c)^{\frac{1}{3}}} \quad \text{or} \quad h_2(\phi) = \frac{\phi}{\phi_c - \phi}. \tag{7}$$

Here, $h_1(\phi)$ is proposed by Frankel & Acrivos (1995) and $h_2(\phi)$ by Harris & Crighton (1995). M is the particle viscosity coefficient.

2.3. Constitutive equations for the fluid-particle interaction force

The fluid-particle interaction force is modelled based on the analysis develloped by Sobral & Cunha (2001a) for spherical particles. Three drag contributions considered here are: a viscous linear drag term, an inertial drag and a transient drag (virtual mass), namely:

$$\mathbf{f} = \alpha(\phi)\frac{\mu_f}{a^2}(\mathbf{u}-\mathbf{v}) + \kappa(\phi)\frac{\rho_f}{a}(\mathbf{u}-\mathbf{v})|\mathbf{u}-\mathbf{v}| + \beta(\phi)\rho_f(\dot{\mathbf{u}}-\dot{\mathbf{v}}), \tag{8}$$

where a is the particle radius and the overdot ($\dot{}$) represents the material derivative operator. In order to evaluate the effect of the quadratic drag term on the stability of the bed, two different correlations are proposed for $\mathbf{f}$. The first model, which considers only viscous effects, is based on the Richardson & Zaki (1954) correlation for the terminal falling velocity of a particle in a suspension. This is given by:

$$\alpha_{rz}(\phi) = \frac{9}{2}\frac{\phi}{(1-\phi)^n} \quad \text{and} \quad \kappa_{rz}(\phi) = 0, \tag{9}$$

where n is a constant parameter that varies between 3 and 5 for fluidized beds (Harris & Crighton, 1994; Paiva et al., 2001). The second models takes into account the effect of steady quadratic drag. The Ergun (1952) correlation, valid for $\phi > 0.2$, is used. The corresponding expressions for $\alpha_e(\phi)$ and $\kappa_e(\phi)$ are written as:

$$\alpha_e(\phi) = \frac{75}{2}\frac{\phi^2}{(1-\phi)^3} \quad \text{and} \quad \kappa_e(\phi) = \frac{7}{8}\frac{\phi}{(1-\phi)^3}. \tag{10}$$

For the coefficient of the transient drag, the Zuber (1964) expression is used in both cases, say:

$$\beta(\phi) = \frac{1+2\phi}{2(1-\phi)}. \tag{11}$$

2.4. Dimensionless equations

The final equations for the proposed model can be written by introducing Eq. (4), Eq. (5) and Eq. (8) into Eq. (2) and into Eq. (3). Before that, we should make these equations non-dimensional by setting the following dimensionless quantities:

$$\mathbf{u}^* = \frac{\mathbf{u}}{u_0}, \quad \mathbf{v}^* = \frac{\mathbf{v}}{u_0}, \quad \mathbf{x}^* = \frac{\mathbf{x}}{a}, \quad t^* = \frac{tu_0}{a}, \quad M^* = \frac{M}{\mu_f}, \quad p_f^* = \frac{p_f}{\rho_f u_0^2}, \quad \sigma^* = \frac{\sigma}{\rho_f u_0^2}. \tag{12}$$

In (12), u_0 denotes the fluidization velocity. Finally, dropping the stars that indicate dimensionless quantities to avoid a heavy notation, the linear momentum equations for the fluid and particulate phases can be written, respectively, as:

$$\begin{aligned}(1-\phi)\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right) &= -\nabla p_f + \frac{1}{Re}\left(\nabla^2\mathbf{u} + \frac{1}{3}\nabla(\nabla\cdot\mathbf{u})\right) - (1-\phi)Fr^{-1}\mathbf{e_z} \\ &\quad -\frac{\alpha(\phi)}{Re}(\mathbf{u}-\mathbf{v}) - \beta(\phi)(\dot{\mathbf{u}}-\dot{\mathbf{v}}) - \kappa(\phi)(\mathbf{u}-\mathbf{v})|\mathbf{u}-\mathbf{v}|,\end{aligned} \tag{13}$$

$$\begin{aligned}\phi\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) &= -\chi\sigma\nabla G(\phi) + \frac{\chi M h(\phi)}{Re}\left(\nabla^2\mathbf{v} + \frac{1}{3}\nabla(\nabla\cdot\mathbf{v})\right) - \phi Fr^{-1}\mathbf{e_z} \\ &\quad +\frac{\alpha(\phi)\chi}{Re}(\mathbf{u}-\mathbf{v}) + \chi\beta(\phi)(\dot{\mathbf{u}}-\dot{\mathbf{v}}) + \chi\kappa(\phi)(\mathbf{u}-\mathbf{v})|\mathbf{u}-\mathbf{v}|.\end{aligned} \tag{14}$$

