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FORCED CONVECTION HEAT TRANSFER TO POWER-LAW NON-NEWTONIAN FLUIDS IN ISOSCELES TRIANGULAR DUCTS

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Abstract. *A hybrid analytical-numerical approach based on the Generalized Integral Transform Technique (GITT) is employed to simulate thermally developing laminar flow of power-law non-Newtonian fluids inside isosceles triangular ducts. Constant wall temperature is used as thermal boundary condition. Reference results are established for quantities of practical interest (such as dimensionless average temperature and Nusselt numbers) within the thermal entry region for the range of flow behavior index* $0.5 \le n \le 1.25$ *and different apex angles. Critical comparisons are made with results available in the literature for direct numerical and approximate approaches.*

Keywords. *isosceles triangular duct, power-law fluids, laminar forced convection, integral transform solution.*

1. Introduction

Analytical and numerical studies has been carried out to obtain heat transfer solutions for thermally fully developed and thermally developing laminar flow inside non-circular ducts. This kind of results are of considerable interest, mainly, to the design of compact heat exchangers and several other low Reynolds number flow heat exchange devices, as pointed out in different articles and textbooks (Shah and London, 1978; Sundén and Faghri, 1998). Beside this, triangular cross-sectioned passages have been used widely in heat exchangers because of their compactness, cost effectiveness and suitability, where this devices, with triangular passages, are characterized by high ratios of heat transfer surface area to core volume (Kays and London, 1984). In industrial applications, heat transfer to purely viscous non-Newtonian fluids are commonly encountered in different industries, i. e. chemical, food processing and pharmaceutical (Shah and Focke, 1988), where, in these applications, the power-law model can describe adequately the rheology of such fluids.

A literature survey reveals that the study of the forced convection of non-Newtonian power-law fluids flowing inside isosceles triangular ducts has been done theoretically by employing purely numerical techniques and approximate approaches in the solution of appropriate energy transport equations. An important review about the laminar heat transfer of Newtonian fluids inside a isosceles triangular ducts have been made by Shah and London (1978) and the case of the hydrodynamically and thermally fully-developed laminar flows have been analyzed by Haji-Sheikh et al. (1983). Numerical results for the steady laminar heat transfer under hydrodynamically and thermally developed as well as simultaneously developing conditions of power-law non-Newtonian fluids flowing through equilateral triangular ducts were presented by Etemad et al. (1996) and Etemad (1998).

The present study aims at applying the so-called Generalized Integral Transform Technique (GITT) (Cotta, 1993) to solve the energy equation for thermally developing laminar flow of power-law fluids inside isosceles triangular channels subjected to a constant wall temperature. An analysis of convergence is made and a set of benchmark results is established for quantities of practical interest, such as dimensionless average temperature and local Nusselt numbers, within a wide range of the dimensionless axial coordinate, different power-law indices and apex angles. Comparisons are then critically performed with previously reported results (Shah and London, 1978; Etemad et al., 1996) from direct numerical approaches and from Galerkin-type functions (Haji-Sheikh et al., 1983) for both, fully developed and thermally developing regions.

2. Mathematical analysis

Laminar flow of a non-Newtonian power-law fluid inside a isosceles triangular duct of sides a and b, according to Fig. 1 (showed bellow), is considered.

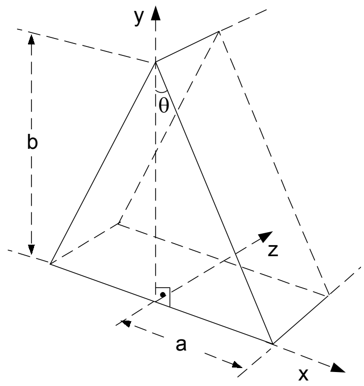

Figure 1 - Geometry and coordinates system for thermally developing isosceles triangular duct flow.

