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MATHEMATICAL MODELING OF TURBULENCE IN SATURATED POROUS MEDIA

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Abstract. *Turbulence models proposed for flow through permeable structures depend on the order of application of time and volume average operators. Two developed methodologies, following the two orders of integration, lead to different governing equations for the statistical quantities. The flow turbulence kinetic energy resulting in each case is different. This paper reviews recently published mathematical models developed for such flows. The concept of double decomposition is discussed and models are classified in terms of the order of application of time and volume averaging operators, among other peculiarities. A total of four major classes of models are identified and a general discussion on their main characteristics is carried out. Proposed equations for turbulence kinetic energy following time-space and space-time integration sequences are derived and similar terms are compared. Treatment of the drag coefficient and closure of the interfacial surface integrals are discussed.*

Keywords. *porous media, turbulent flows, periodic boundary conditions, low Reynolds k-ε model, macroscopic turbulence model.*

1. Introduction

Environmental flows of extreme importance, such as turbulent atmospheric boundary layer over thick rain forests, may benefit from more realistic mathematical models. In this sense, proper mathematical modeling of such flows could bring insight to scientists about overall exchange rates of mass and energy between the soil and the atmosphere. Accordingly, flow over layers of dense vegetation can be characterized by some sort of porous structure through which a fluid permeates. Also, fluidized bed combustors and chemical catalytic reactors are subjected to pressure loss variation due to changes in the flow regime inside the pores. In petroleum extraction, the flow accelerates towards the pumping well while crossing regions of variable porosity. Turbulent regime eventually occurs, affecting overall pressure drop and well performance. In all cases, better understanding of turbulence through adequate modeling can more realistically simulate real-world environmental and engineering flows.

Classical theory when the pore Reynolds number Re_p is less than about 150 is based on the concept of a Representative Elementary Volume - REV (Darcy, 1856, Forchheimer, 1901, Brinkman, 1947, Ward, 1964, Whitaker, 1969, Bear, 1972, Vafai & Tien, 1981, Whitaker, 1999). For high Reynolds number (Re_p >300), turbulence occurs within the pores (Prausnitz & Wilhelm, 1957, Mickley *et al.,* 1965, Takatsu & Masuoka, 1998) and, as such, a turbulence model is necessary in order to close the mathematical problem. For that purpose, models following the *space-time* integration sequence (Lee & Howell, 1987, Wang & Takle, 1995, Antohe & Lage, 1997, Getachewa *et al.,* 2000) and *time-space* averaging of governing equations (Masuoka & Takatsu, 1996, Kuwahara *et al.,* 1998, Kuwahara & Nakayama, 1998, Takatsu & Masuoka, 1998, Nakayama & Kuwahara, 1999) have been proposed. These two different approaches lead to different sets of governing equations. A morphology-based turbulence model has also been suggested (Travkin & Catton, 1992, Travkin *et al.,* 1993, Gratton *et al.,* 1994, Travkin & Catton, 1995, Travkin & Catton, 1998, Travkin *et al.,* 1999) based on the Volume Average Theory. Use of such methodology, however, is regarded by many as of little practical use in engineering applications (Lage, 1998, pg. 23).

Another route closer to the time-space methodology was proposed in the work of Pedras & de Lemos, 1998. Subsequently, a study on the different views in the literature has lead to the proposition of the *double-decomposition* concept (Pedras & de Lemos, 1999a) and to further development of the earlier preliminary model (Pedras & de Lemos, 1999b). The double-decomposition idea allowed for a better characterization of the flow turbulent kinetic energy (Pedras & de Lemos, 2000a) and was also extended to heat transfer analysis (Rocamora & de Lemos, 2000a).

The development of this concept was a step before detailed numerical solution of the flow equations were carried out in order to establish a working version of the model (de Lemos & Pedras, 2000a, Pedras & de Lemos, 2001a). Calculations were needed for adjusting the proposed model and considered either the high Re k-ε closure (Rocamora & de Lemos, 1998) as well as the Low Reynolds version of it (Pedras & de Lemos, 2000b, Pedras & de Lemos, 2001b).

Heat transfer analysis was also the subject of additional research (Rocamora & de Lemos, 1999). One of the main motivations for this development was the ability to treat hybrid computational domains with a single mathematical tool for either the flow field (de Lemos & Pedras, 2000b) or when solving for the energy equation. Non-isothermal recirculating flows in channels past a porous obstacle (Rocamora & de Lemos, 2000b, Rocamora & de Lemos, 2000c) and through a porous insert have been calculated (Rocamora & de Lemos, 2000d) using the work described in Pedras & de Lemos, 2001a.

The objective of this paper is to classify and compare turbulence models for porous medium presented in the literature. The comparison herein is sole based on the distinct propositions for the flow governing equations, which, in fact, involve substantially different concepts. The main idea underlining this work is to focus on the propositions of the transport equations rather than showing values obtained with their numerical solution. As such, proposed equations for the turbulent kinetic energy are reviewed and the interrelationship between correspondent terms is discussed. It is expected that the contribution herein provide some insight to turbulence modelers devoted to analyze engineering systems and environmental flows which can be modeled as a porous structure having a fluid flowing in turbulent regime

2. The Double Decomposition Concept

For the sake of completeness, the definition of volume and time averaging is included below. Also, for clarity, the idea of double decomposition is here quickly reviewed. More details on this matter can be found in Pedras & de Lemos, 2000a, and Rocamora & de Lemos, 2000a.

