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Estimation of Upstream Velocity Profiles in an Incompressible Turbulent Boundary Layer via Universal Laws

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Abstract. *An inverse problem is solved for the estimation of upstream velocity profiles in an incompressible turbulent boundary layer over a smooth flat plate. The procedure is based on hotwire velocity measurements obtained at several different locations in the stream. The paper also presents a direct problem approach for the solution of the turbulent boundary layer equations. The direct approach resorts to a finite difference method and to the Cebeci-Smith turbulence model. The unknown upstream velocity profile is constructed by using the composite Coles's law of the wall, law of the wake formulation. The friction velocity, Von Kármán constant, law of the wall constant, Coles's wake-strength parameter and boundary layer thickness for the initial profile are determined as unknown parameters by the Levenberg-Marquardt algorithm. The effects on solution about the location of the measurement station are examined. The results provided by the direct numerical simulation of the flow are also validated by data obtained through the hotwire anemometry technique in a low-speed wind tunnel. The estimated upstream velocity profiles are shown to compare favourably with hotwire anemometry measurements at the same location.*

Keywords: *turbulence, boundary layer, inverse problem, law of the wall.*

1. Introduction

This work is concerned with the estimation of upstream velocity profiles in an incompressible turbulent boundary layer over a smooth flat plate. This problem is of utmost importance for the experimentalist who would like to have analytical tools for the establishment of upstream profiles that would conform downstream in the test section to some pre-desired condition. Large wind tunnels in particular, which make use of vortice generators for the establishment of very thick boundary layers would greatly benefit from such tools. Often the experimentalist has to resort to trial and error procedures to propose the design of vortice generators; these procedures are necessarily difficult and, in the end, costly. Here we give the first steps in developing a useful procedure that can be used in the future as a helping tool for the design of large wind tunnels. The solution procedure will resort to the inverse problem technique coupled with the well known morphology of the turbulent boundary layer expressed in its two universal laws, the law of the wall and the law of the wake.

A wide variety of inverse heat conduction problems have been solved in the last two decades for the estimation of initial or boundary conditions, physical properties, geometric parameters, or heat source intensities. Özisik and Orlande(2000) and Su and Silva Neto(2001) among others, present some of the methods developed for the solution of such problems. Despite many potential applications, inverse convection problems have only recently received some attention. Moutsoglou(1989) apparently was the first to address an inverse convection problem that has used a sequential function specification algorithm for the estimation of the asymmetric heat flux in mixed convection in a vertical channel. The same author (Moutsouglou, 1990) has also applied the whole domain regularization technique in an inverse analysis to estimate wall heat flux in an elliptic laminar forced convection problem. Raghunath(1993) applied the quasi-Newton conjugate gradient method, which is a special case of the conjugate gradient method, to obtain the temperature profile at the entrance of a thermally developing hydrodynamically developed laminar flow between parallel plates. Huang and Özisik(1992) have applied the regular and modified conjugate gradient methods for the estimation of a steady state wall heat flux in a hydrodynamically developed laminar flow in a parallel plate duct. The same method has been applied by Bokar and Özisik(1995) to estimate the time dependence of inlet temperature in similar flow conditions. Liu and Özisik, 1996 have used the Levenberg-Marquardt algorithm for the minimization procedure for estimation of the thermal conductivity and thermal capacity of laminar flow through a circular duct by using transient temperature readings at a single downstream location. Machado and Orlande(1997) have used the conjugate gradient method with an adjoint equation to estimate the timewise and spacewise variation of the wall heat flux in a parallel plate channel. An inverse problem for estimating the heat flux to a power-law non-Newtonian fluid in a parallel plate channel flow was solved by Machado and Orlande(1998) by using the same method. Hsu et al.(1998) applied the linear least-squares method for simultaneous estimation of the inlet temperature and wall heat flux in a laminar circular duct flow. Huang and Chen(2000) have applied the conjugate gradient method in a three-dimensional inverse forced convection problem to estimate a surface heat flux. Li and Yan(1999) applied the conjugate gradient method for the estimation of the space and time dependent wall heat flux for unsteady laminar forced convection between parallel flat plates, similar to that studied by Machado and Orlande(1997). Cho et. al.(1999) developed an optimization procedure to find the inlet concentration profile for uniform deposition in a cylindrical chemical vapor deposition chamber using local random search technique. In a similar work, Cho et. al.(1999) solved an optimization problem to find the inlet velocity profile that yields as uniform an epitaxial layer as possible in a vertical metalorganic chemical vapor deposition (MOCVD) reactor.