The non-Newtonian viscosity expression (η) can be described according to a Ostwald-de Waele model (or Power Law model) as following (Bird et al., 1987):

$$\eta = K\,\dot{\gamma}^{\,n-1} \tag{1}$$

where K is the fluid consistency index (give in $N.s^n/m^2$), n is the power-law index (dimensionless) and $\dot{\gamma}$ is the rate-of-deformation tensor. The fluids classification is (according to the n value): pseudoplastic fluids ($n < 1$), Newtonian fluids ($n = 1$ and $K = \mu$) and dilatant fluids ($n > 1$). A defined expression for η as one used for the hydrodynamic problem related to this work (Chaves, 2001; Chaves et al., 2001) can be obtained following the steps found out in specific works (Bird et al., 1987 and Bird et al., 2002).

The velocity profile is taken as fully developed and the duct walls are subjected to a constant temperature, so that the dimensionless energy equation for constant property flow, neglecting axial conduction and viscous dissipation, in thermally developing flow is written as:

$$U(X,Y)\frac{\partial\theta(X,Y,Z)}{\partial Z} = \frac{\partial^2\theta(X,Y,Z)}{\partial X^2} + \frac{\partial^2\theta(X,Y,Z)}{\partial Y^2}, \text{ in } \; Z>0,\; 0<X<X_1(Y), 0<Y<\beta \tag{2.a}$$

with inlet and boundary conditions given, respectively, as follows:

$$\theta(X,Y,0)=1, \quad 0 \le X \le X_1(Y), \quad 0 \le Y \le \beta \tag{2.b}$$

$$\frac{\partial\theta(0,Y,Z)}{\partial X} = 0; \;\; \theta(X_1(Y),Y,Z) = 0, \;\; Z>0 \tag{2.c,d}$$

$$\theta(X,0,Z)=0; \;\; \theta(X,\beta,Z)=0, \;\; Z>0 \tag{2.e,f}$$

where in Eqs. (2) above the following dimensionless groups were employed:

$$\theta(X,Y,Z) = \frac{T(x,y,z)-T_w}{T_i - T_w}; \;\; X = \frac{x}{D_h}; \;\; Y = \frac{y}{D_h}; \;\; Z = \frac{z}{D_h Pe}; \tag{3.a-d}$$

$$Pe = Re\,Pr = \frac{\rho cp}{k} u_m D_h; \;\; U(X,Y) = \frac{u(x,y)}{u_m}; \tag{3.e,f}$$

$$\alpha = \frac{a}{D_h}; \;\; \beta = \frac{b}{D_h}; \;\; \gamma = \frac{2\beta}{2\alpha}; \;\; X_1(Y) = \frac{x_1(y)}{D_h} = \alpha\left(1-\frac{Y}{\beta}\right) \tag{3.g-j}$$

The main dimensionless groups in Eqs. (3) above are: Pe (Péclet number), γ (aspect ratio), Re (Reynolds number) and Pr (Prandtl number). D_h is the hydraulic diameter defined as $D_h = 2\gamma a \Big/ \left(1+\sqrt{1+\gamma^2}\right)$. The dimensionless velocity profile is given from the solution of momentum equations, for a non-Newtonian power-law fluid flowing within isosceles triangular ducts, as an infinite series in the form (Chaves, 2001; Chaves et al., 2001):

$$U(X,Y) = \frac{1}{um^*}\sum_{k=1}^{\infty} K_k(X,Y)\,\overline{U}_k(Y); \tag{4.a}$$

$$um^* = \frac{4\sqrt{2}}{\alpha\beta\pi}\sum_{k=1}^{\infty}\tilde{h}_k\tilde{I}_k\,;\;\; \tilde{h}_k = \frac{(-1)^{k+1}}{(2k-1)};\;\; \tilde{I}_k = \int_0^{\beta}\sqrt{X_1(Y)}\,\overline{U}_k(Y)dY; \tag{4.b-d}$$

$$K_k(X,Y) = \frac{\cos\left(\mu_k(Y)X\right)}{\sqrt{X_1(Y)/2}};\;\; \mu_k(Y) = \frac{(2k-1)\pi}{2X_1(Y)} \tag{4.e,f}$$

In Eqs. (4), the quantities $\overline{U}_k(Y)$ represent the transformed potentials for the velocity field, which were numerically obtained by the application of the GITT approach (Chaves, 2001; Chaves et al., 2001), so that the integral in Eq. (4.d) must also be numerically obtained through appropriate subroutines to evaluate integrals of a cubic spline such as the CSITG from the IMSL Library (1991).

