IX CONGRESSO BRASILEIRO DE ENGENHARIA E CIÊNCIAS TÉRMICAS

9th BRAZILIAN CONGRESS OF THERMAL ENGINEERING AND SCIENCES

Paper CIT02-0897

FLOW OF A HIGHLY SHEAR-THINNING MATERIAL PAST A CONTRACTION-EXPANSION: APPLICATION TO BLOOD FLOW THROUGH STENOTIC VESSELS

Erick Fabrizio Quintella
Departament of Mechanical Engineering, Pontifícia Universidade Católica do Rio de Janeiro.
Rua Marquês de São Vicente, 225. Gávea, Rio de Janeiro, 22453-900, Brazil.
erick@mec.puc-rio.br

Paulo Roberto de Souza Mendes
Departament of Mechanical Engineering, Pontifícia Universidade Católica do Rio de Janeiro.
Rua Marquês de São Vicente, 225. Gávea, Rio de Janeiro, 22453-900, Brazil.
pmendes@mec.puc-rio.br

Abstract. *Stenosis is a constriction or narrowing of a duct or passage. This word is often used to refer to constrictions that occur in blood vessels, especially arteries. A stenotic vessel generally causes blood flow stagnation and hence formation of thrombus, which may be rather harmful to the human physiology. In this work, we extend the methodology to analyze the flow of viscoplastic materials through tubes with constant cross-section to study the flow of blood (a highly shear-thinning, or yield stress, material) through a tube in the neighborhood of a constriction. The rheology of blood is chosen to be the one of blood with 54% hematocrit. We calculated Reynolds and* Yield number *values characteristic to different arteries. Also, for comparison purposes, we obtained results for a Newtonian modelling of blood. The results indicate that the non-Newtonian rheology may cause a significant change in the flow pattern in the neighborhood of the stenosis. Because the conditions for thrombus formation are directly related to the flow pattern, studies of the flow through stenotic vessels that assume a Newtonian rheology may lead to erroneous conclusions.*

Keywords: *stenosed arteries, blood flow, viscoplastic materials, blood rheology.*

1. Introduction

Modern scientific suggestions and proposals that fluid mechanical forces, normal or tangential, play an important role in the pathogenesis and pathophysiology of atherosclerosis go back at least a half century. These speculations undoubtedly arose, at least in part, because of the observations that atherosclerotic lesions preferentially occurred in arteries and arterioles in regions of high curvature or where there were bifurcations and junctions, places where there would be major changes in flow patterns and consequently large changes in fluid loading on vessel walls.

Studies for both normal and stenosed vessels have been carried out for idealized arteries and idealized arterial bifurcations and branchings, and for specific, clinically important cases such as the aortic arch, the carotid artery, and the coronary arteries. Most analyses model blood as a Newtonian material (Cavalcanti et al., 1992, Dash et al., 1999, Engvall et al., 1991, Lim et al., 1998), a generally assumed approximation for the rheological behavior of blood in the larger blood vessels. This Newtonian modelling employs a constant viscosity of the same order of magnitude of the plasma viscosity ($\approx 4 \times 10^{-3}$ Pa.s). There has been some work which model blood as a non-Newtonian material (Luo and Kuang, 1992, Srivastava and Saxena, 1994, Srivastava, 1996). Most analyses have assumed symmetric stenoses, to make the problem more tractable, but as numerical methods and computers became more capable, asymmetric stenoses began to be investigated. Nearly all these analyses have assumed the plaque to be rigid, with the plaque playing a passive role, changing the lumen of the vessel so that the fluid flows through a contraction followed by an expansion (Berger and Jou, 2000).

In short, studies of normal arteries, their junctions, bifurcations, curvature, etc., tell us where atherosclerotic lesions are likely to originate. Once formed, two important, related consequences follow. One is that the lumina of affected vessels are altered, generally narrowed, and the flow through them compromised, reducing the supply of blood to vessels downstream. The second, related consequence is that the alteration in flow pattern in these affected vessels will change the loading, shear stresses and pressure, of the fluid on the arterial wall, and as a result affect both further progression of the lesion and its tendency to fracture and rupture. Remember that the mortality associated with atherosclerotic disease may not be correlated simply with the degree of stenosis but also with the tendency of parts of the plaque to break off from the wall and block smaller vessels downstream.

