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AN ANALYSIS OF TWO-PHASE FLOW INSTABILITIES IN AN OPEN NATURAL CIRCULATION LOOP

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Abstract. *Instability maps for density wave and excursive instabilities that may occur in open two-phase flow natural circulation loop are presented. The one-dimensional homogeneous equilibrium model of two-phase flow is assumed. Linear system stability criteria are used for the analysis. Heater power, liquid inlet subcooling, inlet and exit restrictions, heater length and loop height are taken as parameters for sensitivity analysis. The stable region appears at the lower left part of the* N_{sub} *vs.* $N_{pch}Fr^{0.5}$ *dimensionless plane, bounded by two boundaries whereas the excursive instability region appears at the upper right part of the map. It has been confirmed that the density wave stability criterion automatically satisfies the excursive one.*

Keywords*. two-phase flow, loop instabilities, density-wave instability, excursive instability, linear stability analysis*

1. Introduction

In the design of thermosyphon reboilers, solar collectors, and various cooling systems, the concept of two-phase natural circulation is usually adopted. The two-phase natural circulation mode is expected in hypothetical loss of coolant accident in a nuclear reactor as well as in some modern nuclear concepts and passive safety systems. In such a two-phase natural circulation mode, various types of flow instabilities occur depending on the system geometry, fluid properties and operating conditions, and often lead to abnormal behaviors such as limit cycle oscillations or premature burnout. That is, the self-sustained flow oscillations may cause mechanical vibrations of components, and affect the local heat transfer characteristics which can induce a boiling crisis. Thus the prediction of stable operating conditions in two-phase natural circulation is of primary concern.

When the condenser section has a very large cross section compared to the other parts, the liquid level inside the condenser does not change during operations. Therefore, the velocity and enthalpy fluctuations at the exit of the riser section damp out and do not directly affect the flow behavior in the downcomer section. Thus the system is considered to be "open", and the flow is very similar to the forced natural circulation with the constant driving head. Therefore the liquid level inside the condenser is taken as a parameter in addition to the heat flux, inlet and exit-restriction, and the inlet subcooling.

The methods of stability analysis can be divided into two majors groups: one is based on linearized models and the other on full nonlinear models. The main advantage of the former method (Yadigaroglu, 1981; Lee et all, 1990,1991; Fukuda et all, 1989; Lahey et all, 1989 and Podowski et all, 1981) is that by perturbing the governing equations around a steady-state operating point, and converting the resultant linear model from time to frequency domain, exact analytical solutions can be obtained for the transfer functions. These transfer functions can be analyzed using well established quantitative criteria, to obtain rigorous conditions for the system stability. In fact, such an approach has been successfully applied to several boiling systems, from small-scale experimental facilities to commercial boiling water reactors (BWR) and other thermal power systems. Whereas the linear approach proves very useful in determining if a particular operating point of the system is stable (more specifically - linearly stable, since the equilibrium point may be unstable for sufficiently large perturbations) and in establishing the onset-of-instability conditions, it does not provide information about the properties of the response of an unstable system. To obtain such information, full nonlinear models should be used. In this case, the most common practice is via direct integration of the governing equations in the time domain (Rosa et all, 1994; Podowski et all, 1997; Moberg et all, 1986 and March-Leuba, 1986).

This work aims to present instability maps, for an open two-phase natural circulation loop, which show both static (excursive) and dynamic (density wave) instability boundaries, with the various parametric effects taken into account. The one-dimensional homogeneous thermodynamic equilibrium two-phase flow model is used in the analysis. In addition to the purpose of understanding the mechanisms leading to these kinds of instabilities, this work has also the objective of showing the importance in performing extensive parametric analysis regarding the stability of the system not only for the design but also for the operation of such a system. Whereas commercial systems are designed to operate in the stable region with a fair safety margin, experimental apparatus built for two-phase flow stability studies should be designed to operate also in the unstable regions. The loop reference parameters used in this work are the ones for a two-phase flow experimental apparatus to be built at the "Instituto de Estudos Avançados".

2. Loop Modeling

Let us consider the boiling loop shown in Fig. 1. The loop consists of five regions, namely, the adiabatic liquid region, heated liquid region, heated two-phase region, adiabatic two-phase region and the condenser region. Uniform heat flux is applied at the heater section. Since the cross-sectional flow area of the condenser is much larger than that of the other parts of the loop, the liquid level is assumed to remain constant during each operation. For the purpose of the stability analysis, the following simplifying assumptions were also adopted:

- one-dimensional flow;
- homogeneous two-phase flow;
- no subcooled boiling;
- constant system pressure;
- constant inlet subcooling.

