IX CONGRESSO BRASILEIRO DE ENGENHARIA E CIÊNCIAS TÉRMICAS

9th BRAZILIAN CONGRESS OF THERMAL ENGINEERING AND SCIENCES

Paper CIT02-0855

A MATHEMATICAL MODEL FOR DIRECT AND INDIRECT EVAPORATIVE COOLING AIR CONDITIONING SYSTEMS

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Abstract. *Air conditioning systems are responsible for the increase in man's efficiency at the work due to a more comfortable working environment. Presently, the most used system is the mechanical vapor compression system. However, in many cases, evaporative cooling can be an economical alternative in place of conventional system, under several conditions, or as a pre-cooler in the conventional systems. This leads to a reduction in operational cost, in comparison with systems using only mechanical refrigeration. Evaporative cooling operates using natural phenomena, through induced processes, where water and air are the working fluids. It consists in water evaporation, induced by the passage of an air flow, thus decreasing the air temperature. The main characteristic of this process is the fact that it is more efficient when the temperatures are higher, that is, when more cooling is necessary for thermal comfort. This work presents, initially, the basic principles of air conditioning systems based on the evaporative cooling process for human thermal comfort. It also presents the principles of operation for direct and indirect evaporative cooling systems and the mathematical development of the equations of thermal exchanges, allowing the determination of heat transfer convection coefficients for primary and secondary air flow. In addition an analysis of the cooling effectiveness for direct and indirect systems is performed, concluding that such effectiveness can be determined if primary and secondary air flow and their specific heats are known.*

Key-words*: evaporative cooling, air conditioning, heat and mass transfer.*

1. Introduction

Evaporative cooling operates using natural phenomena, through induced processes, where water and air are the working fluids. It consists, specifically, in water evaporation, induced by the passage of an air flow, thus decreasing the air temperature. When water evaporates into the air to be cooled, simultaneously humidifying it, it is called direct evaporative cooling and the thermal process is the adiabatic saturation. When the air to be cooled is kept separated from the evaporation process, and therefore is not humidified while it is cooled, it is called indirect evaporative cooling. The main characteristic of this process is the fact that it is more efficient when the temperatures are higher, that is, when more cooling is necessary for thermal comfort.

2. Direct evaporative cooler analysis

The principle underlying direct evaporative cooling is the conversion of sensible to latent heat. Nonsaturated air is cooled by exposure to free and colder water, both thermally isolated from other influences. Some of the air' sensible heat is transfered to the water and becomes latent heat by evaporating some of the water. The latent heat follows the water vapor and diffuses into the air (Watt and Brown, 1997).

In the study of the psychrometric process dry air is considered as a single gas characterized by an average molecular mass equal to 28.9645. In this work the humid air is considered as a mixture of two gases: the dry air and water vapour.

Considering the flow of humid air close to a wet surface, according to Fig. (1), the heat transfer will occur if the surface temperature T_s is different from the draft temperature T. If the absolute humidity (concentration) of the air close the surface w_s is different from the humidity of the draft w will also occur a mass transfer.

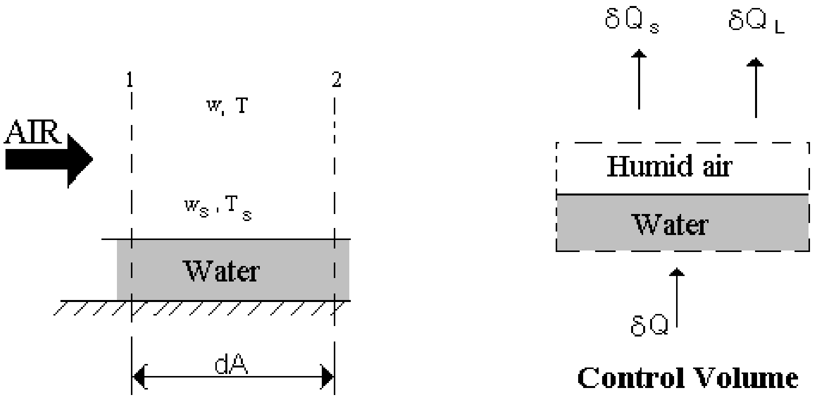

Figure 1 – Schematic direct evaporative cooler

The elementary sensible heat is

$$\delta Q_s = h_c \; dA \, (T_s - T) \tag{1}$$

where h_c is the convective heat transfer coefficient, A is the area of the heat transfer surface, T_s is the surface temperature and T is the temperature far from the surface.