Due to the repeatedly mentioned important role played by the blood flow patterns in atherosclerosis disease, it can be seen that a rational method to investigate these flow patterns would be useful. In the present work, we extend the methodology proposed by Soares et al., 1999, to analyze the flow of viscoplastic materials through tubes with constant cross-section to tubes with a constriction, in order to simulate the flow of blood through arteries with a stenosis. The analysis intends to set the proper conditions to perform the investigation of the flow patterns in the region past the constriction.

2. Analysis

As mentioned before, most of the works on this subject regard blood as a Newtonian material (constant viscosity, absence of yield stress), and only a few works take into account the fact that, in several situations, blood shows non-Newtonian properties (shear-thinning viscosity, presence of yield stress). We intend to investigate whether, in such a complex geometry like a stenosis, the arguments which disregard the non-Newtonian properties of blood are still valid.

2.1. Comments on the dimensionless parameters

Despite the reasonable simplicity of steady flow in tubes of constant cross-section, the phenomenon occurring in the region past a constriction is rather complex. A blood cell in this region is subjected to many different mechanical solicitations. Since Newtonian and non-Newtonian materials, by their very nature, present distinct mechanical behavior, i.e., respond differently to equal mechanical solicitations, we believe that, mainly due to the presence of a possible constriction, blood should not be modelled indifferently as a Newtonian or a non-Newtonian material. Actually, for deciding one should follow the reasoning: if, for the same set of forces, the response of different materials is the same, it means that either way of modelling is satisfactory; however, if, for the same set of forces, the response of different materials is distinct, we should evaluate how harmful this difference may be for blood, and after doing this we may decide to include non-Newtonian properties for modelling blood.

We identify in the region past the constriction the existence of inertia forces and viscous forces imposing themselves on the blood cells. Although we acknowledge the difficulty of mapping these forces, it is usual to say that the flow pattern is determined by the action of these forces on the different materials. We would like to say that these flow patterns must be governed by some dimensionless quantities.

Following this reasoning, we observe that an interesting dimensionless quantity is one which relates inertia forces to viscous forces (a Reynolds number) occurring in the flow. Another interesting dimensionless quantity to mention is one which is capable of quantifying the local shear stress relative to the characteristic shear stress necessary for a viscoplastic material to flow (a dimensionless yield stress).

The challenge is to define these dimensionless numbers in such a way to ensure that, when they are evaluated for different flows and their values are equal, this must mean that the ratio of relevant forces is the same for both flows. This may seem obvious, but, if inappropriate values of the characteristic quantities are chosen, the evaluation of the forces involved will be in error, leading to meaningless dimensionless numbers.

In particular, the non-Newtonian viscosity varies significantly in flows with complex kinematics, which is the case of the flow in the stenosed region. Therefore, it would be difficult and arbitrary to choose as characteristic viscosity a viscosity value which occurs in this region. For this reason, we advocate using as characteristic quantities the ones that occur at the wall upstream the stenosis, where the flow is fully developed.

In this manner, equal values of the dimensionless parameters mean equal ratios of relevant forces in the region upstream of the stenosis. Thus, for two rheologically different materials flowing through the same stenotic vessel at equal dimensionless numbers evaluated in the region upstream the stenosis, all differences in flow pattern observed at the stenotic region will be due to local changes of the ratio of forces caused by the different rheology of the materials.

2.2. Derivation of the dimensionless parameters

In the derivation performed henceforth, we give a relation between inertia forces and viscous forces which are present in the flow of a Generalized Newtonian Liquid through a duct of constant cross-section area. In the analysis, we assume the flow to be steady, axisymmetric and fully developed. It can be seen, immediately, that the dimensionless parameter arising from this derivation cannot represent well the phenomenon occurring at the stenosis region, since, although it is reasonable to assume an axisymmetric geometry, the flow is not steady, neither is fully developed and does not happen in a duct of constant cross-section area. However, as already mentioned, it is desirable to apply the following calculations to a region before the stenosis which fulfills all the assumptions required for this derivation, and this is the region we choose as characteristic of the flow.