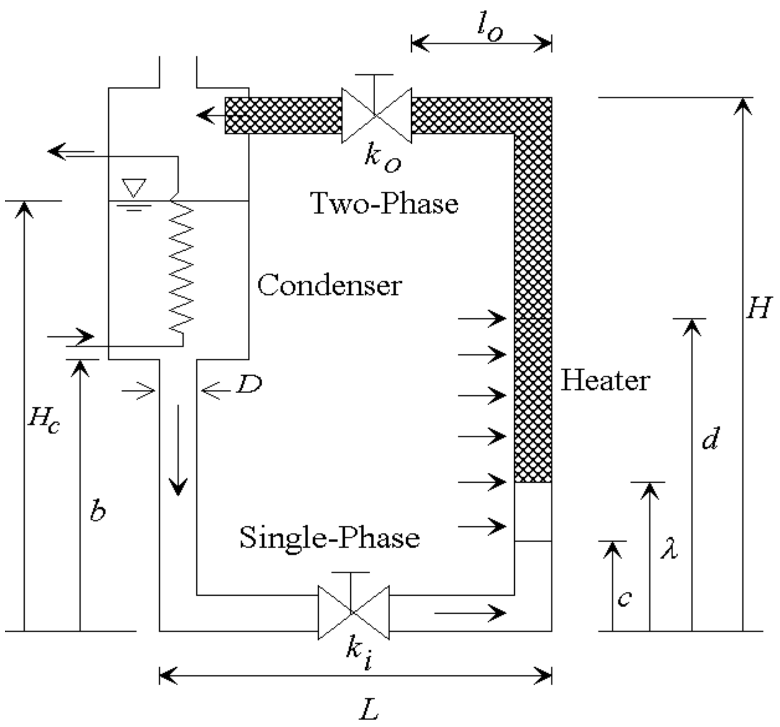

Figure 1: Natural circulation open loop schematic.

Conservation equations of mass, energy and momentum for each region are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial z} = 0 \tag{1}$$

$$\frac{\partial \rho h}{\partial t} + \frac{\partial u \rho h}{\partial z} = \frac{q'}{A_{xs}} \tag{2}$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u^2}{\partial z} + \frac{f}{2D} \rho u^2 + \frac{k_i}{2} \rho u^2 \delta(z - z_i) + \frac{k_o}{2} \rho u^2 \delta(z - z_o) + g \rho = -\frac{\partial p}{\partial z} \tag{3}$$

The linear heat, q', becomes zero in the adiabatic regions and, k_i and k_o are the local loss coefficients at the inlet and exit line restrictions of the loop, respectively. The "δ " is the Dirac function and z_i and z_o represent the locations of the restrictions.

Also needed is the equation of state to relate the density (or the specific volume) and the enthalpy as,

$$\frac{1}{\rho} = v = \begin{cases} v_f + \dfrac{v_{fg}}{h_{fg}} (h - h_f), & \text{for } h > h_f \\ v_f , & \text{for } h \le h_f \end{cases} \tag{4}$$

The pipe friction factor in the liquid regions, f_l, was obtained from the following correlations:

$$f_l = \begin{cases} 64/\mathrm{Re}_l\,, & \text{for laminar flow} \\ 0.316/\mathrm{Re}_l^{0.25}\,, & \text{for turbulent flow} \end{cases} \tag{5}$$

where $\mathrm{Re}_l = \rho_f\, u_l\, D/\mu_l$. The same correlations were used in the two-phase regions but with the two-phase mixture viscosity, μ , instead of μ_l, defined as

$$\frac{1}{\mu} = \frac{x}{\mu_g} + \frac{1-x}{\mu_f} \tag{6}$$

which was proposed by McAdams (Collier, 1981).

The quality required for the evaluation of the mixture density and viscosity in the adiabatic two-phase region is obtained from the following relationship,

$$q'(d-c) = W_l(\Delta h_{sub} + x_e\, h_{fg})\,, \tag{7}$$

and the quality increases linearly along the heated two-phase region.

The model formed by the set of equations just described is nonlinear. The stability of such a system in the small, i.e. for small perturbations to the system equilibrium point, can be studied using techniques which determine the nature of the poles of a system transfer function. Such techniques have been developed for systems of linear equations, therefore, first, perturbation techniques about a steady-state operating point are normally used to linearize the time domain nonlinear equations [Eqs. 1 through 7], then the resultant linear equations are Laplace transformed and integrated along the loop in order to obtain suitable transfer functions to be analyzed, in the form

$$\delta\,\Delta P_{loop} = G(s)\,\delta\, u_l \tag{8}$$

for

$$u_l = \overline{u}_l + \delta\, u_l \tag{9}$$

where $\overline{u}_l$ is the steady (time-averaged) liquid velocity, $\delta\, u_l$ and $\delta\,\Delta P_{loop}$ are the Laplaced transformed perturbations to the liquid velocity and loop pressure drop, and $G(s)$ is the transfer function relating these quantities.

The liquid velocity, $\overline{u}_l$, should satisfy the loop system of equations at steady-state condition subjected to the open loop pressure drop boundary condition, $\Delta P_{loop} = 0$. Therefore,

$$\Delta\overline{P}_{loop}(\overline{u}_l) = \Delta\overline{P}_{loop,f}(\overline{u}_l) + \Delta\overline{P}_{loop,g}(\overline{u}_l) + \Delta\overline{P}_{loop,acc}(\overline{u}_l) = 0 \tag{10}$$

where the bars over the variables represent steady-state values.