For a general fluid property, φ, the intrinsic and volumetric averages are related through the porosity ϕ as (Bear, 1972),

$$\langle\varphi\rangle^i = \frac{1}{\Delta V_f}\int_{\Delta V_f}\varphi dV;\ \langle\varphi\rangle^v = \phi\langle\varphi\rangle^i;\ \phi = \frac{\Delta V_f}{\Delta V} \tag{1}$$

where ΔV_f is the volume of the fluid contained in ΔV. The property φ can then be defined as the sum of $\langle\varphi\rangle^i$ and a term related to its spatial deviation within the REV, ${}^i\varphi$ (Whitaker, 1969), as

$$\varphi = \langle\varphi\rangle^i + {}^i\varphi \tag{2}$$

The relationship between the volumetric average of derivatives and the derivatives of the volumetric average is presented in a number of works. Examples are Slattery, 1967, Whitaker, 1969, Gray & Lee, 1977, and is known as the **Theorem of Local Volumetric Average** or Volume Averaging Theory, VAT. Further, the need for considering time fluctuations occurs when turbulence effects are of concern. The microscopic time-averaged equations are obtained from the instantaneous microscopic equations. For that, the time-average value of property, φ, associated with the fluid is given as:

$$\overline{\varphi} = \frac{1}{\Delta t}\int_t^{t+\Delta t}\varphi dt \tag{3}$$

where Δt is the integration time interval. The instantaneous property φ can be defined as the sum of the time average, $\overline{\varphi}$, plus the fluctuating component, φ':

$$\varphi = \overline{\varphi} + \varphi' \tag{4}$$

being $\overline{\varphi'} = 0$.

From the work in Pedras & de Lemos, 2000a, and Rocamora & de Lemos, 2000a, one can write for any flow property φ combining (2) and (4),

$$\langle\varphi\rangle^i = \langle\overline{\varphi}\rangle^i + \langle\varphi'\rangle^i \tag{5}$$

$$\overline{\varphi} = \langle\overline{\varphi}\rangle^i + {}^i\overline{\varphi} \tag{6}$$

$${}^i\varphi = {}^i\overline{\varphi} + {}^i\varphi' \tag{7}$$

$$\varphi' = \langle\varphi'\rangle^i + {}^i\varphi' \tag{8}$$

where ${}^i\varphi'$ can be understood as either the *time fluctuation of the spatial deviation* or the *spatial deviation of the time fluctuation*. Also, $\langle {}^i\varphi' \rangle^i = \overline{{}^i\varphi'} = 0$. After some manipulation, one can prove that (Pedras & de Lemos, 2001a, Pedras & de Lemos, 2001b)

$$\overline{\langle \varphi \rangle^i} = \langle \overline{\varphi} \rangle^i \quad \textbf{(9)}$$

$$\langle \varphi \rangle^{i\prime} = \langle \varphi' \rangle^i \quad \textbf{(10)}$$

$$ {}^i\overline{\varphi} = \overline{{}^i\varphi} \quad \textbf{(11)}$$

Finally, one can have for any variable a full decomposition as:

$$\begin{aligned} \varphi &= \langle \overline{\varphi} \rangle^i + \langle \varphi' \rangle^i + {}^i\overline{\varphi} + {}^i\varphi' \\ &= \overline{\langle \varphi \rangle^i} + \langle \varphi \rangle^{i\prime} + \overline{{}^i\varphi} + {}^i\varphi' \end{aligned} \quad \textbf{(12)}$$

Equation (12) entails the so-called double decomposition concept.

3. Macroscopic Turbulence Equation for *k*

Equations for the flow turbulent kinetic energy following both *space-time* and *time-space* integration of local instantaneous equations have been presented in de Lemos & Pedras, 2001. For completeness of the work herein, part of that yet unpublished paper is reproduced below.

In all developments found in the literature, the starting point for an equation for the flow turbulent kinetic energy is the microscopic velocity fluctuation $\mathbf{u}'$. An equation for it reads (Hinze, 1959, Warsi, 1998),

$$\rho\left\{\frac{\partial \mathbf{u}'}{\partial t} + \nabla\cdot[\overline{\mathbf{u}}\,\mathbf{u}' + \mathbf{u}'\overline{\mathbf{u}} + \mathbf{u}'\mathbf{u}' - \overline{\mathbf{u}'\mathbf{u}'}]\right\} = -\nabla p' + \mu\nabla^2\mathbf{u}' . \quad (13)$$

Now, the volumetric average of (13) using the Theorem of Local Volumetric Average will give for each term,

$$\langle \frac{\partial \mathbf{u}'}{\partial t} \rangle^v = \frac{\partial}{\partial t}(\phi\langle \mathbf{u}' \rangle^i) , \quad (14)$$

$$\langle \nabla\cdot(\overline{\mathbf{u}}\,\mathbf{u}') \rangle^v = \nabla\cdot(\phi\langle \overline{\mathbf{u}}\,\mathbf{u}' \rangle^i) + \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\cdot(\overline{\mathbf{u}}\,\mathbf{u}')dS \quad (15)$$

$$\langle \nabla\cdot(\mathbf{u}'\overline{\mathbf{u}}) \rangle^v = \nabla\cdot(\phi\langle \mathbf{u}'\overline{\mathbf{u}} \rangle^i) + \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\cdot(\mathbf{u}'\overline{\mathbf{u}})dS \quad (16)$$