We assume that the mechanical behavior of blood is well modelled by a Generalized Newtonian Liquid

(GNL) (Bird et al., 1987). In this model, the extra-stress tensor $\underline{\underline{\tau}}$ is given by

$$\underline{\underline{\tau}} = \eta(\dot{\gamma})\underline{\underline{\dot{\gamma}}}, \tag{1}$$

where $\underline{\underline{\dot{\gamma}}}$ is the rate-of-deformation tensor, defined as $\nabla\mathbf{v} + (\nabla\mathbf{v})^T$, and $\mathbf{v}$ is the velocity vector. The quantity η is the viscosity function, assumed here to be of the Herschel-Bulkley form

$$\eta = \begin{cases} \tau_0/\dot{\gamma} + K\dot{\gamma}^{n-1} & \text{if } \tau > \tau_0, \\ \infty & \text{otherwise,} \end{cases} \tag{2}$$

where τ_0 is the yield stress, $\tau \equiv \sqrt{\frac{1}{2}\mathrm{tr}\underline{\underline{\tau}}^2}$ is a measure of the magnitude of $\underline{\underline{\tau}}$, $\dot{\gamma} \equiv \sqrt{\frac{1}{2}\mathrm{tr}\underline{\underline{\dot{\gamma}}}^2}$ is a measure of the magnitude of $\underline{\underline{\dot{\gamma}}}$, K is the consistency index, and n is the power-law exponent. The parameters included in the viscosity function (τ_0, K and n) are normally determined via least squares fits to experimental shear (τ x $\dot{\gamma}$) data.

In addition of assuming the flow to be steady and axisymmetric, we further assume that heat transfer is negligible. Then, the flow is governed by the mass and momentum equations, in conjunction with the GNL constitutive equation and appropriate boundary conditions. In order to obtain representative dimensionless governing equations and dimensionless parameters, the choice of characteristic quantities is essential. The characteristic quantities are evaluated at the characteristic region, namely the fully developed region before the stenosis.

The characteristic shear rate, $\dot{\gamma}_c$, is defined as

$$\dot{\gamma}_c \equiv \left(\frac{\tau_{\mathrm{R,fd}} - \tau_{0\mathrm{ref}}}{K_{\mathrm{ref}}}\right)^{1/n}, \tag{3}$$

where $\tau_{0\mathrm{ref}}$ and K_{ref} are the values of τ_0 and K at a reference temperature, which was chosen to be 25°C in accordance with the data available for blood in the literature (Picart et al., 1998). The shear stress at the wall in the fully developed region is denoted by $\tau_{\mathrm{R,fd}}$, which can be easily related to the pressure drop by means of a force balance:

$$\tau_{\mathrm{R,fd}} \equiv -[\tau_{rx}(R)]_{\mathrm{fd}} = -\left(\frac{dp}{dx}\right)_{\mathrm{fd}}\frac{R}{2} = \frac{\Delta p}{L_{\mathrm{fd}}}\frac{R}{2}, \tag{4}$$

where p is the pressure, x is the axial coordinate, R is the tube radius and L_{fd} is a tube length in the fully developed region to whose ends the pressure difference Δp corresponds.

The characteristic viscosity is chosen as

$$\eta_c \equiv \eta(\dot{\gamma}_c) = \frac{\tau_{\mathrm{R,fd}}}{\dot{\gamma}_c}. \tag{5}$$

The dimensionless shear rate and viscosity are respectively defined as

$$\dot{\gamma}' \equiv \frac{\dot{\gamma}}{\dot{\gamma}_c}, \eta' \equiv \frac{\eta}{\eta_c}. \tag{6}$$

Using the above definitions, the dimensionless viscosity function can be written as

$$\eta' = \begin{cases} r_0'/\dot{\gamma}' + (1 - r_0')\dot{\gamma}'^{n-1} & \text{if } \tau' > r_0', \\ \infty & \text{otherwise,} \end{cases} \tag{7}$$

where

$$\tau' \equiv \frac{\tau}{\tau_{\mathrm{R,fd}}}, \tag{8}$$

and

$$r_0' \equiv \frac{r_0}{R} = \frac{\tau_{0\mathrm{ref}}}{\tau_{\mathrm{R,fd}}}. \tag{9}$$

Equation (9) shows a dimensionless yield stress, known as *Yield number* (r_0'). This dimensionless number relates the yield stress to the shear stress at wall in the characteristic region. It is greatly helpful for the investigation of materials which, such as blood, necessitate to overcome a certain stress to flow. It takes a minimum value of zero, when the shear stress at wall is so larger than τ_0 that the material can be modelled by viscosity functions that do not require a yield stress; and it takes a maximum value of one, when the shear stress is equal to τ_0, and hence there is no flow.