In these equations, the non-dimensional physical parameters

$$Re = \frac{\rho_f u_0 a}{\mu_f}, \quad Fr = \frac{u_0^2}{ag}, \quad \chi = \frac{\rho_f}{\rho_s}, \tag{15}$$

are the Reynolds number, the Froud number and the specific mass ratio, respectively. Non-dimensional continuity equations keep the same form of Eqs. (1)

It should be stressed out that Eq. (13) is independent of Eq. (1) and of Eq. (14), since these three equations compose a closed system in terms of $\mathbf{u}$, $\mathbf{v}$ and ϕ. Once these three variables are determinated, Eq. (13) should be used to calculate the fluid phase pressure p_f.

We believe that the present model of a fluidized bed contains the physics needed to explain the behaviour of both liquid and gas fluidized beds and the observed differences between them.

3. Hyperbolic equation and modal analysis

The behaviour of the bed can be analysed with respect to a small disturbance imposed to all independent variables on Eq. (1) and Eq. (14). The initial condition for the stability analysis is the homogeneous fluidization state. This state is defined, in terms of non-dimensional variables and considering only the z direction, as follows:

$$u(z,t) = 1, \quad v(z,t) = 0, \quad \phi(z,t) = \phi_0. \tag{16}$$

A small amplitude disturbance can be introduced in the system as:

$$u(z,t) = 1 + u_1(z,t), \quad v(z,t) = 0 + v_1(z,t), \quad \phi(z,t) = \phi_0 + \phi_1(z,t). \tag{17}$$

Equation (17) is applied to the one dimensional Eq. (1) and Eq. (14) and after retaining only the first order terms in disturbances, the resulting system of linearized equations is found to be composed by the continuity equations for the fluid and the particulate phases, respectively:

$$-\frac{\partial \phi_1}{\partial t} + (1-\phi_0)\frac{\partial u_1}{\partial z} - \frac{\partial \phi_1}{\partial z} = 0 \text{ and } \frac{\partial \phi_1}{\partial t} + \phi_0 \frac{\partial v_1}{\partial z} = 0, \tag{18}$$

and by the linear momentum equation for the particulate phase:

$$\begin{aligned} \phi_0 \frac{\partial v_1}{\partial t} &= -\chi\sigma G'(\phi_0)\frac{\partial \phi_1}{\partial z} + \frac{4}{3}\frac{\chi M h(\phi_0)}{Re}\frac{\partial^2 v_1}{\partial z^2} - \phi_1 Fr + \frac{\chi}{Re}\alpha(\phi_0)(u_1 - v_1) + \frac{\chi}{Re}\alpha'(\phi_0)\phi_1 \\ &+ \chi\beta(\phi_0)\left(\frac{\partial u_1}{\partial t} - \frac{\partial v_1}{\partial t} + \frac{\partial u_1}{\partial z}\right) + 2\chi\kappa(\phi_0)(u_1 - v_1) + \chi\kappa'(\phi_0)\phi_1. \end{aligned} \tag{19}$$

One single equation in terms of ϕ_1 can be obtained by taking the divergence $(\partial/\partial z)$ of Eq. (19) and using Eq. (18) to obtain expressions for the derivatives of velocity disturbances in terms of derivatives of concentration disturbances. After a hard algebraic manipulation, one can write:

$$A\frac{\partial^2 \phi_1}{\partial t^2} + B\frac{\partial^3 \phi_1}{\partial t \partial z^2} + C\frac{\partial \phi_1}{\partial t} + D\frac{\partial^2 \phi_1}{\partial t \partial z} + E\frac{\partial^2 \phi_1}{\partial z^2} + F\frac{\partial \phi_1}{\partial z} = 0, \tag{20}$$

where

$$\begin{aligned} &A = -1 - \frac{\chi\beta(\phi_0)}{\phi_0(1-\phi_0)}, \quad B = \frac{4}{3}\frac{\chi M h(\phi_0)}{\phi_0 Re}, \quad C = -\chi\frac{\alpha(\phi_0) + 2Re\kappa(\phi_0)}{Re\phi_0(1-\phi_0)}, \\ &D = \frac{-2\chi\beta(\phi_0)}{1-\phi_0}, \quad E = \chi\sigma G'(\phi_0) - \frac{\chi\beta\phi_0}{1-\phi_0}, \\ &F = Fr - \chi\kappa'(\phi_0) - \chi\frac{\alpha'(\phi_0)}{Re} - \chi\frac{\alpha(\phi_0)}{Re(1-\phi_0)} - \chi\frac{2\kappa(\phi_0)}{1-\phi_0}. \end{aligned} \tag{21}$$

It should be noted that the approach discussed is similar to that proposed by Anderson & Jackson (1967) and followed by Garg & Pritchett (1975). It differs from the one discussed in Sobral & Cunha (2001b) and Homsy et al. (1980), where the perturbated governing equations are written in a matrix form, the modal analysis being carried out using the expression obtained by imposing the singularity of the modal matrix.