Applying the open loop boundary condition to Eq. 8, this equation can be written as

$$G(s)\,\delta\, u_l = 0 \tag{11}$$

Eq. (11) always has a solution for the steady-state case, $\delta\, u_l = 0$, but can also have a nonzero periodic solution provided that,

$$G(s) = 0 \tag{12}$$

In this case, the loop is self-excited and will undergo self-sustained periodic oscillations at the angular frequency, ω , where $s = j\omega$. Eq. (12) is the characteristic equation of the boiling loop, therefore, the asymptotic stability of the system can be determined from the nature of the roots of this equation. The Nyquist criterion has been applied for this purpose.

The excursive instability occurs if

$$\frac{\partial \Delta \overline{P}_{loop}(\overline{u}_l)}{\partial \overline{u}_l} < 0 \tag{13}$$

3. Results and Discussion

Calculations were performed for water at atmospheric pressure and for the loop reference geometric parameters presented in Table 1. A loop stability parametric analysis was performed for the loop inlet and exit restriction loss coefficients, heater length, and loop height for inlet subcooling varying from 0 to 100 ^oC. In order to generalize the results, nondimensional numbers are used, defined as

$$N_{sub} = \Delta T_{sub} \frac{c_p \, v_{fg}}{h_{fg} \, v_f} \qquad N_{pch} = \frac{P}{u_l} \frac{v_{fg}}{A_{xs} \, h_{fg}} \qquad Fr = \frac{u_l^2}{(d-c)\, g}$$

Table 1: Loop parameters used as reference for all calculations.

Reference Loop Parameters	
D (m)	0.018
c (m)	0.
L (m)	1.15
l_o (m)	0.60
d (m)	1.00
H (m)	4.15
b (m)	3.25
H_c (m)	3.75
k_i	1000
k_0	10

Fig. 2 shows two typical curves of the loop flow rate as function of the heater power for different degree of subcooling of the liquid entering the heater section. In the low power range, the increasing rate of flow driving head due to gravity is larger than that of the frictional pressure drop since the two-phase flow velocity is still very small in this range. Therefore, higher liquid velocity is induced by higher powers. In the higher power range there is an opposite trend since the frictional pressure drop is dominant due to the high two-phase velocity, therefore lower liquid velocity is induced by a higher power. Another important aspect observed in the curve for the higher inlet subcooling in Fig. 2 is that there is a power range where there are three velocity values (points A, B and C) for each power value, P_m, in this range which satisfy the open loop natural circulation condition whereas the curve for the lower inlet subcooling shows that there is only one velocity value for each possible power. Fig. 3 shows the gravity, frictional and total loop pressure drop along the entire loop as function of impressed inlet liquid velocity for the loop parametric condition shown in Fig. 2 for the higher inlet subcooling at the heater power P_m. The corresponding A, B and C points in Fig. 2 are also shown in Fig. 3. As can be seen in this figure, point B satisfies the condition of excursive instability (Eq. 13), so this type of flow instability is possible in the power range which yields three steady-state equilibrium points for each power value. As can be seen in Fig. 2, there is no such a power range for the smaller inlet subcooling curve which indicates that this type of instability can only be observed for specific loop parametric conditions. The minimum power, $P_{\min}$, for which excursive instability is possible for a specific loop parametric condition (fixed inlet subcooling) can be obtained by varying the heater power such that point D (maximum total pressure drop), shown in Fig. 3, touches the zero pressure drop line. The maximum power, $P_{\max}$, can be obtained by increasing the power such that point E touches the zero pressure drop line. Therefore, the excursive instability is possible for $P_{\min} < P < P_{\max}$ and for each inlet subcooling.

The range of power operating conditions for dynamically stable loop can be obtained by inspection of the roots of the characteristic equation, given by Eq. 12. All roots of this equation must have negative real parts for loop stability condition. This analysis can be performed by applying the Nyquist criterion to the transfer function $G(j\omega)$ in the frequency domain. Fig. 4 shows examples of Nyquist plots for different powers and keeping the others loop parametric condition fixed. Curves 1 and 3 represent stable and unstable loop operating conditions, respectively. Curve 2 represents a loop marginal stability operating condition.