In order to define the dimensionless number which relates inertia forces to viscous forces at the characteristic region, three more quantities are needed: the dimensionless velocity profile, the axial average velocity and its corresponding dimensionless counterpart.

The dimensionless shear stress is defined as

$$\tau' = \frac{\tau}{\tau_R} = \frac{r}{R} = r' = \eta' \dot{\gamma}'. \tag{10}$$

Combining the dimensionless viscosity function (Eqs. (7)) and the dimensionless shear stress (Eq. (10)) the following form for the dimensionless shear rate is obtained:

$$\dot{\gamma}' = \left(\frac{r' - r'_0}{1 - r'_0} \right)^{1/n}. \tag{11}$$

Now we define the dimensionless axial velocity as

$$u' = \frac{u}{\dot{\gamma}_c R}. \tag{12}$$

Then, the dimensionless shear rate (Eq. (11)) can be used, together with the integration of the definition of shear rate

$$\dot{\gamma} = -\frac{du}{dr}, \tag{13}$$

to find the expression for dimensionless axial velocity as a function of the Yield number and the power-law exponent:

$$u' = \begin{cases} \frac{n}{n+1}(1 - r'_0)^{-\frac{1}{n}} \left[(1 - r'_0)^{\frac{n+1}{n}} - (r' - r'_0)^{\frac{n+1}{n}} \right] & \text{if } r' > r'_0, \\ \frac{n}{n+1}(1 - r'_0) & \text{if } r' \leq r'_0 (\text{plug flow}). \end{cases} \tag{14}$$

The axial average velocity can be obtained integrating the velocity profile:

$$\overline{u} = \frac{2}{R^2} \left[\int_0^{r_0} u_0 r dr + \int_{r_0}^{R} u r dr \right]. \tag{15}$$

Nondimensionalizing this expression, using the dimensionless axial velocity (Eq. (14)) and performing the integrals, we can reach the expression for the dimensionless average axial velocity:

$$\overline{u}' = \frac{2n}{n+1} \left[\frac{1}{2}(1 - r'_0) - \frac{n}{2n+1}(r'_0)(1 - r'_0)^2 - \frac{n}{3n+1}(1 - r'_0)^3 \right]. \tag{16}$$

From these average values (Eqs. (15) and (16)) the expression for the characteristic shear rate at wall is re-written as

$$\dot{\gamma}_c = \frac{\overline{u}}{\overline{u}' R} = \frac{\overline{u}}{R} \frac{n+1}{2n} \left[\frac{1}{2}(1 - r'_0) - \frac{n}{2n+1}(r'_0)(1 - r'_0)^2 - \frac{n}{3n+1}(1 - r'_0)^3 \right]^{-1}. \tag{17}$$

Using the above defined characteristic quantities, the conservation equations can be reduced to their dimensionless forms. The mass and momentum equations for the entrance region of a steady laminar flow of a Herschel-Bulkley material through a duct are thus given by

$$\frac{\partial u'}{\partial x'} + \frac{1}{r'} \frac{\partial (r' v')}{\partial r'} = 0, \tag{18}$$

$$v' \frac{\partial u'}{\partial r'} + u' \frac{\partial u'}{\partial x'} = -\frac{\partial p'}{\partial x'} + \frac{2\overline{u}'}{Re} \left[\frac{1}{r'} \frac{\partial}{\partial r'} \left(\eta' r' \frac{\partial u'}{\partial r'} \right) + \frac{\partial}{\partial x'} \left(\eta' \frac{\partial u'}{\partial x'} \right) \right], \tag{19}$$

$$v' \frac{\partial v'}{\partial r'} + u' \frac{\partial v'}{\partial x'} = -\frac{\partial p'}{\partial r'} + \frac{2\overline{u}'}{Re} \left[\frac{\partial}{\partial r'} \left(\frac{1}{r'} \frac{\partial}{\partial r'} (\eta' r' v') \right) + \frac{\partial}{\partial x'} \left(\eta' \frac{\partial v'}{\partial x'} \right) \right]. \tag{20}$$

In these equations, $v' = v/R\dot{\gamma}_c$, $x' = x/R$, $r' = r/R$, and $p' = p/\rho(R\dot{\gamma}_c)^2$.