Imposing a plane wave solution for ϕ_1 on Eq. (20):

$$\phi_1 = \hat{\phi}_1 e^{st} e^{-ikz} = \hat{\phi}_1 e^{\xi t} e^{-i(kz - \omega t)}, \tag{22}$$

where $\hat{\phi}_1$ is a small amplitude and $s = \xi + i\omega$ is a complex frequency, Eq. (20) can be written in the k wave-number and s complex-frequency spaces as follows:

$$Q_1 s^2 + Q_2 s + Q_3 + i(Q_4 s + Q_5) = 0, \tag{23}$$

where

$$Q_1 = A, \quad Q_2 = Bk^2 - C, \quad Q_3 = -Ek^2, \quad Q_4 = Dk, \quad Q_5 = -Fk. \tag{24}$$

As an illustration, if the equations of the model proposed on this work is written in the matrix form

$$\mathbf{A} \cdot \mathbf{\Omega} = \mathbf{0}, \tag{25}$$

the modal matrix $\mathbf{A}$ is obtained as:

$$\mathbf{A}=\left(\begin{array}{cccc} -(1-\phi_0)ik & 0 & s+ik & 0 \\ 0 & -ik\phi_0 & -s & 0 \\ \begin{array}{c}(1-\phi_0)(-s-ik)+\\ +\frac{4k^2}{3Re}+\frac{\alpha(\phi_0)}{Re}-\\ -\beta(\phi_0)(s+ik)+2\kappa(\phi_0)\end{array} & \begin{array}{c}-\frac{\alpha(\phi_0)}{Re}+\beta(\phi_0)s-\\ -2\chi\kappa(\phi_0)\end{array} & \begin{array}{c}-Fr^{-1}+\frac{\alpha'(\phi_0)}{Re}+\\ +\kappa'(\phi_0)\end{array} & -ik \\ \begin{array}{c}-\frac{\chi\alpha(\phi_0)}{Re}+\chi\beta(\phi_0)(s+ik)-\\ -2\chi\kappa(\phi_0)\end{array} & \begin{array}{c}-\phi_0 s+\frac{4\chi Mh(\phi_0)k^2}{3Re}+\frac{\chi\alpha(\phi_0)}{Re}\\ -\chi\beta(\phi_0)s+2\chi\kappa(\phi_0)\end{array} & \begin{array}{c}-\chi\sigma G'(\phi_0)ik+Fr^{-1}\\ -\frac{\chi\alpha'(\phi_0)}{Re}-\chi\kappa'(\phi_0)\end{array} & 0 \end{array}\right). \quad (26)$$

The first line of the matrix states for the fluid phase continuity equation and the second line for the particulate phase continuity equation. The third line represents the momentum equation for the fluid phase and the fouth line the momentum equation for the particulate phase. In Eq. (25),

$$\mathbf{\Omega} = [\hat{u}_1 \;\; \hat{v}_1 \;\; \hat{p}_1 \;\; \hat{\phi}_1]^T \quad (27)$$

is the column vector composed by the amplitudes of the disturbances of all variables.

To complete the calculation on the matrix approach, the singularity of the modal matrix presented on Eq. (26) is obtained when

$$det(\mathbf{A}) = \mathbf{0}, \quad (28)$$

from where Eq. (23) is obtained with the same coefficients.

The solution of Eq. (23) in terms of the complex frequency s gives a dispersion relation $s = s(k)$ for the modes of perturbations. The real part $\xi(k)$ represents the amplification growth rate of the disturbances, while the complex part $\omega(k)$ represents the frequency of the modes and is associated to the speed of propagation of disturbances, defined by the relation:

$$V(k) = \frac{\omega(k)}{k}. \quad (29)$$

A tedious algebraic manipulation allows one to write the following expressions for $\xi(k)$ and for $\omega(k)$:

$$\xi(k) = \frac{-Q_2 \pm \sqrt{\frac{\Gamma+R}{2}}}{2Q_1} \quad \text{and} \quad \omega(k) = \frac{-Q_4 \pm \frac{\Theta}{\Gamma+R}\sqrt{\frac{\Gamma+R}{2}}}{2Q_1}, \quad (30)$$

where $R = \sqrt{\Gamma^2 + \Theta^2}$, $\Gamma = Q_2^2 - Q_4^2 - 4Q_1Q_3$ and $\Theta = -4Q_1Q_5 + Q_2Q_4$.