Few works have been carried out on inverse problems in turbulent flow despite its obvious technological relevance. Liu and Özisik(1996) applied the conjugate gradient method with an adjoint equation for solving the inverse turbulent convection problem of estimating the timewise varying wall heat flux in parallel plate ducts. Su et al.(2000) applied the Levenberg-Marquardt method to estimate nonuniform wall heat flux in a thermally developing, hydrodynamically developed turbulent flow in a circular pipe based on temperature measurements obtained at several different locations in the stream. Su and Silva Neto(2001) solved an inverse heat convection problem to estimate simultaneously the inlet temperature profile and the wall heat flux distribution in a thermally developing, hydrodynamically developed turbulent flow in a circular pipe based on temperature measurements obtained at several different positions in the stream, using the Levenberg-Marquardt method.

While all above mentioned works are dedicated to internal flow and heat transfer problems, Alekseev(1997) has shown the feasibility of estimation of free stream parameters in a compressible laminar boundary layer which is governed by the parabolized Navier-Stokes (PNS) equations. In this work, we solve an inverse problem for the estimation of an upstream velocity profiles in an incompressible turbulent boundary layer over a smooth flat plate. The solution procedure is based on velocity measurements obtained at several different locations in the stream; the measurements were obtained through the hotwire anemometry technique. The direct problem of a turbulent boundary layer is solved with a finite difference method and uses the Cebeci-Smith turbulence model. The unknown upstream velocity profile is represented by the composite Coles's law of the wall, law of the wake profile. The friction velocity, Von Kármán constant, law of the wall constant, Coles's wake-strength parameter and boundary layer thickness for the initial profile are determined as unknown parameters by the Levenberg-Marquardt algorithm. The effects on solution about the location of the measurement station are examined.

2. Mathematical Formulation of the Direct Problem

The Reynolds averaged governing equations for a steady, incompressible and two-dimensional turbulent boundary layer can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\partial}{\partial y}(\nu\frac{\partial u}{\partial y} - \overline{u'v'}). \tag{2}$$

The notation is classical. The eddy viscosity concept is used, that implies that the turbulent stress term is related to the mean rate of strain by

$$-\overline{u'v'} = \nu_t\frac{\partial u}{\partial y}. \tag{3}$$

In the Cebeci-Smith model, the eddy viscosity model of Boussinesq is invoked together with the mixing length concept of Prandtl and the Van Driest damping function for the characteristic length of the flow. Thus, near the wall

$$\nu_t = l^2|\frac{\partial u}{\partial y}|, \tag{4}$$

where

$$l = \kappa y \big[1 - \exp{(-\frac{y^+}{A^+})} \big], \tag{5}$$

and $y^+ = yu_\tau/\nu$, u_τ is the friction velocity , ν is the kinematic viscosity, and $\kappa = 0.41$ (Von K*á*rm*á*n constant).
Constant A^+ is defined by

$$A^+ = 26 \Big(1 + yp^+ \Big)^{-\frac{1}{2}}, \tag{6}$$

where $p^+ = (dp/dx)/\rho(u_\tau)^2$.
The defect region is described through an eddy viscosity of the type

$$\nu_t = C_1 u_e \delta_1 \gamma, \tag{7}$$

where δ_1 is the boundary layer displacement thickness, $C_1 = 0.0168$, and γ is the intermittency factor of Klebanoff, given by