Finally, the dimensionless number is defined as the following Reynolds number:

$$Re \equiv \frac{2\rho\overline{u}R}{\eta_c} = \frac{2\rho(\overline{u}')^{n-1}(\overline{u})^{2-n}R^n}{K_{\rm ref} + \tau_{0\rm ref}\left(\frac{\overline{u}'R}{\overline{u}}\right)^n} \tag{21}$$

It is interesting to observe that, when $\tau_{0\rm ref} = 0$, $K_{\rm ref} = \mu$ and $n = 1$, the Newtonian material case is recovered; when $K_{\rm ref} = \mu_p$ and $n = 1$ the Bingham material case is recovered; and when $\tau_{0\rm ref} = 0$ the power-law material case is recovered.

The boundary conditions are the usual no-slip condition at the wall, the symmetry condition at the centerline, uniform velocity profile at the tube inlet, and locally parabolic flow at the characteristic region:

$$\frac{\partial u'}{\partial r'}(0,x') = 0, v'(0,x') = 0, u'(1,x') = 0, v'(1,x') = 0, \tag{22}$$

$$u'(r',0) = \overline{u}', v'(r',0) = 0, \frac{\partial u'}{\partial x'}(r',L') = 0, \frac{\partial v'}{\partial x'}(r',L') = 0, \tag{23}$$

where $L' \equiv L/R$ is the dimensionless tube length.

2.3. Procedure for evaluating the flow parameters *in vivo*

In order to perform calculations related to the flow of blood in arteries, it is important to determine what are the characteristic flow rates, as well as to determine what are the shear stresses experienced by blood in these flows. After obtaining these blood flow parameters, the Yield number for blood and the Reynolds number at the corresponding arteries can be determined by means of the following procedure.

Equating the expression which relates the characteristic shear rate calculated by using the constitutive equation (Eq. (3)) to its definition as a function of the average velocity values (Eq. (17)), the dimensionless axial average velocity can be re-written as

$$\overline{u}' = \frac{\overline{u}}{R}\left[\frac{\tau_o}{Kr_0'}(1-r_0')\right]^{-1/n} \tag{24}$$

Equating this expression to the dimensionless average axial velocity (Eq. (16)), the following polynomial is obtained:

$$\frac{2n}{n+1}\left[\frac{1}{2}(1-r_0') - \frac{n}{2n+1}(r_0')(1-r_0')^2 - \frac{n}{3n+1}(1-r_0')^3\right] - \frac{\overline{u}}{R}\left[\frac{\tau_o}{Kr_0'}(1-r_0')\right]^{-1/n} = 0. \tag{25}$$

We can observe that, for a given flow rate of a given viscoplastic material in a certain geometry, the solution of Eq. (25) gives the Yield number, i.e., the root of this equation is the ratio between the yield stress and the shear stress at wall due to the flow of the material at the characteristic region.

Thus, this Yield number can be substituted in Eq. (16) to calculate the dimensionless axial average velocity, and then to finally calculate the relevant Reynolds number through Eq. (21).

3. Results and Discussion

Human blood is a two-phase system and consists mainly of an aqueous polymeric and ionic solution of low viscosity, the plasma, in which a 45% to 50% concentrated cellular fraction (hematocrit) is suspended. Red blood cells are the most numerous cell elements and represent 99% of the hematocrit with regard to less than 1% for white cells and platelets. Blood's mechanical behavior varies according to the imposing forces, showing typically two behaviors: a viscoelastic behavior, caused by the elastic membrane properties of red cells and by the liquids inside and outside the cells, and a viscoplastic behavior, caused by the great amount of suspended particles in plasma that grant it a yield stress and a shear-thinning viscosity.