Actually, Eqs. (30) admit two solutions: one being the desired dispersion relation, obtained for $+$ before the square root, and the other, obtained for $-$ before the square root, being of no interest, since it represents an always stable response mode. Locally stable state for a specific wave number k' disturbance will be achieved when $\xi(k') < 0$. When $\xi(k) < 0$, $\forall k$, we say that the fluidized bed is asymptotically stable, or stable. On the other hand, if $\xi(k) > 0$ for any mode k, the corresponding disturbance in the bed will be amplified, growing exponetially with time until it is so large that non-linearities become significant. The bed is said to be unstable in this case.

When $\xi = 0$, the mode is said to be neutrally stable. If all the modes with neutral growth rate response are collected and plotted with respect to any physical parameter of the system, a neutral stability line can be determinated, defining two specific zones: a stability zone and an instability zone. In order to determine the neutral lines expression with respect to Fr, Eq. (30) is solved for $\xi(k) = 0$. The expressions obtained in our model are:

$$Fr_n = \frac{-c_1 \pm \sqrt{c_1^2 - 4a_1c_2}}{2a_1}. \quad (31)$$

In eq.(31):

$$a_1 = -16Q_1^2, \quad c_1 = -(2a_1b + a_2), \quad c_2 = a_1b^2 + a_2b + a_3, \quad a_2 = 8Q_1Q_2Q_4, \quad (32)$$
$$a_3 = 3Q_2^2Q_4^2 + 16Q_2^2Q_1Q_3, \quad b = -\chi\kappa'(\phi_0) - \chi\frac{\alpha'(\phi_0)}{Re} - \chi\frac{\alpha(\phi_0)}{Re(1-\phi_0)} - \chi\frac{2\kappa(\phi_0)}{1-\phi_0}.$$

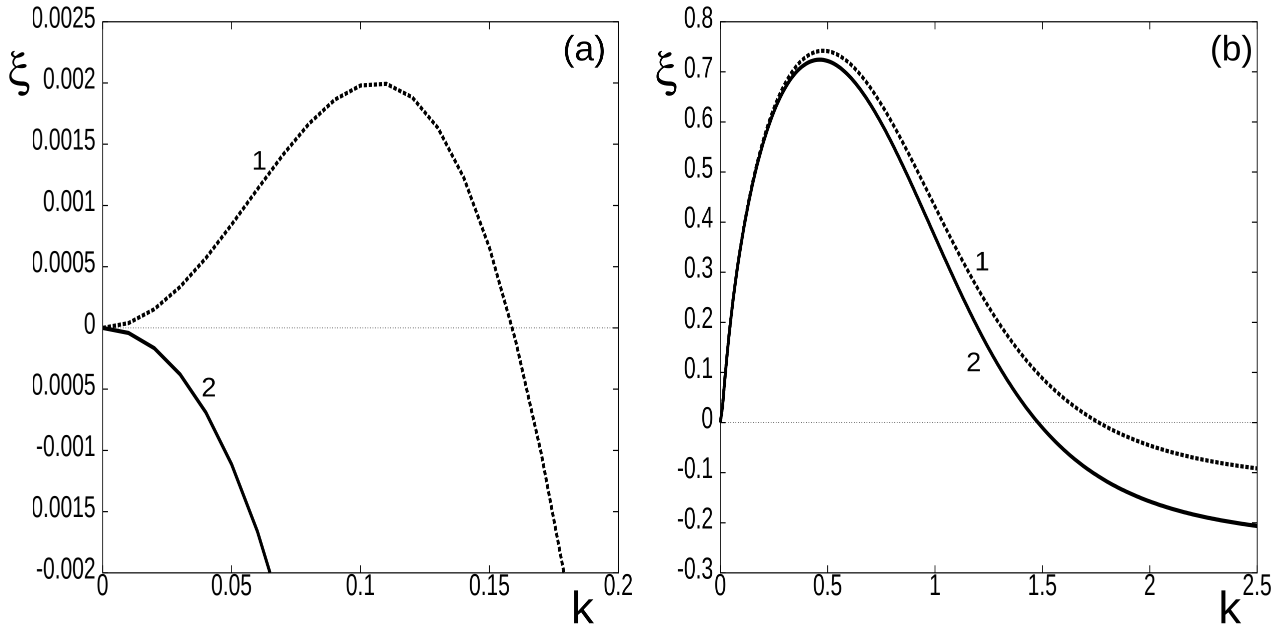

Figura 1: Particle pressure effects on the amplification growth rate for $\chi = 0.5$ (a) and for $\chi = 0.01$ (b). Lines with flag 1 were plotted with the Hernández & Jiménez expression and lines with flag 2 were plotted with the Harris & Crighton expression. $Re = 4$; $Fr^{-1} = 10$; $n = 3.65$; $\phi_0 = 0.57$; $r = 0.3$; $M = 50$; $\sigma = 1$ and $\phi_c = 0.65$.