Due to the non-separable nature of the velocity profile given in Eq. (4.a) and consequently, of the related eigenvalue problem needed to solve the energy equation through well-known analytical methods such as the classical integral transform technique (Mikhailov and Ösizik, 1984), an exact solution of problem (2) is not possible. On the other hand, with the advances on the so-called GITT approach for the hybrid analytical-numerical solution of this class of non-separable eigenvalue problem, it is possible to avoid these difficulties as now demonstrated (Aparecido and Cotta, 1992; Cotta, 1993, Chaves et al., 2002). For this purpose, in order to alleviate the difficulties related to the eigenvalue problem and to permit the employment of the generalized integral transform technique, the following auxiliary eigenvalue problems are chosen:

$$\frac{\partial^2\psi_i(X,Y)}{\partial X^2} + \mu_i^2(Y)\psi_i(X,Y) = 0;\;\; 0 < X < X_1(Y) \tag{5.a}$$

$$\frac{\partial\psi_i(0,Y)}{\partial X} = 0; \qquad \psi_i(X_1(Y),Y) = 0 \tag{5.b,c}$$

and

$$\frac{d^2\phi_m(Y)}{dY^2} + \lambda_m^2\phi_m(Y) = 0;\;\; 0 < Y < \beta \tag{6.a}$$

$$\phi_m(0) = 0; \qquad \phi_m(\beta) = 0 \tag{6.b,c}$$

which are readily solved to yield eigenfunctions, eigenvalues, and normalization integrals as

$$\psi_i(X,Y) = \cos(\mu_i(Y)X); \qquad \phi_m(Y) = \sin(\lambda_m Y) \tag{7.a,b}$$

$$\mu_i = \frac{(2i-1)\pi}{2X_1(Y)}; \qquad \lambda_m = \frac{m\pi}{\beta} \tag{7.c,d}$$

$$N_i = \frac{X_1(Y)}{2}; \qquad M_m = \frac{\beta}{2}, \qquad i = 1,2,\ldots,N;\;\; m = 1,2,\ldots,N^*. \tag{7.e,f}$$

Eigenvalue problems (5) and (6) allow the development of the following integral transform pair:

$$\tilde{\tilde{\theta}}_{im}(Z) = \int_0^{\beta}\int_0^{X_1(Y)} K_i(X,Y)\,Z_m(Y)\theta(X,Y,Z)dXdY\,, \qquad \text{transform} \tag{8.a}$$

$$\theta(X,Y,Z) = \sum_{i=1}^{\infty}\sum_{m=1}^{\infty} K_i(X,Y)Z_m(Y)\tilde{\tilde{\theta}}_{im}(Z)\,, \qquad \text{inversion} \tag{8.b}$$

where,

$$K_i(X,Y) = \frac{\cos(\mu_i(Y)X)}{\sqrt{N_i}} \text{ and } Z_m = \frac{\sin(\lambda_m Y)}{\sqrt{M_m}} \tag{8.c,d}$$

Equation (2.a) is now operated on with $\int_0^\beta \int_0^{X_1(Y)} K_i(X,Y)\, Z_m(Y) dXdY$ to yield, after employing the inversion formula (8.b), the following truncated system of coupled differential equations to compute the transformed potentials $\tilde{\tilde{\theta}}_{im}(Z)$:

$$\sum_{j=1}^{N}\sum_{n=1}^{N^*} D_{ijmn} \frac{d\tilde{\tilde{\theta}}_{jn}(Z)}{dZ} + H_{ijmn}\tilde{\tilde{\theta}}_{jn}(Z) = 0, \quad Z > 0\,; \;\; j = 1, 2, ..., N; \;\; n = 1, 2, ...\, N^* \tag{9.a}$$

where

$$D_{ijmn} = \int_0^\beta \int_0^{X_1(Y)} K_i(X) K_j(X)\, Z_m(Y) Z_n(Y) U(X,Y) dXdY \tag{9.b}$$