$$\langle \nabla\cdot(\mathbf{u}'\mathbf{u}') \rangle^v = \nabla\cdot(\phi\langle \mathbf{u}'\mathbf{u}' \rangle^i) + \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\cdot(\mathbf{u}'\mathbf{u}')dS , \quad (17)$$

$$\langle \nabla\cdot(\overline{\mathbf{u}'\mathbf{u}'}) \rangle^v = \nabla\cdot(\phi\langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i) + \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\cdot(\overline{\mathbf{u}'\mathbf{u}'})dS , \quad (18)$$

$$\langle \nabla p' \rangle^v = \nabla(\phi\langle p' \rangle^i) + \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\,p'dS , \quad (19)$$

$$\langle \nabla\cdot\nabla\mathbf{u}' \rangle^v = \nabla^2(\phi\langle \mathbf{u}' \rangle^i) + \nabla\cdot\left[\frac{1}{\Delta V}\int_{A_i} \mathbf{n}\mathbf{u}'dS\right] + \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\cdot(\nabla\mathbf{u}')dS . \quad (20)$$

Note that at the interface, A_i , all velocities are null due to the non-slip condition resulting in,

$$\begin{aligned} \rho\frac{\partial}{\partial t}(\phi\langle \mathbf{u}' \rangle^i) + \rho\nabla\cdot\{\phi[\langle \overline{\mathbf{u}}\,\mathbf{u}' \rangle^i + \langle \mathbf{u}'\overline{\mathbf{u}} \rangle^i + \langle \mathbf{u}'\mathbf{u}' \rangle^i - \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i]\} = \\ -\nabla(\phi\langle p' \rangle^i) + \mu\nabla^2(\phi\langle \mathbf{u}' \rangle^i) + \mathbf{R}' \end{aligned} \quad \textbf{(21)}$$

where,

$$\mathbf{R}' = \frac{\mu}{\Delta V}\int_{A_i} \mathbf{n}\cdot(\nabla \mathbf{u}')dS - \frac{1}{\Delta V}\int_{A_i} \mathbf{n}\, p' dS \tag{22}$$

is the fluctuating part of the total drag due to the porous structure.

Expanding further the divergent operators in equations (15), (16), (17) and (18) by means of equations (6) and (8), one ends up with an equation for $\langle \mathbf{u}' \rangle^i$ as,

$$\begin{aligned} \rho\frac{\partial}{\partial t}(\phi\langle \mathbf{u}'\rangle^i) + \rho\nabla\cdot\{\phi[\langle \overline{\mathbf{u}}\rangle^i\langle \mathbf{u}'\rangle^i + \langle \mathbf{u}'\rangle^i\langle \overline{\mathbf{u}}\rangle^i + \langle \mathbf{u}'\rangle^i\langle \mathbf{u}'\rangle^i + \\ \langle {}^i\overline{\mathbf{u}}\,{}^i\mathbf{u}'\rangle^i + \langle {}^i\mathbf{u}'\,{}^i\overline{\mathbf{u}}\rangle^i + \langle {}^i\mathbf{u}'\,{}^i\mathbf{u}'\rangle^i - \overline{\langle \mathbf{u}'\rangle^i\langle \mathbf{u}'\rangle^i} - \langle \overline{{}^i\mathbf{u}'\,{}^i\mathbf{u}'}\rangle^i]\} = \\ -\nabla(\phi\langle p'\rangle^i) + \mu\nabla^2(\phi\langle \mathbf{u}'\rangle^i) + \mathbf{R}' \end{aligned} \tag{23}$$

The determination of the flow macroscopic turbulent kinetic energy follows two different paths in the literature. Lee & Howell, 1987, Wang & Takle, 1995, Antohe & Lage, 1997, and Getachewa *et al.,* 2000, based their turbulence models on $k_m = \overline{\langle \mathbf{u}'\rangle^i\cdot\langle \mathbf{u}'\rangle^i}/2$. They started with a simplified form of equation (23) neglecting the 4th, 5th, 6th. and 8th terms (dispersions). Then they took the scalar product of it by $\langle \mathbf{u}'\rangle^i$ and applied the time-average operator in order to obtain an equation for k_m.

On the other hand, if one starts out with equation (13) and proceed with time-averaging first, one ends up, after volume averaging, with $\langle k\rangle^i = \langle \overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$. This was the path followed by Masuoka & Takatsu, 1996, Kuwahara *et al.,* 1998, Kuwahara & Nakayama, 1998, Takatsu & Masuoka, 1998 and Nakayama & Kuwahara, 1999.

Equation for $\langle k\rangle^i$. The other procedure for composing the flow turbulent kinetic energy is to take the scalar product of (13) by the microscopic fluctuating velocity $\mathbf{u}'$. Then apply both time and volume-operators for obtaining an equation for $\langle k\rangle^i = \langle \overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$. Note that in this case the order of application of both operations is immaterial since no additional mathematical operation (the scalar product) is conducted in between the averaging processes. Therefore, this is the same as applying the volume operator to an equation for k, which for an incompressible constant property flow reads (Hinze, 1959, Warsi, 1998):

$$\rho\left[\frac{\partial k}{\partial t} + \nabla\cdot(\overline{\mathbf{u}}k)\right] = -\rho\nabla\cdot\overline{\left[\mathbf{u}'\left(\frac{p'}{\rho} + \frac{\mathbf{u}'\cdot\mathbf{u}'}{2}\right)\right]} + \mu\nabla^2 k - \rho\overline{\mathbf{u}'\mathbf{u}'}:\nabla\overline{\mathbf{u}} - \rho\varepsilon \tag{24}$$

where $\varepsilon = \nu\overline{\nabla\mathbf{u}':(\nabla\mathbf{u}')^T}$. The volumetric average of (24) has been carried out in detail by de Lemos & Pedras, 2000a, and Pedras & de Lemos, 2001a, and for that only the final modeling equation is here presented.