$$\gamma = \big[1 + 5.5 (\frac{y}{\delta})^6 \big]^{-1}, \tag{8}$$

and δ is the boundary layer thickness.
The partial differential equations have to be solved with appropriate boundary conditions,

$$u = 0, \quad \text{for} \quad y = 0 \tag{9}$$

$$v = 0, \quad \text{for} \quad y = 0 \tag{10}$$

$$u = u_e(x), \quad \text{as} \quad y \to \infty \tag{11}$$

$$u = u_0(y), \quad \text{for} \quad x = x_0 \tag{12}$$

$$v = v_0(y). \quad \text{for} \quad x = x_0 \tag{13}$$

If all fluid properties, coefficients of turbulent modelling, and boundary conditions are known, the direct problem given by Eq. (1) to Eq. (13) can be solved to obtain the velocity field of the fluid in the turbulent boundary layer. In this work, the direct problem defined by equations Eq. (1) to Eq. (13) is solved through an implicit finite difference method.

3. Solution of the Inverse Problem

In the inverse problem considered in this work, we are looking for the unknown upstream velocity profile $u_0(y)$; this must be evaluated from velocity measurements taken at several downstream points in the flow field.
The unknown upstream velocity profile is represented by the composite Coles's law of the wall, law of the wake formulation

$$u_0(y) = u_\tau \left[\frac{1}{\kappa} \ln y^+ + A + \frac{2\tilde{\pi}}{\kappa} \sin^2 \left(\frac{\pi}{2} \frac{y}{\delta} \right) \right], \tag{14}$$

where κ (Von Kármán constant), A (law of the wall constant), u_τ (friction velocity), $\tilde{\pi}$ (Cole's wake-strength) and δ (boundary layer thickness) are parameters to be determined.
Upon the parameterization given by Eq. (14), the inverse problem has been formulated as a parameter estimation problem. The solution of this inverse problem for the estimation of the five unknown parameters is based on the minimization of the ordinary least squares norm defined by

$$R(\vec{P}) = \sum_{m=1}^{M} \big[u_m(\vec{P}) - Z_m \big]^2, \tag{15}$$

where $u_m(x_m, y_m)$ are the calculated velocities and $Z_m(x_m, y_m)$ are the measured velocities at points (x_m, y_m), $m = 1, 2, ..., M$, with M being the total number of measurement points.
The vector of unknown parameters is formed by

$$\vec{P}^T = [p_1, p_2, p_3, p_4, p_5] = [\kappa, A, u_\tau, \tilde{\pi}, \delta]. \tag{16}$$

Equation (15) can be written in the following form,

$$R(\vec{P}) = [\vec{u}(\vec{P}) - \vec{Z}]^T[\vec{u}(\vec{P}) - \vec{Z}] = \vec{F}^T\vec{F} \tag{17}$$

with $\vec{F}$ being the difference vector between calculated and measured velocities, $F_m = u_m - Z_m$, $m = 1, 2, ..., M$.

As the inverse problem is solved as an optimization problem, our objective is to minimize the norm $R(\vec{P})$,

$$\frac{\partial R}{\partial p_n} = \frac{\partial}{\partial p_n}(\vec{F}^T\vec{F}) = 0, \quad n = 1, ..., 5. \tag{18}$$

Considering a Taylor expansion,

$$F(\vec{P}^{k+1}) = F(\vec{P}^k + \Delta\vec{P}^k) = F(\vec{P}^k) + \sum_{n=1}^{N_P} \frac{\partial F(\vec{P})}{\partial p_n}\Delta p_n + O(\Delta p_n^2), \tag{19}$$

keeping only the terms up to the first order terms in Eq. (19), and plugging the resulting expression into Eq. (18), we obtain the normal equation,

$$J^T J \Delta\vec{P}^k = -J^T\vec{F}, \tag{20}$$

where the elements of the Jacobian matrix are

$$J_{mn} = \frac{\partial u_m}{\partial p_n}, \quad m = 1, 2, ..., M \quad \text{and} \quad n = 1, .., 5. \tag{21}$$

Summing up with a damping factor λ to improve the convergence behaviour we have the Levenberg-Marquardt method,

$$(J^T J + \lambda D)\Delta\vec{P} = -J^T\vec{F}, \tag{22}$$

where D represents the diagonal matrix.