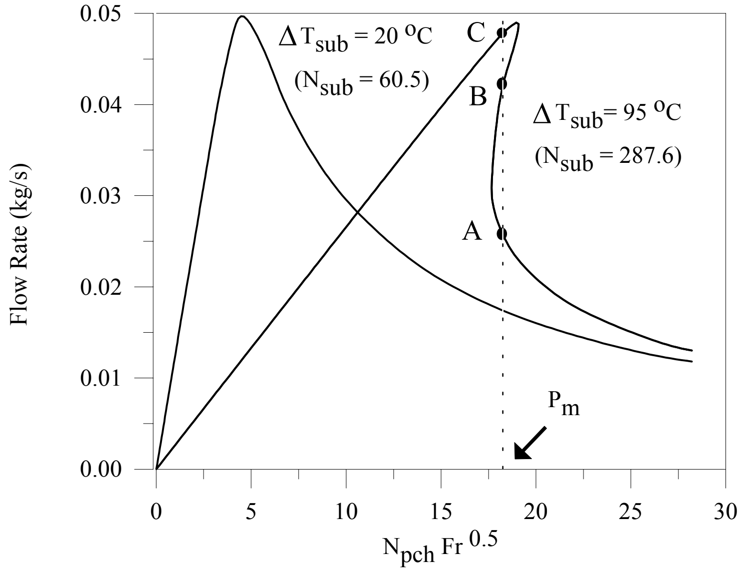

Figure 2: Natural circulation inlet liquid velocity as function of the heater power for different liquid inlet subcoolings using the loop parameters in Table 1.

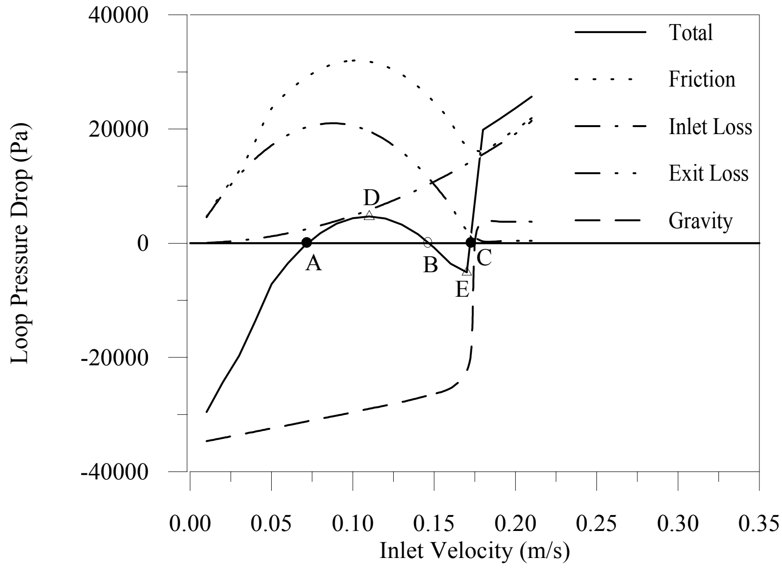

Figure 3: Loop pressure drop components for the higher subcooling, $\Delta T_{sub} = 95\ ^oC$, and power P_m shown in Fig. 2.

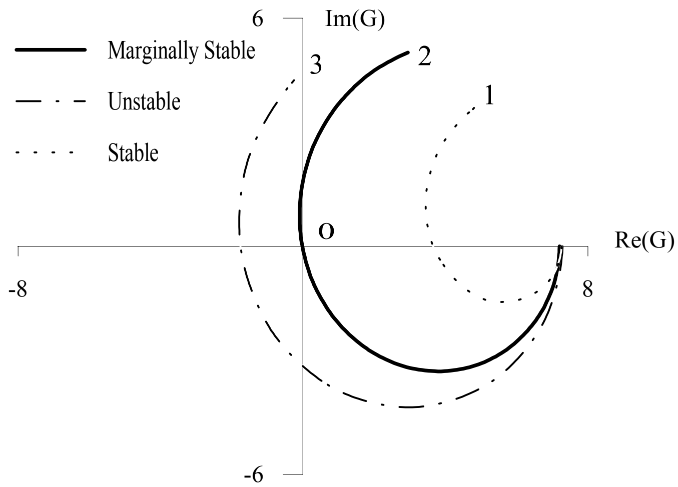

Figure 4: Typical Nyquist plots for dynamically stable, unstable and marginally stable loop operating conditions.

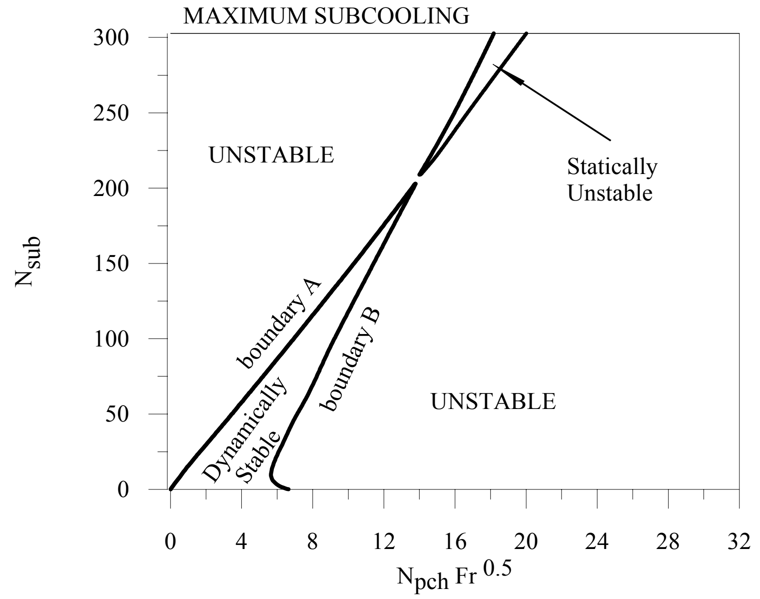

Figure 5: Typical non-dimensional static and dynamic instability maps.