Application of the operator (1) to (24) takes into consideration that at the interface A_i. (see for example **(15)**) the non-slip condition gives $\overline{\mathbf{u}} = \mathbf{u}' = k = 0$. As a consequence, all integrals over A_i are zero giving for the volumetric average of (24),

$$\begin{aligned} \rho\left[\frac{\partial}{\partial t}(\phi\langle k\rangle^i) + \nabla\cdot[\phi(\langle\overline{\mathbf{u}}\rangle^i\langle k\rangle^i + \langle {}^i\overline{\mathbf{u}}\,{}^ik\rangle^i)]\right] = \\ -\rho\nabla\cdot\left\{\phi\left\langle \overline{\mathbf{u}'\left(\frac{p'}{\rho} + \frac{\mathbf{u}'\cdot\mathbf{u}'}{2}\right)}\right\rangle^i\right\} + \mu\nabla^2(\phi\langle k\rangle^i) \\ -\rho\phi[\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\langle\nabla\overline{\mathbf{u}}\rangle^i + \langle {}^i(\overline{\mathbf{u}'\mathbf{u}'}):{}^i(\nabla\overline{\mathbf{u}})\rangle^i] - \rho\phi\langle\varepsilon\rangle^i \end{aligned} \tag{25}$$

where the space decomposition (2) has been applied to the convection and production terms.

The first term on the right of (25) is normally modeled by a gradient diffusion-like expression of the form:

$$-\rho\nabla\cdot\left\{\phi\left\langle \overline{\mathbf{u}'\left(\frac{p'}{\rho} + \frac{\mathbf{u}'\cdot\mathbf{u}'}{2}\right)}\right\rangle^i\right\} = \rho\nabla\cdot\left[\frac{\mu_{t_\phi}}{\rho\sigma_k}\nabla(\phi\langle k\rangle^i)\right] \tag{26}$$

The third and seventh terms in equation (25) where combined in Pedras & de Lemos, 2001a, for reflecting the influence of a porous matrix on turbulence level as

$$-\nabla\cdot(\phi\langle {}^i\overline{\mathbf{u}}\,{}^ik\rangle^i) - \rho\phi\langle {}^i(\overline{\mathbf{u}'\mathbf{u}'}):{}^i(\nabla\overline{\mathbf{u}})\rangle^i = c_k\rho\phi\frac{\langle k\rangle^i|\overline{\mathbf{u}}_D|}{\sqrt{K}} \tag{27}$$

Further, from the **Theorem of Local Volumetric Average** one has,

$$\langle \nabla \overline{\mathbf{u}} \rangle^i = \frac{1}{\phi} \langle \nabla \overline{\mathbf{u}} \rangle^v = \frac{1}{\phi} \left[\nabla (\phi \langle \overline{\mathbf{u}} \rangle^i) + \frac{1}{\Delta V} \int_{A_i} \mathbf{n} \overline{\mathbf{u}} \, dS \right] = \frac{1}{\phi} \nabla (\phi \langle \overline{\mathbf{u}} \rangle^i) \quad \textbf{(28)}$$

Equation (28) can be used to modify the third term on the left of (25) as,

$$-\rho \phi \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i : \langle \nabla \overline{\mathbf{u}} \rangle^i = -\rho \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D \quad \textbf{(29)}$$

Finally, the use of the Dupuit-Forchheimer expression $\overline{\mathbf{u}}_D = \phi \langle \overline{\mathbf{u}} \rangle^i$ gives for the $\langle k \rangle^i$ transport equation:

$$\rho \left[\frac{\partial}{\partial t} (\phi \langle k \rangle^i) + \nabla \cdot (\overline{\mathbf{u}}_D \langle k \rangle^i) \right] = \nabla \cdot \left[(\mu + \frac{\mu_{t_\phi}}{\sigma_k}) \nabla (\phi \langle k \rangle^i) \right] + P_i + G_i - \rho \phi \langle \varepsilon \rangle^i \quad \textbf{(30)}$$

where

$$P_i = -\rho \langle \overline{\mathbf{u}'\mathbf{u}'} \rangle^i : \nabla \overline{\mathbf{u}}_D \, , \quad G_i = c_k \rho \phi \frac{\langle k \rangle^i |\overline{\mathbf{u}}_D|}{\sqrt{K}} \quad \textbf{(31)}$$

are the production rate of $\langle k \rangle^i$ due to mean gradients of the seepage velocity and the generation rate of intrinsic TKE due the presence of the porous matrix. Equation (30) has been proposed by Pedras & de Lemos, 2001a, where more details on its derivation can be found. The term G_i takes into consideration the generation rate due to the presence of the porous structure. It vanishes as the ratio $\phi/\sqrt{K}$ tends to infinity as in the case of clear fluid flow. The constant c_k was numerically determined in Pedras & de Lemos, 2001b. For a wide range of porosity and Reynolds number a value of 0.28 was found to be suitable for most calculations

Table 1 - Classification of turbulence models for porous media.