Equation (22) is then written in a form convenient to be used in an iterative procedure,

$$\Delta P^k = -(J^{kT}J^k + \lambda^k D^k)^{-1}J^{kT}\vec{F}^k, \tag{23}$$

where k is the iteration index.

A new estimation of the parameters, $\vec{P}^{k+1}$, is calculated by

$$\vec{P}^{k+1} = \vec{P}^k + \Delta\vec{P}^k. \tag{24}$$

Please, note that the problem given by Eq. (22) is different from that given by Eq. (20). Nevertheless, the procedure aims at reducing the value of the damping factor with the iterations so that when convergence is achieved, the obtained solution is about the same as that for the original problem. The iterative procedure starts with an initial guess for parameters, $\vec{P}^0$, and new estimates, $\vec{P}^{k+1}$ are sequentially obtained using Eq. (24) with $\Delta\vec{P}^k$ given by Eq. (23) until the convergence criterion

$$|\frac{\Delta p_n^k}{p_n^k}| < \epsilon, \quad n = 1, ..., 5 \tag{25}$$

is satisfied, where ϵ is a small real number, such as 10^{-8}. The elements of the Jacobian matrix as well as the right hand term of Eq. (22) are calculated by using the solution of the direct problem defined by Eq. (1) to Eq. (13), as described in the previous section.

4. Experimental Apparatus and Instrumentation

The experiments were carried out in a low-speed wind tunnel located at the Laboratory of Turbulence Mechanics of COPPE/UFRJ. The wind tunnel is of open circuit type and has a 5 m long test section with square cross section of 0.67 m x 0.67 m. Wind speed is continuously variable from 0.5 to 3.5 m/s. The turbulent intensity level in the freestream was about 1.0%. The streamwise pressure gradient was closely set to zero by adjusting the roof of the tunnel according to readings of eight equally spaced pressure taps. Mean velocity profiles and turbulent intensity levels were measured by using a DANTEC hotwire anemometer series 55M with a standard P11 probe. A Pitot tube, a high precision inclined multi-tube manometer, and a computer controlled traverse gear were also used. Output signals of the hotwire anemometer were transmitted to a PC through a 16-bit data aquisition card.

Six longitudinal velocity profiles were measured at stations 3.20m, 3.25m, 3.30m, 3.35m, 3.40m and 3.45m from the beginning of the test section. All profiles were measured over the central line of the test section, sufficiently far from the side walls. Around 60 mean velocity measurement points were taken for each profile.

The friction velocity (u_τ), the Coles's wake-strength parameter ($\tilde{\pi}$), the boundary layer thickness (δ), the Von Kármán constant (κ) and the law of the wall constant (A) for each velocity profile measured were obtained by means a program developed in Mathematica software for treatment of the experimental data.

5. Results

The study was developed in three steps. The objective of the first step was to validate the numerical solution for the direct problem by comparison with the experimental data. The experimental profile taken at station $3.20m$ was used as the inlet boundary condition and velocity profiles at the same stations of the experimental measurements were calculated numerically. Figure 1 shows good agreement between the velocity profiles obtained by the numerical simulation and the experimental profiles. In Fig. 2, the same comparison is shown in inner variables.