In a similar way the rate of water vapour transfer dm_V between the draft and the air close to the surface will be

$$dm_v = h_m \, \rho_a \; dA \, (w_s - w) \tag{2}$$

where h_m is the mass transfer coefficient by convection and ρ_a is the density of the water.

Analysing the interface air-liquid, the latent heat δQ_L is determined by the law of energy conservation.

$$\delta Q_L = \delta Q - \delta Q_s = h_{Lvs} \; dm_v \tag{3}$$

where δQ is the flow of total heat and h_{LVS} is the specific enthalpy of vaporization of water at surface temperature. Rearranging Eqs. (1), (2) and (3), the total differencial heat flow is

$$\delta Q \; = [h_c \, (T_s - T) + \rho_a \; h_{Lvs} \, h_m \, (w_s - w)\,]\, dA \tag{4}$$

Equation (4) indicates that the total heat transfer is the result of a combination of a portion originating from temperature difference and other portion originating from the difference of the absolute humidities. The total heat is caused by two potentials and these potentials can be combined by the Lewis relationship so that the total heat flow will be expressed by a single potential, that is the enthalpy difference between the air close to the wet surface and the free current of the air.

Using the specific enthalpy of the mixture as the sum of the individual enthalpies (Moreira, 1999, p.99) gives

$$h_s - h = (h_{sa} - h_a) + (w_s h_{vs} - w \, h_v) \tag{5}$$

where h_{VS} is the vapour enthalpy at surface temperature, h_{sa} is the entalpy of the leaving air, h_a is the air enthalpy and h_v is the vapour entalpy. With the hypothesis that air and vapour are perfect gases it follows that

$$h_s - h = c_{pu} \, (T_s - T) + h_{vs} \, (w_s - w) \tag{6}$$

where the humid specific heat is $c_{pu} = c_{pa} + w \, c_{pv}$, and, therefore

$$T_s - T = \frac{(h_s - h) - h_{vs} \, (w_s - w)}{c_{pu}} \tag{7}$$

Combining Eq. (4) and Eq. (7) gives

$$\delta Q = \frac{h_c\, dA}{c_{pu}} \left[(h_s - h) + \frac{(w_s - w)}{R_{Le}} \, (h_{Lvs} - R_{Le}\, h_{vs}) \right] \tag{8}$$

where R_{Le} is the Lewis relationship. In the above deduction the density of the humid air was approximated by the density of the dry air. Taking the Lewis relationship as being unitary, gives $(h_{Lvs} - h_{vs}) \approx h_{Ls}$. It is also verified that the term $(w - w_s) h_{Ls}$ is usually negligible in the presence of difference of the specific enthalpies $(h_s - h)$, so that only the first term inside brackets is significant. In the same way, the total heat flow is caused by the difference of specific enthalpies of the air and of the saturated air close to the wet surface and is given by

$$\delta Q = \frac{h_c\, dA}{c_{pu}} (h_s - h) \tag{9}$$

The sensible heat transferred is

$$\delta Q_s = m_a\, c_{pu}\, dT \tag{10}$$

Therefore by combining this with Eq. (1) gives

$$h_c\, dA\, (T_s - T) = m_a\, c_{pu}\, dT \tag{11}$$

which can be integrated, resulting in

$$\frac{h_c}{m_a\, c_{pu}} \int_0^A dA = \int_{T_1}^{T_2} \frac{dT}{(T_s - T)} \tag{12}$$

The integration yields

$$1 - \frac{T_1 - T_2}{T_1 - T_s} = \exp\left(- \frac{h_c\, A}{m_a\, c_{pu}} \right) \tag{13}$$

In the same way, the *effectiveness* of a direct evaporative cooling equipment is defined as

$$\varepsilon = \frac{T_1 - T_2}{T_1 - T_s} = 1 - \exp\left(- \frac{h_c\, A}{m_a\, c_{pu}} \right) \tag{14}$$

Analysing the above equation it is verified that an effectiveness of 100% corresponds to air leaving the equipment at the wet bulb temperature of entrance. This requires a combination of great area of heat transfer with a high heat transfer coefficient and low mass flow.