4. Results

The first result that should be commented is the one concerning the different expressions for the particle pressure and viscosity. Figures (1)a,b show that Harris & Crighton's expression tends to overestimate the effect of the particle pressure with respect to Hernández & Jiménez's expression. In the case of a liquid fluidized bed, Fig. (1)a, the former predicts always an stable fluidized bed while some instability is expected by the later. The stabilizing effect of the particle pressure, however, is present in both expressions. Figures (1)a,b also allow one to verify that the amplification growth rates for liquid-solid fluidized beds are lower than the growth rates obtained for gas-solid fluidized beds. This indicates that liquid-solid fluidized beds are more stable than gas-solid fluidized beds. Similar comments hold to the particle viscosity, Fig. (2)a,b. Frankel & Acrivos' expression is a more powerful filter, since it reduces the region of instability and the associated amplification growth rates of the unstable modes. In Fig. (2)a, for example, for $\xi = 0$, the value of k obtained by Frankel & Acrivos' expression is $k \approx 1.8$, while for the Harris & Crighton's expression this value is $k \approx 2.5$. Harris & Crighton's expression predicts that stable disturbances will be dissipated slower, as it can be seen in Fig.(2)b for short waves.

The effect of a inertial drag is explored in Fig.(3). This term is responsible to consider the effects of the wakes behind the particles when Re increases. Fortes et al. (1987) described a non-linear mechanism in liquid-solid fluidization called kissing effect: the wake of a particle would cause a decrease in the fluid pressure in this region and this would capture a surrounding particle to the wake region. The particles should collide and then separate. This effect is expected to stabilize the bed against voidage disturbances. Figures (3)a,b show that stabilization actually takes place when a quadratic velocity dependence is used to model a full drag contribution in the fluid particle interaction force. On the other hand, an opposite effect is shown in Fig. (3)c,d. The inertial drag increased the region of instability and predicted higher amplification rates that those predicted by the Richardson & Zaki (1954) correlation. The difference between the two pairs of figures is that Fig. (3)a,b were obtained by an aggregative fluidized bed, while Fig. (3)c,d were obtained for a particulate fluidized bed. The distinction between particulate and aggregative fluidized beds can be done by the analysis of the Froude number and will be discussed on the following paragraph. When the fluidization regime is aggregative, the kissing effect actually takes place and the flow is stabilized by the mechanism described above. However, on the particulate regime, where lower values of Re take place, the disturbances introduced by the wakes act as a new external perturbation in the system. Actually, for small values of Re, particles would be caputured by the wakes of other particles but would not collide, gathering in a cloud of particles that would be the origin a concentration wave. The consequence of the extra perturbation introduced by the inertial drag is the increasing of the growth rates of the disturbances. It should be noted, however, that mainly short wave-lenght disturbances are affected by the wake effects introduced by a inertial drag.

The effect of Fr can be evaluated with the diagrams on Fig.(4). In Fig. (4)a,b the neutral lines with respect