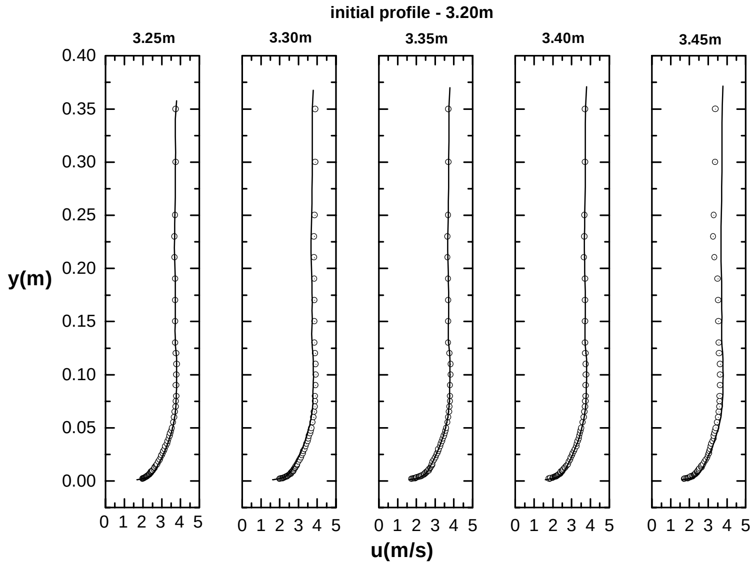

Figure 1: Validation of the numerical solution for the direct problem. Comparison of calculated and measured velocity profiles. Points are the experimental data and line is the simulation.

The second step aimed at estimating the upstream velocity profile, at station $x = 3.20m$, by the inverse method and to compare it with the experimentally obtained profile. A single experimental profile at a downstream station was used in the inverse analysis. We successfully estimated the upstream velocity profile using the measured profiles at stations $x = 3.25m$, $x = 3.35m$ and $x = 3.45m$. Figure 3 shows that the estimated upstream velocity profiles match smoothly the measured upstream profile. Figure 4 shows the same comparison in inner variables.

In the third step, we checked the precision of the numerical simulation of the turbulent boundary layer, as a direct problem, if the estimated initial profile could be used as the inlet boundary condition. We used the estimated values of parameters u_τ, κ, A, $\tilde{\pi}$ and δ to construct the initial boundary condition and compared the results with that obtained by using directly the measured initial profile. Figure 5 shows a comparison of the velocity profiles at all stations when was used an initial profile obtained by inverse method known the experimental profile at $3.25m$ station, the known profile nearest of the initial profile. Figure 6 shows a comparison of these results in inner variable.

The direct simulation with the initial profile obtained by inverse method known each single experimental profile was done. Then Fig. 7 shows a comparison of the velocity profiles at all stations when was used an initial profile obtained by inverse method known the experimental profile at $3.45m$ station, the known profile farther of the initial profile. Figure 8 shows a comparison of these results in inner variable.

The friction velocity is always difficult to determine experimentally. In this work, the friction velocity was determined by means a non linear regress program developed in Mathematica software for treatment of the experimental data. Table 1 shows values of friction velocity estimated by the inverse method compared with the measured values of friction velocity at station $x = 3.20m$, that was 0.160. As can be seen, the relative errors were less than 5% for all estimations, which shows that the inverse analysis can be used to determine the friction velocity based on mean velocity measurements in the flow field.

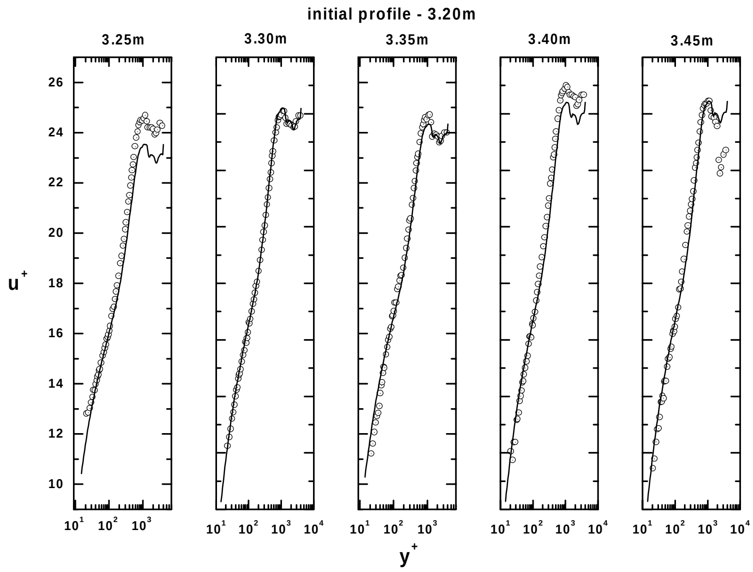

Figure 2: Validation of the numerical solution for the direct problem. Comparison of calculated and measured velocity profiles in inner variable.