Fig. 5 shows the stability maps in the N_{sub} *vs.* $N_{pch}Fr^{0.5}$ plane for excursive (static) and density wave (dynamic) instabilities of an open natural circulation loop for the reference parameters in Tab. 1. As is shown in this figure, the two instability boundaries, boundaries A and B of the dynamically stable region get close to each other as the inlet subcooling increases and finally merge depending on the operating conditions imposed. At boundary A, the location of the transition from single to two-phase flow takes place near the top of the heater and varies very little along this boundary, therefore the gravitational and frictional two-phase flow pressure drops are basically given by the ones in the adiabatic section. The unstable region in the left side of boundary A in Fig. 5 has a very small exit heater quality, mainly at small powers leading to larger changes in the gravitational two-phase flow pressure drop than that in the frictional one, since the two-phase flow velocity is small. Therefore, the gravitational pressure drop component in the unheated riser plays the dominant role for small powers. As can be seen in Fig. 6.a, as the power is increased over the boundary A, the rate of change in the frictional pressure drop overcomes the gravitational one, therefore the frictional pressure drop component becomes the dominant one, although the gravitational effect is still important. On the other hand, the exit quality in the unstable region in the right side of boundary B is high, this is because for such a high power, evaporation starts at lower locations of the heater and also the rate of increase of quality in the two-phase part of the heater is higher leading to a high two-phase flow velocity in the unheated riser, therefore the two-phase frictional pressure drop plays the dominant role in this region all over boundary B where the gravitational effect is always negligible, as can be seen in Fig. 6.b. As also can be seen in Fig. 5, the excursive instability region lies outside of the density wave one, therefore, the analysis of the density wave stability is sufficient for the purpose of stability analyses.

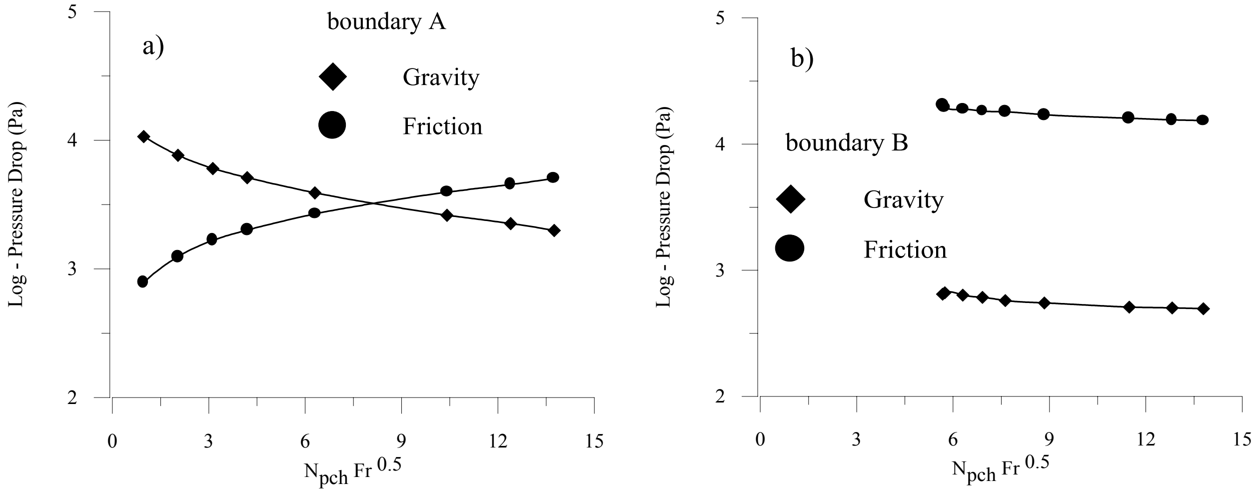

Figure 6: Two-phase region gravitational and frictional pressure drops along (a) boundary A; and (b) boundary B.