$$H_{ijmn} = E_{ijmn}\delta_{ij} - F_{ijmn} - G_{ijmn} + \lambda_m^2 \delta_{ij}\delta_{mn} \tag{9.c}$$

and

$$E_{iimn} = \int_0^\beta \mu_i^2(Y) Z_m(Y) Z_n(Y) dY \tag{9.d}$$

$$F_{ijmn} = 2\int_0^\beta \int_0^{X_1(Y)} K_i(X,Y) \frac{\partial K_j(X,Y)}{\partial Y} Z_m(Y) \frac{dZ_m(Y)}{dY} dXdY \tag{9.e}$$

$$G_{ijmn} = \int_0^\beta \int_0^{X_1(Y)} K_i(X,Y) \frac{\partial^2 K_j(X,Y)}{\partial Y^2} Z_m(Y) Z_n(Y) dXdY \tag{9.f}$$

while the transformed inlet condition becomes

$$\tilde{\tilde{\theta}}_{im}(0) = \tilde{\tilde{g}}_{im} = \frac{4(-1)^{i+1}}{(2i-1)\pi\sqrt{\beta}} \int_0^\beta \sqrt{X_1(Y)}\, \mathrm{sen}(\lambda_m Y) dY \tag{9.e}$$

In Eq. (8.b) each summation is associated with the eigenfunction expansion in a corresponding spatial coordinate, for computational purposes, the series solution given by Eq. (8.b) is, in general, truncated to a finite number of terms for it summation, in order to compute the potential $\theta(X,Y,Z)$. The solution convergence is verified by comparing the values for the potential obtained with the truncated series for different numbers of retained terms. Such number of terms is commonly user-supplied and even taken as the same for each summation.

Then, the indices i and m related to the temperature field are reorganized into the single index p, while the indices j and n are collapsed into the new index q. The associated double sums are then rewritten as:

$$\sum_{i=1}^{N}\sum_{m=1}^{N^*} \rightarrow \sum_{p=1}^{NT} \quad ; \quad \sum_{j=1}^{N}\sum_{n=1}^{N^*} \rightarrow \sum_{q=1}^{NT} \tag{10.a,b}$$

where

i = int[(p–1)/N] + 1, j = int[(q–1)/N*] + 1, m = p – (i–1) N and n = q – (j–1) N* (10.c-f)

The truncated version of system (9) is now rewritten in terms of these new indices as:

$$\sum_{q=1}^{NT} D_{pq} \frac{d\tilde{\tilde{\theta}}_q(Z)}{dZ} + H_{pq}\,\tilde{\tilde{\theta}}_q(Z) = 0, \quad Z > 0\,;\; p = 1, 2, ..., N \times N^*\,;\; q = 1, 2, ..., N \times N^* \tag{11.a}$$

$$\tilde{\tilde{\theta}}_p(0) = \tilde{\tilde{g}}_p \tag{11.b}$$

The coupled system of ordinary differential equations (11) is solved by efficient numerical algorithms for initial value problems, such as in subroutine IVPAG from the IMSL package (1991), with high accuracy. Then, after the transformed potentials are obtained, quantities of practical interest are determined from the analytic inversion formula (8.b), such as the dimensionless average temperature

$$\theta_{av}(Z) = \frac{1}{A_c}\int_{A_c} U(X,Y)\theta(X,Y,Z)dA \tag{12.a}$$

or

$$\theta_{av}(Z) = \frac{2\sqrt{2}}{\alpha\beta um^*\sqrt{\beta}}\sum_{p=1}^{NT}\sum_{k=1}^{NV} Q_{pk}\tilde{\tilde{\theta}}_p(Z) \tag{12.b}$$

where

$$Q_{pk} = \int_0^{\beta} \sin\left[\lambda_{m(p)}Y\right]\overline{U}_k(Y)\delta_{i(p)k}dY; \quad \delta_{i(p)k} = \int_0^{X_1(Y)} K_i(X,Y)\,K_k(X,Y)dX \tag{13.a,b}$$

and the variable um^* is given by Eq. (4.b).