Model Class	Authors	General characteristics and treatment of surface integrals	Sequence of integration	Applications
A-L	Lee & Howell, 1987, Wang & Takle, 1995, Antohe & Lage, 1997, Getachewa *et al.,* 2000.	Surface integrals are not applied since models are based on macroscopic Quantities subjected to time-averaging only.	Space-time	Only theory presented. Numerical results using this model are found in Chan *et al.,* 2000.
N-K	Masuoka & Takatsu, 1996, Kuwahara *et al.,* 1998, Kuwahara & Nakayama, 1998, Takatsu & Masuoka, 1998, Nakayama & Kuwahara, 1999.	Masuoka & Takatsu, 1996, assumed a non-null value in their eqn. (11) for the turbulent shear stress $\mathbf{S_t} = -\rho \overline{\mathbf{u}'\mathbf{u}'}$ along the interfacial area A_i. Takatsu & Masuoka, 1998, assume for their volume integral in eqn. (14) a different form zero value for $\mathbf{d} = (\mu/\rho + \mu_t/\sigma_k \rho)\nabla k$ at the interface A_i.	Time-space	Microscopic computations on periodic cell of square rods. Macroscopic model computations presented.
T-C	Gratton *et al.,* 1994, Travkin & Catton, 1992, ,Travkin *et al.,* 1993, Travkin & Catton, 1995, Travkin & Catton, 1998, Travkin *et al.,* 1999	Morphology-based theory. Surface integrals and volume-average operators depend on media morphology.	Time-space	Only theory presented.
P-dL	Pedras & de Lemos, 2000a, Rocamora & de Lemos, 2000a, Pedras & de Lemos, 2001a, Pedras & de Lemos, 2001b.	Double-decomposition theory. Surface integrals involving null quantities at surfaces are neglected. The connection between ***space-time*** and ***time-space*** theories is unveiled.	Time-space	Microscopic computation on periodic cell of circular rods. Macroscopic computations for porous media presented. Results for hybrid domains are found in de Lemos & Pedras, 2000b, Rocamora & de Lemos, 2000b, Rocamora & de Lemos, 2000c, and Rocamora & de Lemos, 2000d.

Comparison of Macroscopic Transport Equations. A comparison between terms in the transport equation for k_m and $\langle k \rangle^i$ can now be conducted. Pedras & de Lemos, 2000a, have already showed the connection between these two quantities as being,

$$\langle k \rangle^i = \langle \overline{\mathbf{u}' \cdot \mathbf{u}'} \rangle^i / 2 = \overline{\langle \mathbf{u}' \rangle^i \cdot \langle \mathbf{u}' \rangle^i} / 2 + \langle \overline{{}^i\mathbf{u}' \cdot {}^i\mathbf{u}'} \rangle^i / 2 = k_m + \langle \overline{{}^i\mathbf{u}' \cdot {}^i\mathbf{u}'} \rangle^i / 2 \quad (32)$$

Expanding the correlation forming the production term P_i by means of (2), a connection between the two generation rates can also be written as,

$$P_i = -\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i : \nabla\overline{\mathbf{u}}_D = -\rho\left(\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i} : \nabla\overline{\mathbf{u}}_D + \overline{\langle{}^i\mathbf{u}'\,{}^i\mathbf{u}'\rangle^i} : \nabla\overline{\mathbf{u}}_D\right) = P_m - \rho\overline{\langle{}^i\mathbf{u}'\,{}^i\mathbf{u}'\rangle^i} : \nabla\overline{\mathbf{u}}_D \qquad (33)$$

where P_m is the production rate of k_m. One can note that all production rate of k_m due to the mean flow constitutes only part of the general production rate responsible for maintaining the overall level of $\langle k\rangle^i$. Ultimately, equation (33) indicates that models based on the time-space integration sequence seems to be of a greater universality than those based on volume-time integration of the transport equations.

4. General Classification of Turbulence models for Porous Media

Based on the derivations above, one can establish a general classification of the models presented so far in the literature. Table 1 classifies all proposals into four major categories. These classes are based on the sequence of application of averaging operators, on the handling of surface integrals and on the applications reported so far.

The A-L models make use of transport equations for $k_m = \overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}/2$ instead of $\langle k\rangle^i = \langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$. Consequently, this methodology applies only time-averaging procedure to already established macroscopic equations (see for example Hsu & Cheng, 1990, for macroscopic equations). In this sense, the sequence *space-time* integration is employed and surface integrals are not manipulated since macroscopic quantities are the sole independent variables used. Application of this theory is found in Chan *et al.,* 2000.

N-K models constitute the second class of models here compiled. It is interesting to mention that Masuoka & Takatsu, 1996, assumed a non-null value for the turbulent shear stress, $\mathbf{S_t} = -\rho\overline{\mathbf{u}'\mathbf{u}'}$, along the interfacial area A_i in their eqn. (11). With that, their surface integral $\int_{A_i}\mathbf{S_t}\cdot\mathbf{n}\,dA$ was associated with the Darcy flow resistance term. Yet, using the Boussinesq approximation as in their eqn. (7), $\mathbf{S_t} = 2\mu_t\mathbf{D} - \frac{2}{3}k\,\mathbf{I}$, one can also see that both μ_t and *k* will vanish at the surface A_i, ultimately indicating that the surface integral in question is actually equal to zero. Similarly, Takatsu & Masuoka, 1998, assumed for their surface integral in eqn. (14), $\int_{A_i}\mathbf{d}\cdot\mathbf{n}\,dA$, a non-null value where $\mathbf{d} = (\mu/\rho + \mu_t/\sigma_k\rho)\nabla k$. Here also it is worth noting that $\nabla k = \overline{\mathbf{u}'\cdot(\nabla\mathbf{u}')^T}$ and that, at the interface A_i, $\nabla k = 0$ due to the non-slip condition. Consequently, also in this case the surface integral of **d** over A_i is of zero value. In regard to the average operators used, N-K models follow the *time-space* integration sequence. Calibration of the model required microscopic computations on a periodic cell of square rods. Macroscopic results in a channel filled with a porous material were also a test case run by Nakayama & Kuwahara, 1999.