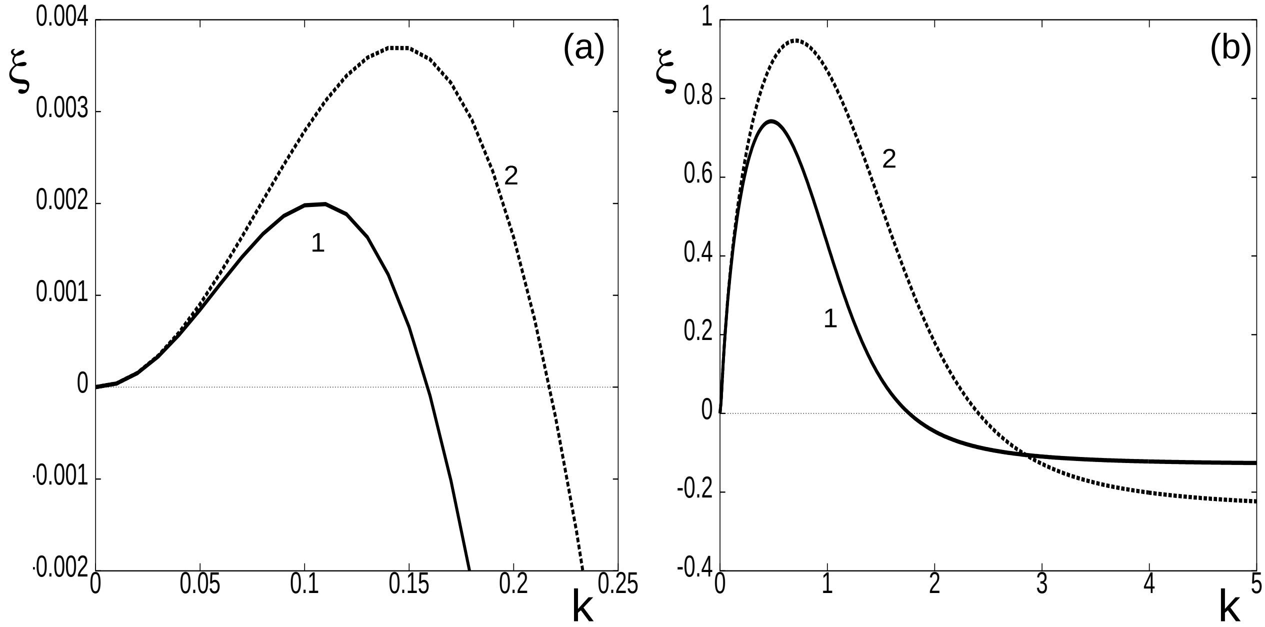

Figura 2: Particle viscosity effect on the amplification growth rate for $\chi = 0.5$ (a) and for $\chi = 0.01$ (b). Lines with flag 1 were plotted with the Frankel & Acrivos expression and lines with flag 2 were plotted with the Harris & Crighton expression. $Re = 4$; $Fr^{-1} = 10$; $n = 3.65$; $\phi_0 = 0.57$; $r = 0.3$; $M = 50$; $\sigma = 1$ and $\phi_c = 0.65$.

to Fr are plotted, for liquid and gas fluidized beds. It can be identified in (a) one stablity zone and two instability zones: a region of particulate instability and a zone of aggregative instability. The aggregative instability regime is identified by fast concentration waves that propagate along the bed, increasing their amplitude until non-linearities dominate and a finite amplitude concentration wave is formed. In Fig. (4)a, the region defined by modes corresponding to $k < 0.6$ and limited by values of $Fr^{-1} < 70$ is unstable. This region should not be called aggregative because the growth rates of disturbances are small, as seen in Fig. (4)c lines 1-4, indicating an unstable sub-inertial instability zone called particulate fluidization. As Fr^{-1} increases from 0 to approximately 70, less unstable bed configurations are obtained, as also seen in Fig. (4)c, until stablility region is reached for $Fr^{-1} > 70$ and a stable fluidized bed configuration is obtained. However, as Fr^{-1} grows so that $Fr^{-1} > 90$, instability is achieved again, defining the aggregative region of the fluidized bed, either for short or long wave disturbances. This region is represented in Fig. (4)c by line 5, and represents aggregative instabilities because the amplification of the disturbances are very high and the disturbances propagate with high velocity, as verified in Fig.(4)e. A similar comment can be done for gas fluidized beds, Fig. (4)b,d,f. The only difference is that in gas fluidized beds, particulate regimes are obtained for a short range of Fr^{-1}, showing that the aggregative regime dominates the dynamic of gas-fluidized beds . Although the scales of velocity may differ in Fig.(4)e,f, indicating that there is a difference of the aggregative regime on liquid (weak) and gas (strong) fluidized beds, velocities will always tend to vanish as $k \to \infty$. This is due to the effect of the particle viscosity, present in the higher order terms of Eq. (20).

5. Conclusion

The proposed model has described the dynamics of fluidized beds in qualitative agreement with experimental observations. The form of the expressions proposed for p_p and μ_p has shown to be unimportant and their application should be conditioned to an adequate calibration of the constants. The inertial drag effect has show to be dependent on the kind of fluidization, either particulate or aggregative, acting as a stabilizing or unstabilizing effect on the fluidized bed. The Froude number has shown to be an important parameter of the systems, since it defines if a aggregative regime is established. The modal analysis develloped in this work has revealed to be an adequate tool to understand the physics envolved in the dynamics of fluidized beds. In future publications, a wave hierarchy approach of the concentration disturbances will be develloped in order to determine a criteria for the transition between particulate and aggregative fluidization.