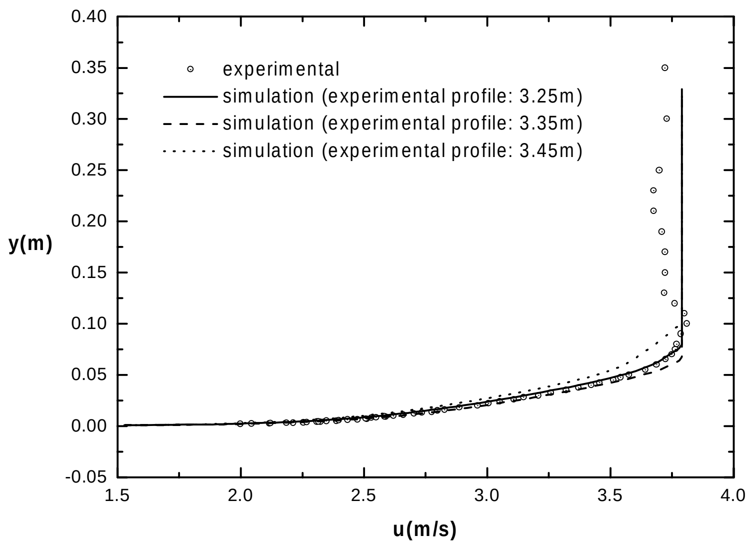

Figure 3: Comparison of estimated initial velocity profile with experimental data using one measured velocity profile at different longitudinal stations.

Table 1: Friction velocity obtained by inverse method.

known velocity profile	$\mathbf{u}_\tau$	**relative error (%)**
$x = 3.25m$ station	0.155	-3.1
$x = 3.30m$ station	0.164	2.5
$x = 3.35m$ station	0.158	-1.3
$x = 3.40m$ station	0.156	-2.5
$x = 3.45m$ station	0.153	-4.4

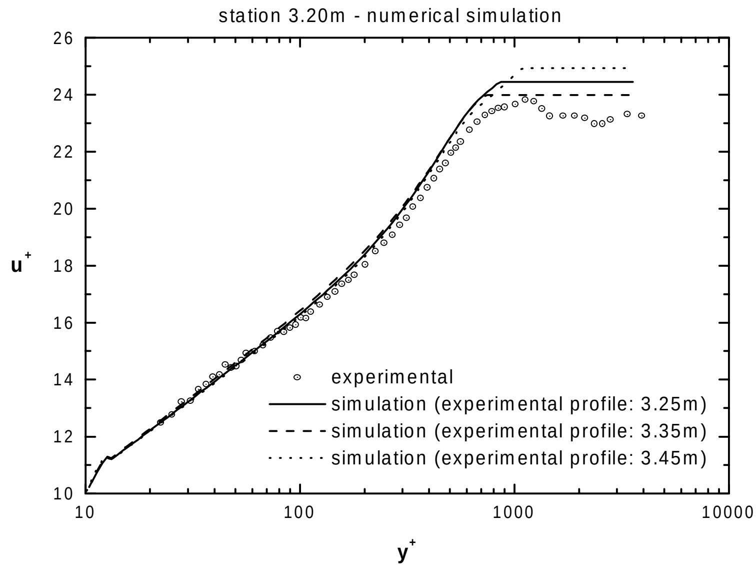

Figure 4: Comparison of estimated initial velocity profile in inner variable with experimental data using one measured velocity profile at different longitudinal stations.