The work developed in a series of papers using a morphology-oriented theory is here group in the T-C model category shown in Table 1. In this morphology-based theory, surface integrals resulting after application of volume-average operators depend on the media morphology. Governing equations set up for turbulent flow, although complicated at first sight, just follow usual volume integration technique applied to standard *k-ε* and *k-L* turbulence models. In this sense, time-space integration sequence is followed. No closure is proposed for the unknowns surface integrals (and morphology parameters) so that practical application of such development in solving *real-world* engineering flows is still a challenge to be overcome. Nevertheless, the developed theory seems to be mathematically correct even though additional *ad-hoc* information is still necessary to fully model the remaining unknowns and medium-dependent parameters.

Lastly, the model group named P-dL uses the recently developed *double-decomposition* theory just reviewed above. In this development, all surface integrals involving null quantities at interface A_i are neglected. The connection between ***space-time*** and ***time-space*** theories is made possible due to the splitting of the dependent variables into four (rather that two) components, as expressed by equation (12) above. For the momentum and energy equations, the double-decomposition approach has proved that either *time-space* or *space-time* order of application of averaging operators is immaterial. For the turbulence kinetic energy equation, however, the order of application of such mathematical operators will define different quantities being transported (Pedras & de Lemos, 2000a, Rocamora & de Lemos, 2000a). Microscopic computation on a periodic cell of circular rods was used in order to calibrate the proposed model (Pedras & de Lemos, 2001a). Pedras & de Lemos, 2001b, further presented macroscopic computations for flow in a channel filled with a porous material. Further results for hybrid domains (*porous medium - clear fluid*) are found in de Lemos & Pedras, 2000b, Rocamora & de Lemos, 2000b, Rocamora & de Lemos, 2000c, and Rocamora & de Lemos, 2000d.

5. Conclusions

This paper presented the views in the literature for characterizing turbulent flow in porous media. The equations for the flow turbulent kinetic energy were derived following both *time-space* and *space-time* integration sequences. A comparison between corresponding terms in these two transport equations was carried out. A general classification of all models published so far lead to four major groups that were commented upon. The treatment of the drag term following both methodologies was also discussed by means of the mechanical energy equation for the flow. This work was intended to provide a general view on the *state-of-the-art* of turbulence modeling in porous media. It is the authors' expectation that the contribution herein stimulates further research on this important area.

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7. References

Antohe, B.V., Lage, J.L., 1997, A general two-equation macroscopic turbulence model for incompressible flow in porous media, *Int. J. Heat Mass Transfer*, vol. 40, pp. 3013-3024.

Awartani, M., Hamdan, M.H., 1999, Non-reactive gas-particle models models of flow through porous media, *Applied Mathematics and Computation*, vol. 100, pp. 93-102.

Bear, J., 1972, *Dynamics of Fluids in Porous Media*, American Elsevier Pub. Co., New York.

Bird, R. B., Stewart, W. E., and Lightfoot, E. N., 1960, *Transport Phenomena*, John Wiley & Sons, New York.

Brinkman, H. C., 1947, A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles, *Appl. Sci. Research*, A 1, pp. 27-34.

Chan, E.C., Lien, F.-S., Yovanovich, M.M., 2000, Numerical Study of Forced Flow in a Back-Step Channel Through Porous Layer, *Proc. of 34th ASME-National Heat Transfer Conference* (on CD-ROM), ASME-HTD-I463CD, Paper NHTC2000-12118, ISBN:0-7918-1997-3, Pittsburgh, Pennsylvania, August 20-22.

Darcy, H., 1856, *Les Fontaines Publiques de la Vile de Dijon*, Victor Dalmond, Paris.

de Lemos, M.J.S., Pedras, M.H.J., 2000a, Modeling Turbulence Phenomena in Incompressible Flow Through Saturated Porous Media, *Proc. of 34th ASME-National Heat Transfer Conference* (on CD-ROM), ASME-HTD-I463CD, Paper NHTC2000-12120, ISBN:0-7918-1997-3, Pittsburgh, Pennsylvania, August 20-22.

de Lemos, M.J.S., Pedras, M.H.J., 2000b, Simulation of Turbulent Flow Through Hybrid Porous Medium-Clear Fluid Domains, *Proc. of IMECE2000 - ASME - Intern. Mech. Eng. Congr.*, ASME-HTD-366-5, pp. 113-122, ISBN:0-7918-1908-6, Orlando, Florida, November 5-10.

de Lemos, M.J.S., Pedras, M.H.J., 2001, Recent Mathematical Models For Turbulent Flow In Saturated Rigid Porous Media, *Journal of Fluids Engineering* (submitted).

Forchheimer, P., 1901, Wasserbewegung durch Boden, *Z. Ver. Deutsch. Ing.*, vol. 45, pp. 1782-1788.