It is also observed that the effectiveness is constant if the mass flow is constant since it controls directly and indirectly the value of the parameters on the right side of Eq. (14).

3. Indirect evaporative cooler analysis.

This section presentes the basic theory for the general development of the effectiveness coefficient for an indirect evaporative cooler using the models presented by Maclaine-Cross and Banks (1983), Chen et al. (1991) and Peterson (1993).

An indirect evaporative cooler (IEC) has two distinct air passages, one called the “primary” and the other called the “secondary” air passage. The primary air is usually outdoor air that is supplied to the room after it has been cooled by air in the secondary air passages through heat transfer. The surface of the secondary air passages is wetted by circulating water, so that heat and mass transfer takes place between the wet surface and the secondary air.

A schematic drawing of indirect evaporative cooler in counterflow is shown in Fig. (2), where h_{se} and h_{ss} are the entalpies of the entering and leaving secondary air and h_w is the vaporized water enthalpy; m_p and m_s are the mass flows of the primary and secondary air and m_w is the mass of the liquid; T_{pe} and T_{ps} are, respectively, the inlet and outlet dry bulb temperatures of the primary air; T_{se} and T_{ss} the inlet and oulet wet bulb temperatures of the secondary air and T_w is the temperature of the liquid; h_{cp} ande h_{cs} are the convection heat transfer coefficients on the sides of the primary and secondary air.

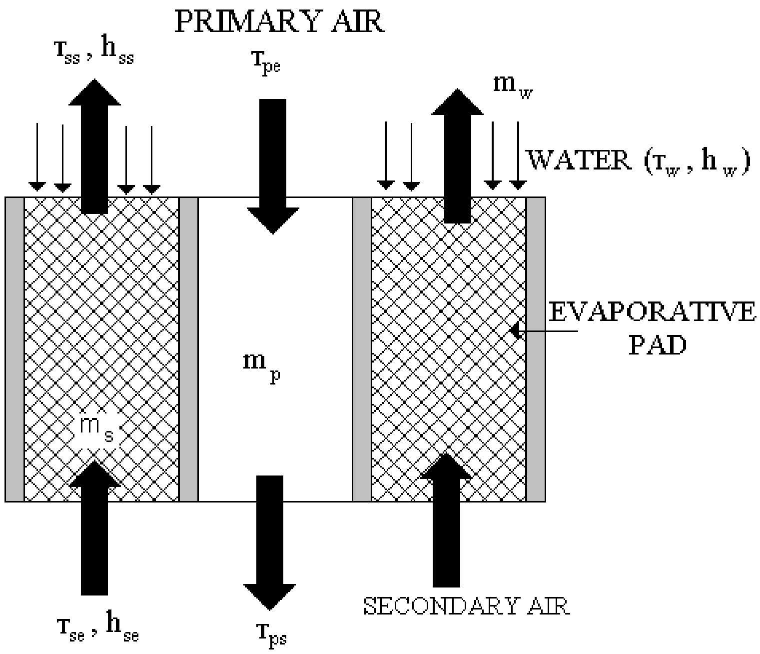

Figure 2 – Schematic counterflow IEC

In order to simplify the analysis, the following assumptions are considered:

1. Heat and mass transfer occurs only in a direction normal to flows
2. The wall of separation of the flows is impermeable to the mass transfer and its tickness is negligible
3. The Lewis number is equal to unity
4. No water loss occurs due to evaporation
5. The energy balance is evaluated only in the dry and humid flows
6. Variable local conditions of temperature and enthalpy at the air/water interface, are reasonably approximated by a constant average
7. Heat and mass transfer coefficients are constant
8. Surface wetting is complete and uniform
9. Saturation enthalpy is linear with respect to wet-bulb temperature.