The local Nusselt number can be calculated by making use of the temperature gradients at the wall integrated over the perimeter, or utilizing the axial gradient of the average temperature,

$$Nu_1(Z) = \frac{h(z)D_h}{k} = -\frac{1}{P_m^*\theta_{av}(Z)}\left[\int_0^{\beta}\left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{X_1(Y)}dY - \int_0^{\alpha}\left.\frac{\partial\theta(X,Y,Z)}{\partial Y}\right|_{Y=0}dX + \int_0^{\beta}\left.\frac{\partial\theta(X,Y,Z)}{\partial X}\right|_{X=X_1(Y)}dY\right] \tag{14.a}$$

or

$$Nu(Z) = -\frac{1}{4\theta_{av}(Z)}\frac{d\theta_{av}(Z)}{dZ} \tag{14.b}$$

3. Results and discussion

Numerical results for thermally developing laminar flow of power-law fluids inside isosceles triangular ducts was obtained by codes developed in FORTRAN 90 programming language. The system given by Eqs. (11) was handled through the subroutine IVPAG from the IMSL Library (1991) with a user-prescribed subroutine tolerance of 10^{-8}. These codes was implemented on PENTIUM – IV 1.3 GHz microcomputer and the complete solution was computed with $NT \le 400$ in the expansion.

The results are presented in terms of dimensionless average temperature and Nusselt numbers along the axial coordinate, within the range of $Z = 10^4 - 1$, for different apex angles (θ = 20, 30 and 45°) and different power-law indices (n = 0.5, 1.0 and 1.25).

Table (1) illustrate the convergence behavior of the approach in terms of the Nusselt number in thermal entry region (i. e., $Z = 10^{-2}$, 10^{-1} and 1) for different power-law indices and θ = 30°. It is observed in this table an excellent convergence ratio, with practically three digits converged for all positions studied.

In Fig. 2 we present results for the local Nusselt number along the thermal entry region from the present work and from Shah and London (1978) and Etemad et al. (1996). It can be noticed that the numerical results presented by these authors cover a range of the axial coordinate smaller than the range presented by the present work. Instead of this, within the applicable ranges, the results are in good agreement.

Table 1. Convergence of the local Nusselt number for a equilateral triangular duct (θ = 30°).

Nu(Z)			
Z = 0.01			
NT	**n = 0.50**	**n = 1.00**	**n = 1.25**
25	4.1545	3.9832	3.9509
100	4.2576	4.0598	4.0229
255	4.2227	4.0259	3.9909
400	4.2196	4.0239	3.9895
a	-	4.00	-
Z = 0.1			
25	2.6935	2.5206	2.4833
100	2.6914	2.5183	2.4809
255	2.6913	2.5182	2.4808
400	2.6912	2.5181	2.4807
a	-	2.57	-
Z = 1			
25	2.0453	2.4963	2.4579
100	2.6709	2.4954	2.4570
255	2.6708	2.4953	2.4570
400	2.6708	2.4953	2.4570
a	-	2.47	-

a - Shah and London (1978)

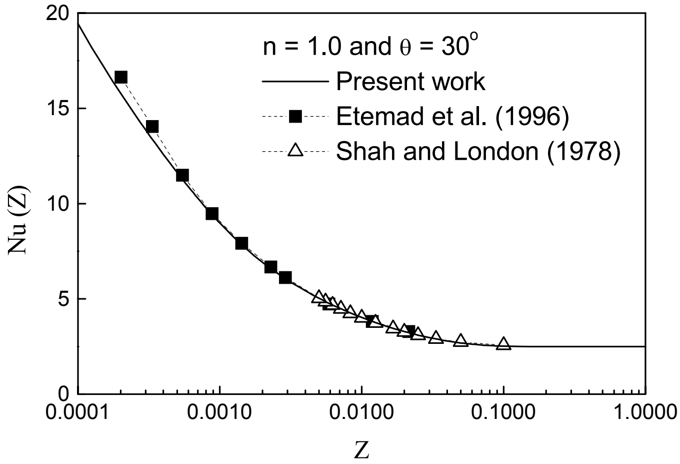

Figure 2. Comparison of local Nusselt number for different authors for a equilateral triangular duct.