Fox, R.W., McDonald, A.T., 1998, *Introduction to fluids mechanics*, John Wiley & Sons, New York, 5th, Ed., p. 156.

Getachewa, D., Minkowycz, W.J., Lage, J.L., 2000, "A modied form of the k-ε model for turbulent flow of an incompressible fluid in porous media", *Int. J. Heat Mass Transfer*, vol. 43, pp. 2909-2915.

Gratton, L., Travkin, V.S., Catton, I., 1994, Numerical solution of turbulent heat and mass transfer in a stratified geostatistical porous layer for high permeability media, ASME Proccedings HTD-vol. 41, pp. 1-14.

Gray, W. G., Lee, P. C. Y., 1977, On the theorems for local volume averaging of multiphase system, *Int. J. Multiphase Flow*, vol. 3, pp. 333-340.

Hinze, J. O., 1959, *Turbulence*, McGraw-Hill, New York.

Hsu, C. T., Cheng, P., 1990, Thermal dispersion in a porous medium, *Int. J. Heat Mass Transfer*, vol. 33, pp. 1587-1597.

Kuwahara, F., Kameyama, Y., Yamashita, S., and Nakayama, A., 1998, Numerical modeling of turbulent flow in porous media using a spatially periodic array, *J. Porous Media*, vol. 1, pp. 47-55.

Kuwahara, F., Nakayama, A., 1998, Numerical modeling of non-Darcy convective flow in a porous medium, *Proc. 11th Int. Heat Transf. Conf.*, Kyongyu, Korea, August 23-28.

Lage, J.L., 1998, The fundamental theory of flow through permeable media from Darcy to turbulence, in TRANSPORT PHENOMENA IN POROUS MEDIA, Eds. D.B. Ingham & I. Pop, Elsevier Science Ltd., ISBN: 0-08-042843-6, 446 pgs.

Lee, K., Howell, J. R., 1987, Forced convective and radiative transfer within a highly porous layer exposed to a turbulent external flow field, *Proc. of the 1987 ASME-JSME Thermal Eng. Joint Conf.*, vol. 2, pp. 377-386.

Lim, I.G., Matthews, R.D., 1993, Development of a model for turbulent combustion within porous inert media, *Transport Phenomena in Thermal Engineering* **1**, (Eds. J.S. Lee, S.H. Chung, K.Y. Kim), pp. 631-636, Begell House, New York.

Masuoka, T., Takatsu, Y., 1996, Turbulence model for flow through porous media, *Int. J. Heat Mass Transfer*, vol. 39, pp. 2803-2809.

Mickley, H.S., Smith, K.A., Korchak, E.I., 1965, Fluid Flow in Packed Beds, *Chemical Engineering Sciences*, v. 20, pp. 237-246.

Nakayama, A., Kuwahara, F., 1999, A Macroscopic Turbulence Model for Flow in a Porous Medium," *ASME Journal of Fluids Engineering,* vol. 121, pp. 427-433.

Pedras, M.H.J., de Lemos, M.J.S., 1998, Results for Macroscopic Turbulence Modeling for Porous Media, *Proc. of ENCIT98- 7th Braz. Cong. Eng. Th. Sci.*, vol. 2, pp. 1272-1277, Rio de Janeiro, Brazil, November 3-6 (in Portuguese).

Pedras, M.H.J., de Lemos, M.J.S., 1999a, On Volume and Time Averaging of Transport Equations for Turbulent Flow in Porous Media, *Proc. of 3rd ASME/JSME Joint Fluids Engineering Conference* (on CDROM), ASME-FED-248, Paper FEDSM99-7273, ISBN 0-7918-1961-2, San Francisco, California, July 18-23.

Pedras, M.H.J., de Lemos, M.J.S., 1999b, Macroscopic Turbulence Modeling for Saturated Porous Media, *Proc. of COBEM99 - 15th Braz. Congr. Mech. Eng.* (on CD-ROM), ISBN: 85-85769-03-3*,* Águas de Lindóia, São Paulo, Brazil, November 22-26 (In Portuguese).

Pedras, M.H.J., de Lemos, M.J.S., 2000a, On the definition of turbulent kinetic energy for flow in porous media, *Intern. Comm. Heat and Mass Transfer*, vol. 27 (2) pp. 211-220.

Pedras, M.H.J., de Lemos, M.J.S., 2000b, Numerical Solution of Turbulent Flow in Porous Media using a Spatially Periodic Cell and the Low Reynolds k-ε Model, *Proc. of CONEM2000 - National Mechanical Engineering Congress* (on CD-ROM), Natal, Rio Grande do Norte, Brazil, August 7-11 (in Portuguese).

Pedras, M.H.J., de Lemos, M.J.S., 2001a, Macroscopic Turbulence Modeling for Incompressible Flow Through Undeformable Porous Media, *Intern. J. Heat and Mass Transfer*, vol. 44, n. 6, pp. 1081-1093.

Pedras, M.H.J., de Lemos, M.J.S., 2001b, Simulation of Turbulent Flow in Porous Media Using a Spatially Periodic Array and a Low Re Two-Equation Closure, *Numerical Heat Transfer - Part A Applications*, vol. 39, n. 1, pp. 35-59.

Prausnitz, J.M., Wilhelm, R.H., 1957, Turbulent concentrations fluctuations in a packed bed, *Industrial and Engineering Chemistry*, v. 49, pp. 237-246.