3.1. Flow curve for blood

Picart et al., 1998, used a Couette-type rheometer with cylindrical walls to investigate the rheometrical shear properties of blood. In a careful work they roughen the surface of the cylindrical walls in order to prevent slip, a phenomenon common to suspensions. Also in this paper it can be found an excellent review of blood yield stress, together with a discussion about the difficulty to measure this quantity.

From the flow curves obtained by Picart, we decided to work with the one associated to a 54% cellular fraction, understanding that this hematocrit is representative of the healthy average human blood hematocrit (range 40% to 55%). The data available relates only viscosity and shear rate, i.e., are of the (η x $\dot{\gamma}$) type. Thus, nothing can be inferred about the elastic properties of blood, and then this analysis can be applied regarding the viscoplastic behavior of blood only.

3.2. Blood as a Herschel-Bulkley material

In an important paper, Barnes, 1999, presents a large review of the parameter known as yield stress, and vehemently question its physical existence, relegating it to a mere coefficient which intends to fit the viscoplastic material flow curves. In the same paper Barnes criticizes Picart's decision of assuming to the blood yield stress the shear stress value at the lowest shear rate measured, i.e., the shear stress at 10^{-3} s^{-1}.

Avoiding the most philosophical discussion about the existence of a yield stress, we understand that such a parameter is still useful in the description of many materials' mechanical behavior in the shear stress range to which they are commonly subjected. Thus, we decided to fit Picart's experimental blood data for 54% hematocrit to a Herschel-Bulkley model. Figure 1 shows the fitted curve.

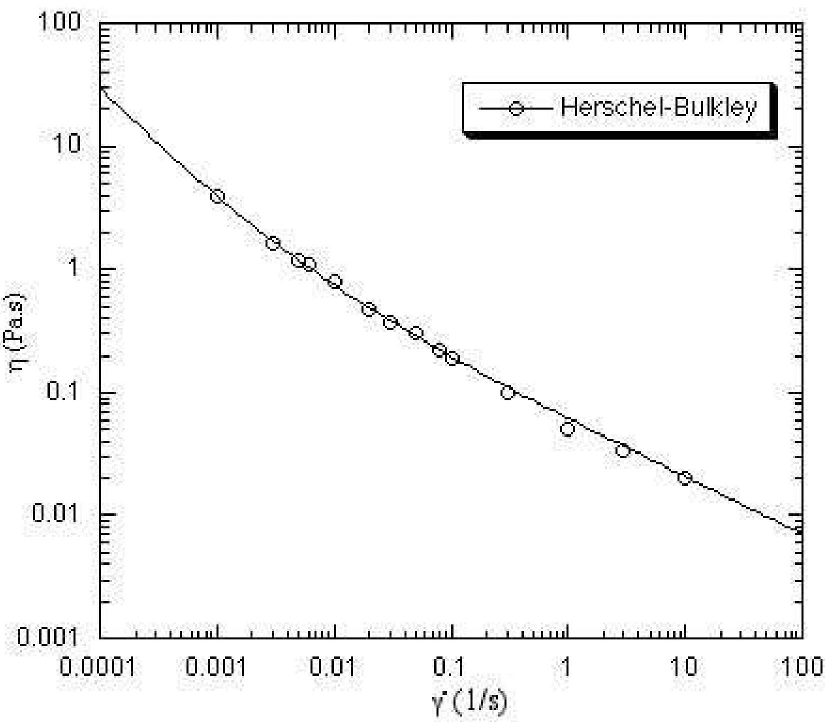

Figure 1: Viscosity versus shear rate curve for 54% hematocrit blood data fitted to a Herschel-Bulkley constitutive model. Experimental data reproduced from Picart et al., 1998.

The curve-fitting parameters are τ_o = 2,55 x 10^{-3} Pa, K = 59,5 x 10^{-3} Pa.s^n and n = 0,539.

3.3. Blood flow parameters for different arteries

In the analysis section we derived two important dimensionless quantities to this complex flow of a viscoplastic material: the Reynolds number and the Yield number. Hence, the next step is to calculate the desired dimensionless quantities.

In a study of Casson fluid flow through stenotic blood vessels, Srivastava and Saxena, 1994, present an interesting table reporting the real values of diameters and flow rates occurring at the arteries most susceptible of the stenosis problem. Table 1 is reproduced below.