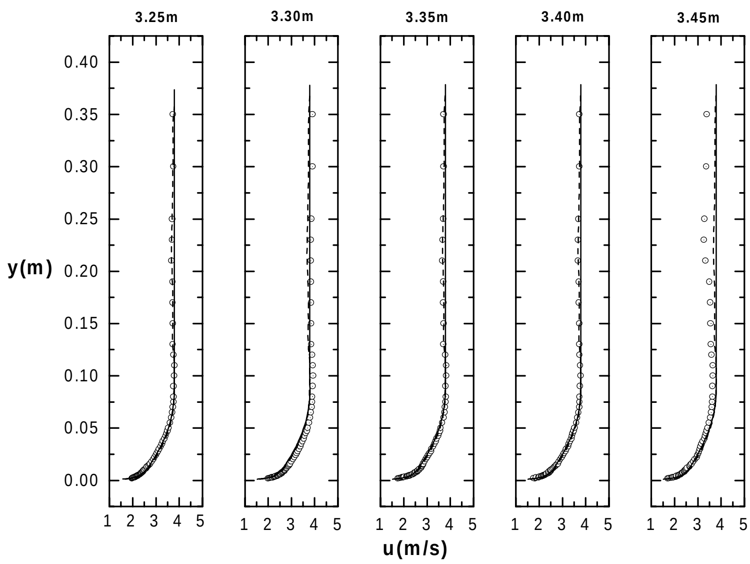

Figure 5: Calculated dowstream velocity profiles using experimental and estimated initial profiles, known profile at 3.25m station, compared with measured velocity profiles. Points are the experimental data, solid line is the simulation with inverse method and dashed line with direct method.

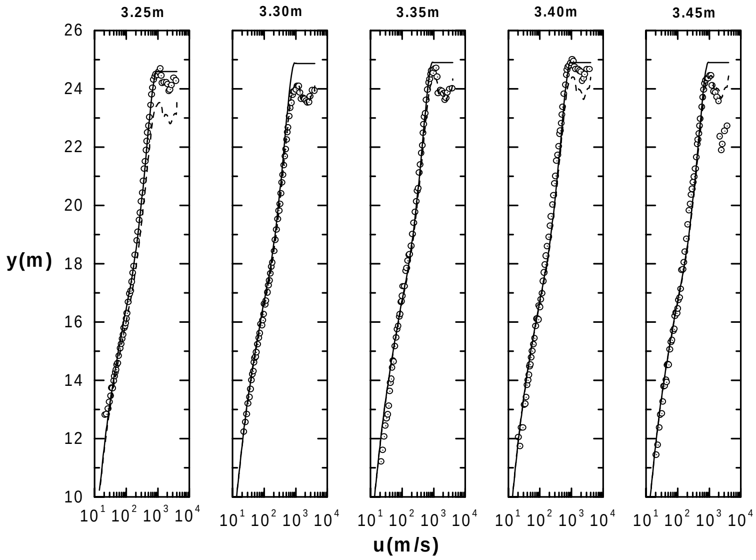

Figure 6: Calculated dowstream velocity profiles using experimental and estimated initial profiles in inner variable, known profile at 3.25m station, compared with measured velocity profiles. Points are the experimental data, solid line is the simulation with inverse method and dashed line with direct method.