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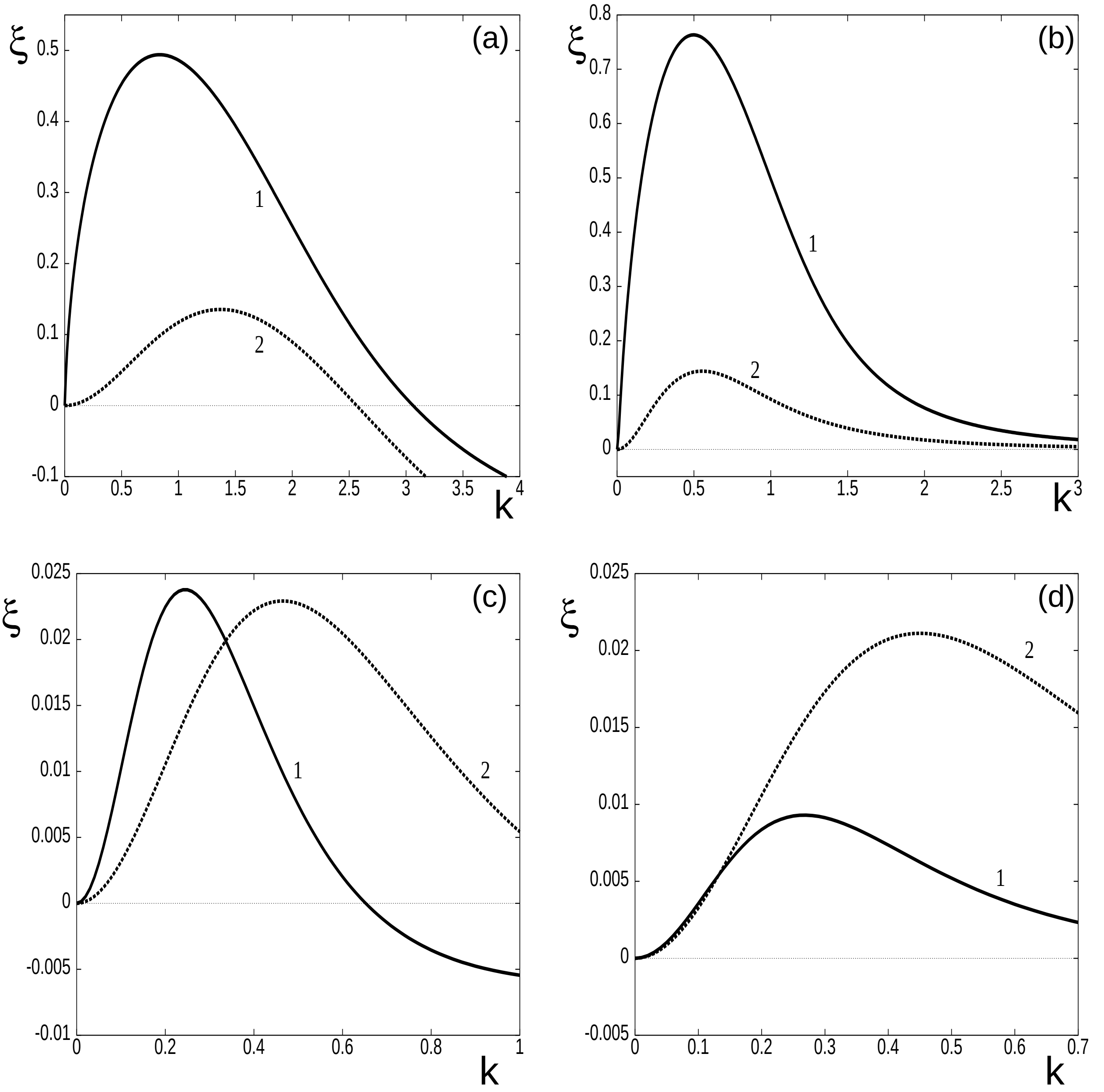

Figura 3: Inertial drag effect on the growth rate of disturbances for $\chi = 0,5$ (a) and (c) and for $\chi = 0,01$ (b) and (d). Lines with flag 1 were plotted with Richardson & Zaki correlation and lines with flag 2 were plotted with Ergun correlation. In (a) $Re = 200$, in (b) and (c) $Re = 4$ and in (d) $Re = 0.3$. $Fr^{-1} = 10$; $n = 3.65$; $\phi_0 = 0.57$; $r = 0.3$; $M = 50$; $\sigma = 0.1$ and $\phi_c = 0.65$.