Table 1: Blood vessels radius and flow rates.

Parameter	Aorta	Femoral	Carotid	Coronary	Arteriole
Radius (cm)	1	0,5	0,4	0,15	0,008
Flow rate (cm^3/s)	71,67	19,63	12,57	3,47	0,00008

Assuming that the blood flowing in these arteries have a 54% hematocrit, we can model it as a Herschel-Bulkley material, and hence its values for yield stress (τ_0), consistency index (K) and power-law exponent (n) are the same obtained from the curve-fitting above. Therefore, it remains to perform the calculations detailed in the dimensional analysis section to find the relevant Reynolds numbers and Yield numbers for the different arteries.

The Reynolds number proposed in this work embodies the non-Newtonian properties of blood. The Reynolds number used in the consulted literature, however, assumes blood is a Newtonian material, and the usual definition is employed:

$$Re = \frac{2\rho\bar{u}R}{\mu}. \tag{26}$$

Here ρ is the blood density ($= 1060$ kg/m^3), $\bar{u}$ is the average axial flow velocity, R is the undeformed radius of the artery under study, and μ is the blood viscosity, assumed to be constant ($\approx 4 \times 10^{-3}$ Pa.s).

In order to perform comparisons, the Yield number, the Reynolds number calculated according to the proposition of this work and the Reynolds number calculated according to the usual manner are shown in Tab. 2.

Table 2: Yield numbers and Reynolds numbers of blood characteristic of different arteries.

Parameter	Aorta	Femoral	Carotid	Coronary	Arteriole
Yield number (τ_0/τ_R)	0,0040	0,0027	0,0024	0,0011	0,0027
$\mathrm{Re}_{proposed}$	816,1	662,8	593,2	1000,8	0,161
Re_{usual}	1209,1	662,3	530,2	390,3	0,169

Excluding from the arterioles, the Reynolds number of blood in the different healthy arteries calculated according to the proposed procedure varies, approximately, between 500 and 1000.

Both Reynolds numbers calculated for the carotid show a slight difference, and the ones calculated for the femoral show virtually no difference, because, coincidentally, $\eta_c \approx \mu$ for these cases. The same cannot be said about the aorta neither about the coronary arteries.

It is interesting to see that the largest Reynolds number does not occur in the artery with the largest diameter and through which the largest flow rate exist (aorta). This would happen only if viscosity, in the Reynolds number evaluation, were assumed constant. Since blood has a shear-thinning viscosity, and this property was included in the proposed Reynolds number calculation, the apparent oddness of this result is just a consequence of the non-Newtonian properties of blood. Note the striking influence of the shear-thinning viscosity of blood for the flow through the coronaries.

Note, also, how small is the magnitude of the Yield number for blood flowing through these healthy arteries. We can even neglect them. Since this dimensionless number represents the competition between a hypothetical shear stress necessary for blood to flow and the real local shear stresses, we can conclude that, in the flow of blood through healthy arteries, the fact that blood holds a yield stress barely differentiates it from a fluid that does not hold a yield stress. Thus, for straight arteries and regarding only this parameter, modelling blood as a material which holds or not a yield stress is indifferent. Therefore, for simplicity, it is advisable modelling blood as a material that does not hold a yield stress, paying attention just to preserve the shear-thinning viscosity properties.

However, these low values of Yield number calculated for the healthy arteries may be highly misleading for the case of stenosed arteries. We know, for instance, that in flow through a stenosed artery there are vortices, and consequently regions where the real local shear stresses are very low. In these regions, the existence of a yield stress in blood flowing can be decisive in thrombus formation. Therefore, in order to obtain data to give information related to devices where complex blood flow patterns will occur, nothing should be disregarded about the rheology of blood.

4. Final Comments

Research on the pathogenesis and pathophysiology of atherosclerosis in the last fifty years suggest that fluid mechanical forces play an important role in the origin and aggravation of the disease, by means of the change in flow patterns and, consequently, changes in fluid loading on vessel walls. The literature also attests that most of the publications regard blood as a Newtonian material. In the present work, we extend a methodology to analyze the flow of viscoplastic materials through tubes with constant cross-section to tubes with a constriction, in order to study the flow of blood through arteries with a stenosis.