Table (2) list a set of benchmark results for fully developed Nusselt numbers ($Z \rightarrow \infty$) and their comparisons with those results found out in the literature. In this table there are comparisons with results obtained with purely numerical approaches presented in the works by Shah and London (1978) and Etemad et al. (1996). An approximated approach also was compared, which was presented in the work by Haji-Sheikh et al. (1983) and employed the Galerkin method. It can be noticed the satisfactory overall agreement in the various predictions, but presenting a better agreement with the results of Haji-Sheikh et al. (1983). Thus, the present global error controlled results are able to be employed in the benchmarking of these previously obtained results from purely numerical methods and approximated approaches.

Table 2. Benchmark results for the fully developed Nusselt number for isosceles triangular ducts.

θ (degrees)	**Nu_∞**							
	n = 0.5		**n = 1.0**				**n = 1.25**	
	a	Present	a	b	c	Present	a	Present
5	-		-	1.702	1.61	1.696	-	-
10	-	-	-	2.053	2.00	2.050	-	-
15	-	-	-	2.272	2.26	2.271	-	-
20	-	2.566	-	2.405	2.39	2.405	-	2.371

Table 2. Benchmark results for the limit Nusselt number for isosceles triangular ducts. (Continued)

θ (degrees)	Nu_∞							
	n = 0.5		n = 1.0				n = 1.25	
	a	Present	a	b	c	Present	a	Present
25	-	-	-	2.475	2.45	2.475	-	-
30	2.594	2.671	2.503	2.495	2.47	2.495	2.471	2.457
35	-	-	-	2.479	2.45	2.478	-	-
40	-	-	-	2.430	2.40-	2.430	-	-
45	-	2.510	-	2.357	2.34	2.357	-	2.325
60	-	-	-	2.027	2.00	2.027	-	-
75	-	-	-	1.600	1.50	1.578	-	-

a Etemad et al. (1996) b Haji-Sheikh et al. (1983) c Shah and London (1978)

From this table the influence of the apex angle and the power-law indices are observable. Related to the influence of the power-law indices, occurs an decrease of the Nusselt number with the increase of the power-law index. This occurs because the apparent viscosity of the fluid increase in the duct wall at the high local velocity gradient regions. So the little apparent viscosity of the fluid at the duct surface favor the heat transfer. Then it can be noticed that the Nusselt numbers a pseudoplastic fluid is greater than a dilatant fluid one. In reference to the influence of the other governing parameter, it can be noticed that occur a maximum value of the Nusselt number at θ = 30° (equilateral isosceles duct). This is happen because the minimum sharp corners effects on the flow (and more particularly on the product Fanning friction factor – Reynolds number) and consequently on the Nusselt number, where a change in the apex angle to one greater, or lower, than θ = 30°, there are a decrease in the Nusselt number due the sharp corners effects on the decrease of the fRe product, where there are a kind of reduction of the effective fluid contact at the duct surface.

Figure (3) and (4) show the behavior of the dimensionless average temperature and local Nusselt numbers along the thermal entry region, respectively, for the case of apex angle θ = 30° and different power-law indices. In Fig. (3) it can be noticed a small influence of the power-law index on this parameter, when it is verified a slight decrease in the average temperature when n > 1.0. However, in Fig. (4), it is observed an apposite behavior for the Nusselt numbers. These aspects can be explained by the fact when n < 1.0 the viscous effects near the wall diminish and, consequently, the thermal exchange is more intensified resulting in greater values for this parameter when compared with those values for n > 1.0. In Fig. (5) are shown the plots of the present results for the local Nusselt along the thermal entry region for the case of apex angle θ = 30°considering different non-Newtonian power-law indices, as well as, those results obtained by Etemad et al. (1996) with finite element method. As can be seen from this figure, the results obtained for n = 1.25 are in agreement with the present results, but those obtained for n = 0.5 diverge abruptly for over Z < 0.005. This analysis demonstrate a kind of limitation of the finite element method, employed for that work, to estimate results for the thermally entry region of pseudoplastic fluids inside equilateral triangular ducts.