Prescott, P.J., Incropera, F.P., 1995, The effect of turbulence on solidification of a binary metal alloy with electromagnetic stirring, *ASME J. Heat Transfer*, **117**, pp.716-724

Rocamora Jr., F.D., de Lemos, M.J.S., 1998, Numerical Solution of Turbulent Flow in Porous Media using a Spatially Periodic Array and the k-ε Model, *Proc. ENCIT-98 - 7th. Braz. Cong. Eng. Th. Sci.*, vol. 2, pp. 1265-1271, Rio de Janeiro, RJ, Brazil, November 3-6.

Rocamora Jr., F.D., de Lemos, M.J.S., 1999, Simulation of Turbulent Heat Transfer in Porous Media Using a Spatially Periodic Cell and the k-ε Model, *Proc. of COBEM99 - 15th Braz. Congr. Mech. Eng.* (on CD-ROM), ISBN: 85-85769-03-3*,* Águas de Lindóia, São Paulo, Brazil, November 22-26.

Rocamora Jr., F.D., de Lemos, M.J.S., 2000a, Analysis Of Convective Heat Transfer For Turbulent Flow In Saturated Porous Media, *Intern. Comm. Heat and Mass Transfer*, vol. 27 (6), pp. 825-834.

Rocamora Jr., F.D., de Lemos, M.J.S., 2000b, Prediction of Velocity and Temperature Profiles for Hybrid Porous Medium-Clean Fluid Domains, *Proc. of CONEM2000 - National Mechanical Engineering Congress* (on CD-ROM), Natal, Rio Grande do Norte, Brazil, August 7-11.

Rocamora Jr., F.D., de Lemos, M.J.S., 2000c, Laminar Recirculating Flow And Heat Transfer In Hybrid Porous Medium-Clear Fluid Computational Domains, *Proc. of 34th ASME-National Heat Transfer Conference* (on CD-ROM), ASME-HTD-I463CD, Paper NHTC2000-12317, ISBN:0-7918-1997-3, Pittsburgh, Pennsylvania, August 20-22.

Rocamora Jr., F.D., de Lemos, M.J.S., 2000d, Heat Transfer In Suddenly Expanded Flow in a Channel With Porous Inserts, *Proc of IMECE2000 - ASME - Intern. Mech. Eng. Congr.*, ASME-HTD-366-5, pp. 191-195, ISBN:0-7918-1908-6, Orlando, Florida, November 5-10.

Rudraiah, N., 1988, Turbulent convection in porous media with non-Darcy effects, ASME-HTD-vol. 96, 1, pp. 747-754.

Rudraiah, N., Siddheshwar, G., Masuoka, T, 1995, Turbulent convection in a high porosity porous medium with convective acceleration, *Journal of Mathematical and Physical Sciences*, 19, pg. 93- 117.

Slattery, J. C., 1967, Flow of viscoelastic fluids through porous media, *A.I.Ch.E.J.*, vol. 13, pp. 1066-1071.

Takatsu, Y., Masuoka, T., 1998, Turbulent Phenomena in flow through porous media, *Journal of Porous Media*, vol. 1, n. 3, pp. 243-251.

Takatsu, Y., Masuoka, T., 1998, Turbulent phenomena in flow through porous media, *J. Porous Media*, vol. 3, pp. 243-251.

Travkin, V.S., Catton, I., 1992, Models of turbulent thermal diffusivity and transfer coefficients for a regular packed bed of spheres, *Proc. 28th National Heat Transfer Conference*, San Diego, C-4, ASME-HTD-193, pp 15-23.

Travkin, V.S., Catton, I., 1995, A two temperature model for turbulent flow and heat transfer in a porous layer, *Journal of Fluid Engineering*, vol. 117, pp. 181-188.

Travkin, V.S., Catton, I., 1998, Porous media transport descriptions - non-local, linear and non-linear against effective thermal/fluid properties, *Advances in Colloid and Interface Science*, vol. 76-77, pp. 389-443.

Travkin, V.S., Catton, I., Gratton, L., 1993, Single-phase turbulent transport in prescribed non-isotropic and stochastic porous media, *Heat Transfer in Porous Media*, ASME-HTD-240, pp. 43- 48.

Travkin, V.S., Hu, K., Catton, I., 1999, Turbulent Kinetic Energy and Dissipation Rate Equation Models for Momentum Transport in Porous Media, *Proc. 3rd ASME/JSME Joint Fluids Engineering Conference* (on CD-ROM), Paper FEDSM99-7275, San Francisco, California, 18-23 July.

Vafai, K., Tien, C.L., 1981, Boundary and inertia effects on flow and heat transfer in porous media, *Int. J. Heat Mass Transfer*, vol. 24, pp. 195-203.

Wang, H., Takle, E.S., 1995, Boundary-layer flow and turbulence near porous obstacles, *Boundary Layer Meteorology*, vol. 74, pp. 73-88.

Ward, J.C., 1964, Turbulent flow in porous media, *J. Hydraul. Div. ASCE*, vol. 90 (HY5), pp. 1-12.

Warsi, Z.U.A., 1998, Fluid Dynamics - Theoretical and Computational Approaches, 2nd. ed., CRC Press, Boca Raton.

Whitaker, S., 1969, Advances in theory of fluid motion in porous media, *Indust. Engng. Chem.*, vol. 61, pp. 14-28.

Whitaker, S., 1999, *The Method of Volume Averaging,* Kluwer Academic Publishers, Dordrecht.