The dimensionless numbers arising from the theoretical calculations are applied to a region before the stenosis, reasoning that, equaling the dimensionless parameters in a region where the different materials are subjected to the same ratio of forces, the differences in flow pattern observed in the constricted region would be an indicative of how important is to regard the non-Newtonian properties of blood in the modelling. These dimensionless numbers are then calculated to blood flowing through the arteries where the probability of occurrence of a stenosis is largest. Here, blood is modelled as a viscoplastic material, with the viscosity function following the Herschel-Bulkley equation. The respective Newtonian dimensionless numbers are calculated for comparison purposes.

We found that, for the flow through non-stenosed blood vessels such as the arterioles, the femoral and the carotid arteries, the dimensionless numbers do not show a significant difference, suggesting that for these vessels modelling blood as a Newtonian or a non-Newtonian material is indifferent. On the other hand, the Reynolds numbers considering and disregarding the non-Newtonian properties of blood show a large difference in magnitude for the flow through the aorta and the coronary arteries. These discrepancies illustrate the importance of employing an appropriate characteristic viscosity when evaluating Re, which ensures meaningful comparisons between flows of equal Reynolds numbers. The non observance of this procedure could compromise the efficiency of the design of biological devices, among others.

5. Acknowledgments

Erick Quintella was supported with a scholarship from FAPERJ. This research was partially funded by CNPq and FINEP.

6. References

Barnes, H. A., 1999, The Yield Stress – A Review, “Journal of Non-Newtonian Fluid Mechanics”, Vol. 81, pp. 133–178.

Berger, S. A. and Jou, L.-D., 2000, Flows in Stenotic Vessels, “Annual Review of Fluid Mechanics”, Vol. 32, pp. 347–382.

Bird, R. B., Armstrong, R. C., and Hassager, O., 1987, “Dynamics of Polymeric Liquids”, John Wiley & Sons, New York, USA, 650 p.

Cavalcanti, S., Bolelli, P., and Belardinelli, E., 1992, Pressure Drops Through Arterial Stenosis Models in Steady-Flow Condition, “Journal of Biomechanical Engineering”, Vol. 114, pp. 416–418.

Dash, R. K., Jayaraman, G., and Mehta, K. N., 1999, Flow in a Catheterized Curved Artery with Stenosis, “Journal of Biomechanics”, Vol. 32, pp. 49–61.

Engvall, J., Ask, P., Loyd, D., and Wranne, B., 1991, Coarctation of the Aorta — A Theoretical and Experimental Analysis of the Effects of a Centrally Located Arterial Stenosis, “Medical & Biological Engineering & Computing”, Vol. 29, pp. 291–296.

Lim, W. L., Chew, Y. T., Chew, T. C., and Low, H. T., 1998, Steady Flow Dynamics of Prosthetic Aortic Heart Valves: A Comparative Evaluation with PIV Techniques, “Journal of Biomechanics”, Vol. 31, pp. 411–421.

Luo, X. Y. and Kuang, Z. B., 1992, Non-Newtonian Flow Patterns Associated with an Arterial Stenosis, “Journal of Biomechanical Engineering”, Vol. 114, pp. 512–514.

Picart, C., Piau, J. M., and Galliard, H., 1998, Human Blood Shear Yield Stress and its Hematocrit Dependence, “Journal of Rheology”, Vol. 42, pp. 1–12.

Soares, M., Naccache, M. F., and Mendes, P. R. S., 1999, Heat Transfer to Viscoplastic Materials Flowing Laminarly in the Entrance Region of Tubes, “International Journal of Heat and Fluid Flow”, Vol. 20, pp. 60–67.

Srivastava, V. P. and Saxena, M., 1994, Two-Layered Model of Casson Fluid Flow Through Stenotic Blood Vessels: Applications to the Cardiovascular System, “Journal of Biomechanics”, Vol. 27, pp. 921–928.

Srivastava, V. P., 1996, Two-Phase Model of Blood Flow Through Stenosed Tubes in the Presence of a Peripheral Layer: Applications, “Journal of Biomechanics”, Vol. 29, pp. 1377–1382.