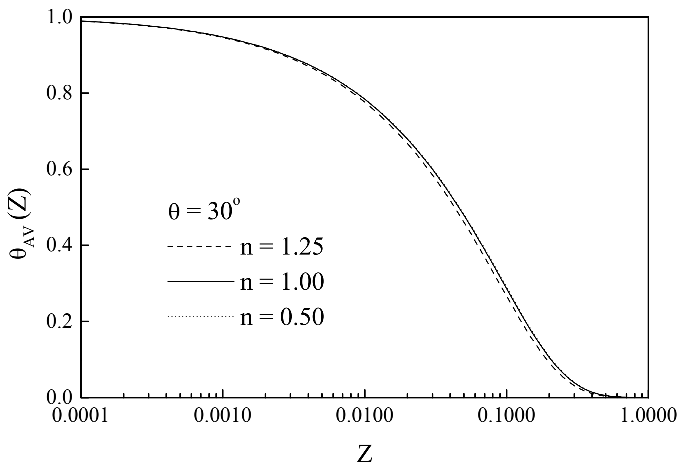

Figure 3. Dimensionless average temperature for different power-law indices and θ = 30°.

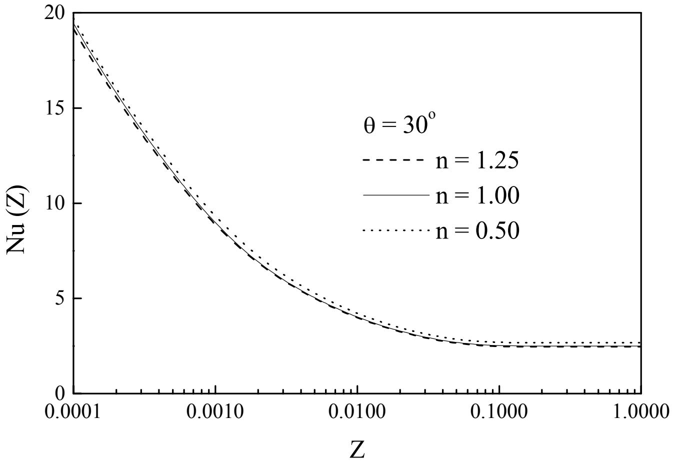

Figure 4. Nusselt numbers along the thermal entry region for different power-law indices and θ = 30°.

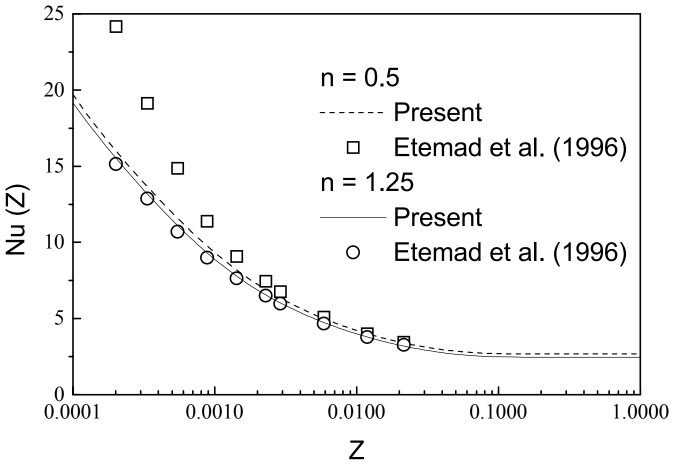

Figure 5. Comparison of Nusselt numbers along the thermal entry region for different power-law indices and θ = 30°.

4. Conclusions

The present study demonstrated the powerfulness of the Generalized Integral Transform Technique (GITT), where the codes created with this approach showed be relatively cheap, within the range of NT considered. Numerical results were tabulated and graphically presented providing a reliable source of benchmark for the local Nusselt numbers and dimensionless average temperature. The next step in the application of the present methodology involving both heat transfer and flow of non-Newtonian fluids will be concerned to the case of others irregularly shaped duct geometries as described by Cotta (1993) and Chaves (2001).

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