3.1. Heat balance of primary air flow

For heat transfer from primary air flow to the air-water interface shown in Fig. (2) one has

$$U_o\,(T - T_w)\,dA = -m_p\,c_p\,dT \tag{15}$$

where U_o is the overall coefficient of heat transfer between the primary air flow and air-water interface and c_p is the specific heat of the primary air (Chen et al., 1991). Integrating Eq. (15) the number of primary transfer units NTU_P can be calculated

$$NTU_p = \frac{U_o A}{m_p c_p} = -\ln\left(\frac{T_{ps} - T_w}{T_{pe} - T_w}\right) \tag{16}$$

where A is the area of heat transfer. From Eq. (16), the primary effectiveness is defined as (Peterson, 1993)

$$\varepsilon_p = 1 - e^{-NTU_p} = \left(\frac{T_{pe} - T_{ps}}{T_{pe} - T_w}\right) \tag{17}$$

When the process of cooling of the primary air flow results from sensible heat, one has

$$m_p\, c_p\,(T_{pe} - T_{ps}) = h_{cs} \frac{(T_{pe} - T_{ps})}{\ln\left(\dfrac{T_{pe} - T_w}{T_{ps} - T_w}\right)} A_p \tag{18}$$

where A_p is the area of heat transfer on the primary air flow side. From Eq. (18) it follows that

$$T_{ps} = T_w + \frac{(T_{pe} - T_w)}{e^{h_{cs} A / m_p c_p}} \tag{19}$$

3.2. Heat balance of secondary air flow

The combined heat and mass transfer from the air-water interface to the secondary air flow is given by

$$h_D\,(h_w - h)\, dA = m_s\, dh \tag{20}$$

where h_D is the convective mass transfer coefficient. Integrating Eq. (20) gives

$$NTU_s = \frac{h_D A}{m_s} = \frac{h_D c_p A}{m_s c_p} = -\ln\left(\frac{h_{ss} - h_w}{h_{se} - h_w}\right) \tag{21}$$

where NTU_S is the number of secondary transfer units (Incropera and de Witt,1990). Rearranging Eq. (21), the secondary effectiveness is defined by

$$\varepsilon_s = 1 - e^{-NTU_s} = \left(\frac{h_{se} - h_{ss}}{h_{se} - h_w}\right) = \left(\frac{T_{se} - T_{ss}}{T_{se} - T_w}\right) \tag{22}$$

Assuming a unitary Lewis number gives

$$NTU_s = \frac{h_D A}{m_s} = -\ln\left(\frac{h_{ss} - h_w}{h_{se} - h_w}\right) = -\ln\left(\frac{T_{ss} - T_w}{T_{se} - T_w}\right) \tag{23}$$

So that, $h_D = \dfrac{h_{cs}}{c_p}$

Substituing Eq. (23) into Eq. (20) gives

$$\frac{h_{cs}}{c_p}\,(h_w - h)\, dA = m_s\, dh \tag{24}$$

From this, it follows that

$$\frac{h_w - h_{se}}{h_w - h_{ss}} = \exp\left(\frac{h_{cs} A_s}{m_s c_p}\right) \tag{25}$$

The equation to determinate h_{ss} can be obtained from the above equation giving

$$h_{ss} = h_w - \frac{h_w - h_{se}}{\exp\left(\dfrac{h_{cs} A_s}{m_s c_p}\right)} \tag{26}$$

where A_s is the area of heat transfer on the secondary side.

3.3. Heat balance between primary and secondary air flow

The heat loss of primary air flow must be equal the heat gain of secondary air flow. Therefore,

$$m_s\,(h_{ss} - h_{se}) = m_p\,c_p\,(T_{pe} - T_{ps}) \tag{27}$$

from where it follows that

$$h_{ss} = h_{se} + \frac{m_p}{m_s} c_p\,(T_{pe} - T_{ps}) \tag{28}$$

Substituting Eq. (18) into Eq. (28) gives

$$h_{ss} = h_{se} + \left(\frac{m_p}{m_s}\right) c_p\,(T_{pe} - T_{ps})\left(1 - \frac{1}{\exp\left(\frac{h_{cp}\,A_p}{m_p\,c_p}\right)}\right) \tag{29}$$

One can, theoretically use Eq. (26) and Eq. (29) to determine t_w. However, the most usual method is to solve it through trial and error, starting with an initial guess for t_w and using the respective h_w. Like this, two values for h_{ss} can be calculed using Eqs. (26) and (28) and compared to each other. If they are not equal , another value of t_w is guessed until the calculated values of h_{ss} converges to a single value. Once t_w is obtained, the dry bulb temperature of the leaving primary air can be obtained using Eq. (19).