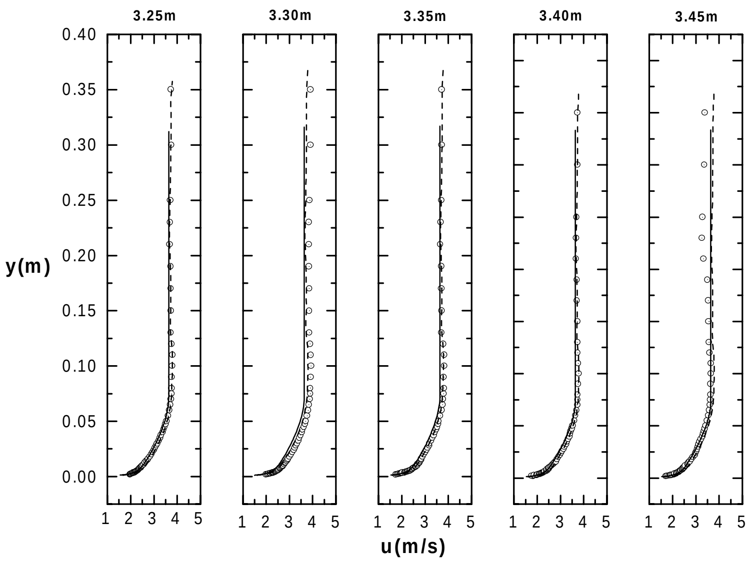

Figure 7: Calculated dowstream velocity profiles using experimental and estimated initial profiles, known profile at 3.45m station, compared with measured velocity profiles. Points are the experimental data, solid line is the simulation with inverse method and dashed line with direct method.

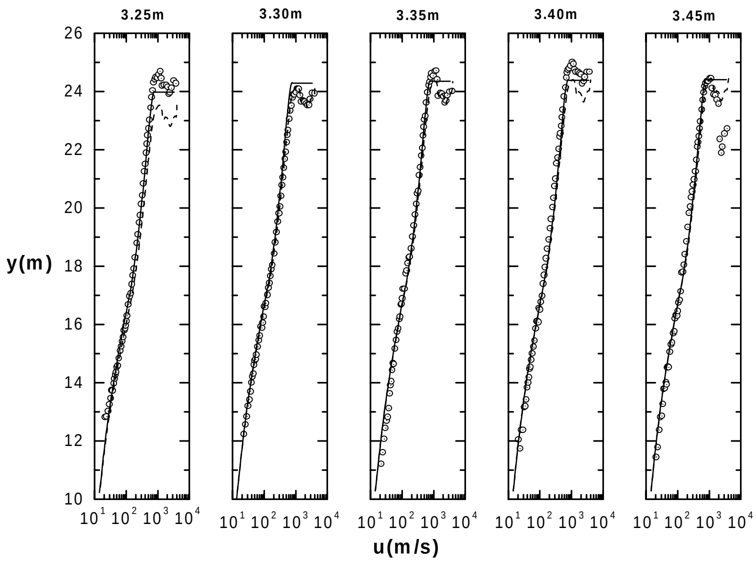

Figure 8: Calculated dowstream velocity profiles using experimental and estimated initial profiles in inner variable, known profile at $3.45m$ station, compared with measured velocity profiles. Points are the experimental data, solid line is the simulation with inverse method and dashed line with direct method.

6. Conclusion

An inverse analysis for the estimation of upstream velocity profiles in an incompressible turbulent boundary layer over a smooth flat plate was carried out. The direct problem of turbulent boundary layer equations with an algebraic turbulence model was solved by using a finite difference method, which was validated against data obtained in a low-speed wind tunnel. Five parameters, the friction velocity, the Von Kármán constant, the law of the wall constant, the Coles's wake-strength parameter and the boundary layer thickness, were used for the analytical construction of the upstream velocity profile by using the Coles's composite law of the wall, law of the wake formulation. Thus, the inverse problem of estimation of initial velocity profiles was formulated as a parameter estimation problem that searched for the friction velocity, the Von Kármán constant, the law of the wall constant, the Coles's wake-strength parameter and the boundary layer thickness at an upstream station in the turbulent boundary layer. We have shown, through comparison with the measured velocity profile at the same station, that the upstream velocity profile could be accurately estimated if experimental data of velocity measurement within 25 cm from the inlet station were used. The proposed inverse analysis can be used to generate an accurate and smooth initial velocity profile for numerical simulation of turbulent boundary layer and to determine accurately some boundary layer parameters, such as the friction velocity and the boundary layer thickness, that are difficult to measure directly.

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