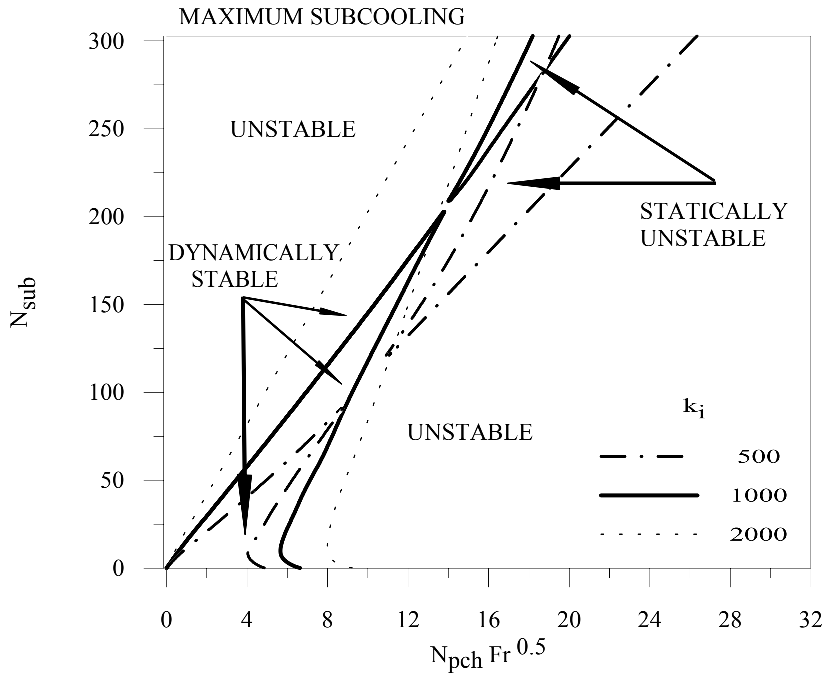

Figure 7: The effect of inlet restriction loss coefficient on the dynamic and static stability regions.

As can be seen in Fig. 7, the increase of the inlet loss coefficient enlarges the dynamically stable region and reduces the statically unstable one, therefore, it has a stabilizing effect on the loop. Increasing the inlet loss coefficient, for the same liquid inlet subcooling, in the dynamically stable region, the power range for stability increases due to a reduction in both unstable regions, i.e., the ones to the left of boundary A and to the right of boundary B. Also can be seen in this figure that increasing the inlet loss coefficient the dynamically stable region extends to higher values of liquid inlet subcooling. On the other hand, increasing the inlet loss coefficient, the excursive instability region becomes bounded by both smaller power and inlet subcooling ranges and may eventually disappear, as is the case for $k_i = 2000$, for instance. The minimum subcooled number of this region is moved to higher values and, for the same subcooled number the power range moves towards lower values, as the inlet loss coefficient is increased, therefore excursive instability may occur at smaller exit qualities.

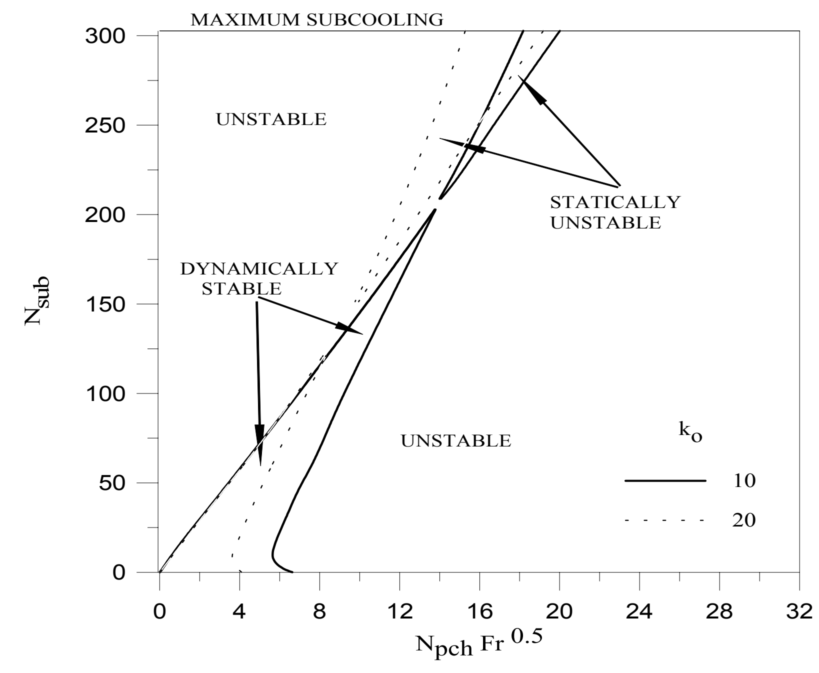

Figure 8: The effect of exit loss coefficient on the dynamic and static stability of the loop.

Opposite to the effect of the inlet loss coefficient, the increase of the exit loss coefficient in the two-phase flow region has the effect of destabilizing the system, that is, the dynamically stable region is reduced and the statically unstable region is enlarged, as can be seen in Fig. 8. The maximum subcooled number and the power range for each inlet subcooling are smaller in the dynamically stable region. Boundary A is basically not affected by changes in the two-phase flow local loss coefficient whereas boundary B moves toward low powers as this parameter is increased. This is because the gravitational pressure drop plays the dominant role on boundary A and the frictional pressure drop is the important one on boundary B, as mentioned earlier. For the statically unstable region, both the subcooled number and power ranges are increased for increasing values of the two-phase local loss coefficient, and the system may become statically unstable for lower powers for the same subcooled number.