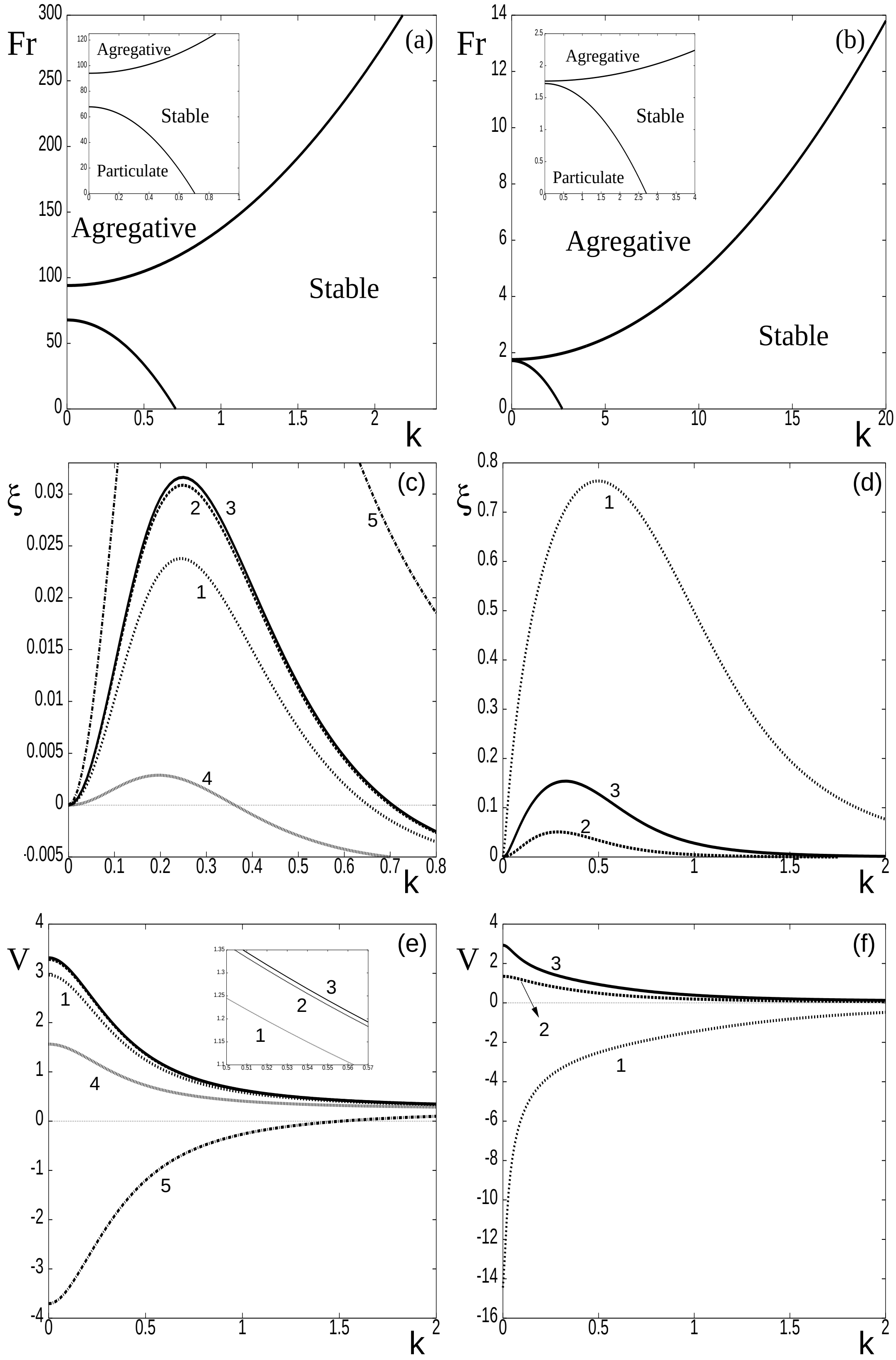

Figura 4: Neutral lines (a) and (b), growth rate of disturbances (c) and (d) and velocity of propagation of disturbances (e) and (f). $\chi = 0.5$ in (a), (c) and (e), and $\chi = 0.01$ in (b), (d) and (f). Lines with flag 1 were plotted for $Fr^{-1} = 10$, lines with flag 2 for $Fr^{-1} = 1$, lines with flag 3 for $Fr^{-1} = 0.1$, lines with flag 4 for $Fr^{-1} = 50$ and lines with flag 5 for $Fr^{-1} = 200$. $Re = 4$; $n = 3.65$; $\phi_0 = 0.57$; $r = 0.3$; $M = 50$; $\sigma = 0.1$ and $\phi_c = 0.65$.

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