The equation to calculate the humidit ratio of the leaving secondary air has the same form of Eq. (26), i.e., (Chen et al. (1991))

$$w_{ss} = w_w - \frac{w_w - w_{se}}{\exp\left(\frac{h_{cs}\,A_s}{m_s c_p}\right)} \tag{30}$$

4. Cooling effectiveness

The performance of an indirect evaporative cooler (IEC) for the transfer of heat from primary air to secondary air is known as *cooling effectiveness* and it is usually represented as

$$\varepsilon = \frac{(T_{pe} - T_{ps})}{(T_{pe} - T_w)} \tag{31}$$

However, this form of cooling effectiveness requires that t_{ps} be known. Since the temperature of primary air leaving the IEC is usually unknown it is desirable to find another representation related entirely to known information about the heat exchange and its operation. Therefore, assuming no additional heat gains, there will be an exchange of energy only between primary and secondary air (Peterson, 1993).

From Eq. (27) it follows that

$$m_p\,c_p\,(T_{pe} - T_{ps}) = m_s\,(h_{ss} - h_{se}) \tag{32}$$

Defining the saturation specific heat as $C_{wb} = (h_{ss} - h_{se})/(T_{ss} - T_{se})$ and substituting this value into Eq. (32) gives

$$T_{ps} = T_{pe} - \frac{C_{max}}{C_{min}}(T_{ss} - T_{se}) \tag{33}$$

where $C_{min} = m_p c_p$ and $C_{max} = m_s c_{wb}$. Therefore

$$\varepsilon_p = \frac{C_{max}}{C_{min}}\left(\frac{T_{ss} - T_{se}}{T_{pe} - T_w}\right) \tag{34}$$

Substituting Eq. (22) into Eq. (34) gives

$$T_w = \frac{\varepsilon_s \left(\frac{C_{max}}{C_{min}} \right) T_{se} + \varepsilon_p \, T_{pe}}{\varepsilon_s \left(\frac{C_{max}}{C_{min}} \right) + \varepsilon_p} \tag{35}$$

Finally, substituting Eq. (35) into Eq. (17) one finds the relationship for cooling effectiveness

$$\varepsilon_c = \frac{1}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_s} \left(\frac{C_{min}}{C_{max}} \right)} \quad \Rightarrow \quad \varepsilon_c = \frac{1}{\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_s} \left(\frac{m_p c_p}{m_s c_{wb}} \right)} \tag{36}$$

The equation above is useful once it is a function of known parameters such as the mass flows and constant pressure specific heat of the primary and secondary air flow, respectively.

5. Conclusion

This paper presents a mathematical model for direct and indirect evaporative cooling air conditioning systems. Initially it presents a mathematical model for a direct evaporative cooler that is obtained by writing the equation of the energy conservation for an elementary control volume and analizes the heat and mass transfer between the humid air and the water. The resulting equation allows the determination of the effectiveness is in agreement with the research developed by Alonso et al. (1995).

Then an indirect counterflow evaporative cooler is analyzed. An energy balance is evaluated to the primary air flow and to the secondary air flow and, still, a heat balance between the primary and the secondary air flow is done too. It provides an equation that allows the determination of the primary air outlet temperature (Eq. (33)) and also an equation (Eq (36)) for the cooling effectiveness. It can be seen that the effectiveness can be calculate if the primary and secondary air mass flows and their constant pressure specific heat are known.

This paper is a theoretical basis for the analysis of evaporative devices. The follow-on papers will be devoted to the application of this mathematical model on some particular evaporative cooling devices.

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