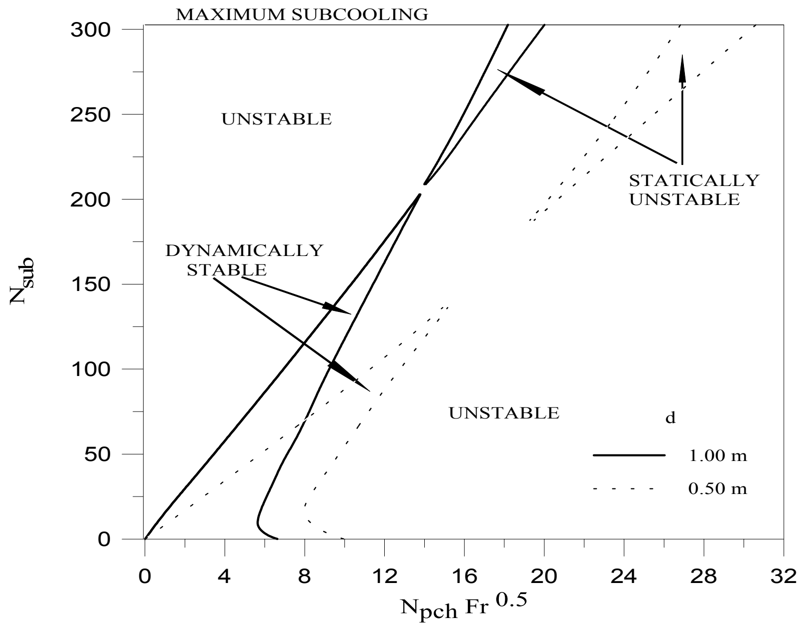

Figure 9 Effect of heater length on the dynamic and static stability of the loop.

As can be seen in Fig. 9, the effect of reducing the heater length on the dynamically stable region is to increase the unstable region to the left of boundary A, similarly to the effect of reducing the inlet local loss coefficient (see Fig. 7), and to decrease the unstable region to the right of boundary B, therefore it has a destabilizing effect to low powers and a stabilizing one to high powers. For the same liquid inlet subcooling, this has the effect of shifting the stable power range to higher values. By reducing the heater length, the maximum possible liquid inlet subcooling for stability is also reduced. For excursive instability, the unstable region is slightly enlarged but the power range for instability is shift to higher values.

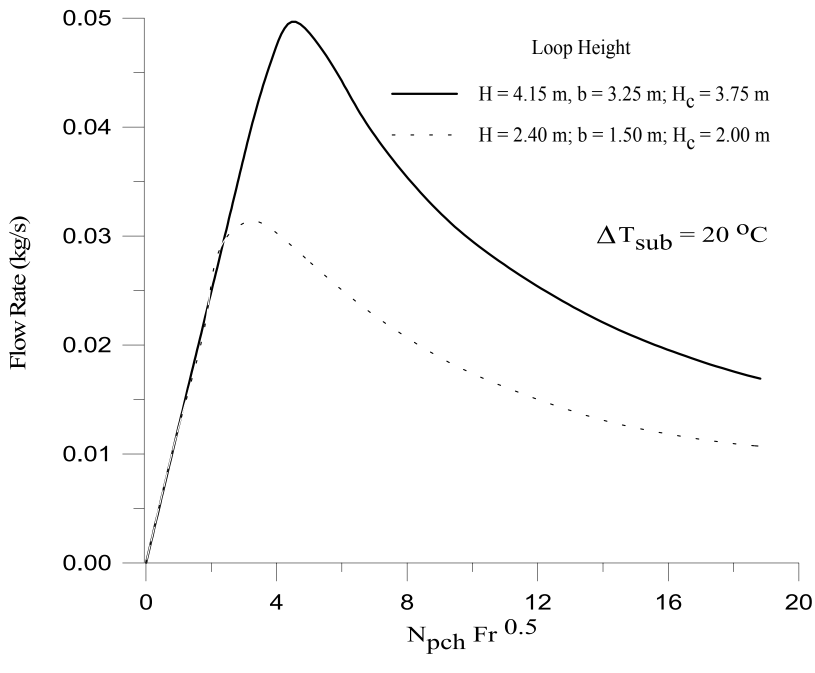

Figure 10: Effect of loop height on the natural circulation liquid velocity for a fixed liquid inlet subcooling.

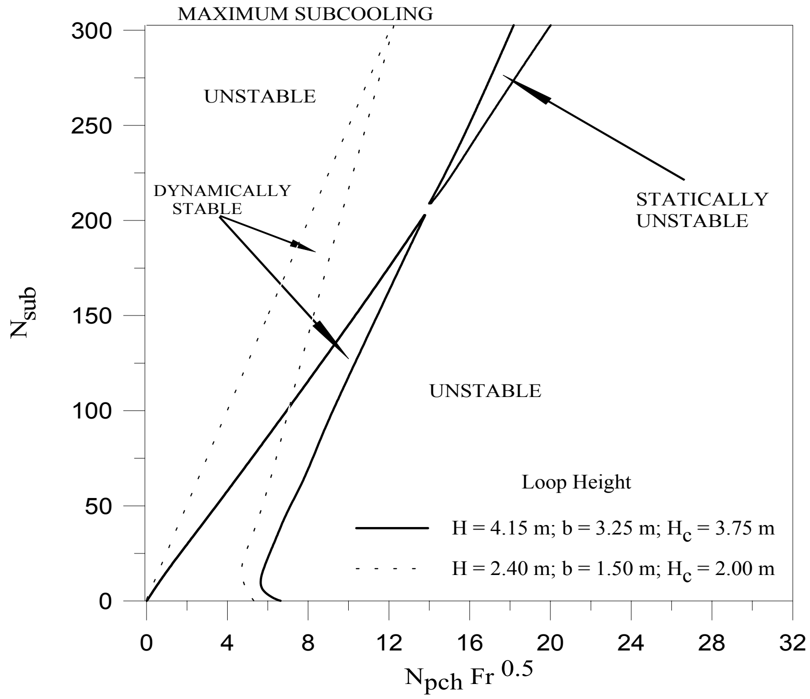

Figure 11: The effect loop height on the dynamic and static stability of the loop.

Here, reduction of loop height means that the unheated riser and liquid downcomer heights are both reduced by the same amount. This is equivalent to increase the heater length and to decrease the liquid pressure head. Therefore, the liquid driving head (buoyancy) decreases what imposes a reduction to the loop liquid velocity as well, as can be seen in Fig. 10. Reduction of the loop height has a stabilizing effect, both dynamically and statically, on the loop as is shown in Fig. 11. For the given loop height reduction shown in this figure, dynamic stability can be achieved for any liquid inlet subcooling for appropriate selections of heater power, also the loop cannot undergo excursive instability in any circumstance unless other loop parameters are changed.

3. Comments and Conclusions

It has been presented a loop modeling and a methodology based on linear criteria for obtaining stability maps for both density wave and excursive instabilities in open two-phase natural circulation loops. It has been confirmed that the density wave stability criterion automatically satisfies the excursive one. The dynamic stability region appears at the lower left part of the N_{sub} vs. $N_{pch}Fr^{0.5}$ dimensionless plane, bounded by two boundaries whereas the excursive instability region appears at the upper right part of the map and eventually touches the line of maximum inlet subcooling. It has been shown that increasing frictional pressure loss in the liquid region of the loop has a stabilizing effect on the system whereas increasing frictional loss in the two-phase region of the loop has an opposite effect. Reduction of the loop height by decreasing both liquid downcomer and adiabatic riser sections in the same amount showed a stabilizing effect on the loop but with the expense of a considerable reduction in the natural circulation flow for high powers.

Comparisons of the analytical results with experimental ones performed by Lee and Lee (1991) using the homogenous equilibrium model and linear stability criteria have showed that the trend of boundary B agrees reasonably with the experimental observations except in the low liquid inlet subcooling range. The same observations were made by Rosa and Podowski (1995) for a boiling channel with fixed Froude number (Fr), using the time domain computer code DYNOBOSS (Rosa and Podowski, 1997). In this work, the authors have showed that subcooled boiling and phasic slip play important roles in the stability of the system as previously reported by Saha and Zuber (1978). Although time domain solutions can be used to calculate threshold of system stability by introducing a small perturbation to the system and analyzing the time response, this is certainly not the most practical way. Therefore, it is important to develop linear models which incorporate such considerations and apply linear stability criteria to improve the results in the low subcooled number range. This will certainly produce a much more powerful tool for stability analysis of such systems.

4. Nomenclature

A_{xs}	flow area
$b,\cdots d$	lengths
c_p	liquid specific heat

D	pipe hydraulic diameter
Fr	Froude number
f_l	liquid friction factor
g	gravity acceleration
G	transfer function
h	mixture enthalpy
h_f	saturated liquid enthalpy
h_{fg}	latent heat of vaporization
H, H_c	lengths
k_i, k_o	inlet and exit friction loss coefficients
l, l_o, L	lengths
N_{pch}	Phase change number
N_{sub}	Subcooled number
p	pressure
P	heater power
q'	linear heat rate
Re_l	liquid Reynolds number
s	Laplace transform variable
t	time
u	mixture velocity
u_l	liquid velocity
v	mixture specific volume
v_f	saturated liquid specific volume
W_l	flow rate
x	quality
x_e	heater exit quality
z	axial direction coordinate
δ	perturbation symbol
λ	boiling boundary transition
ρ	mixture density
μ	mixture viscosity
ω	angular frequency
Δh_{sub}	liquid inlet enthalpy subcooling
ΔP	pressure drop
ΔT_{sub}	liquid inlet temperature subcooling

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