IX CONGRESSO BRASILEIRO DE ENGENHARIA E CIÊNCIAS TÉRMICAS

9th BRAZILIAN CONGRESS OF THERMAL ENGINEERING AND SCIENCES

Paper CIT02-0090

THERMALLY DEVELOPING FLOW OF PHAN-THIEN-TANNER FLUIDS IN DUCTS

André Fonseca Mendes
Nielson Fernando da Paixão Ribeiro
Emanuel Negrão Macêdo
João Nazareno Nonato Quaresma
Chemical and Food Engineering Department, Universidade Federal do Pará – UFPA
Campus Universitário do Guamá, Rua Augusto Corrêa, 01
66075-900, Belém, PA, Brazil
quaresma@ufpa.br

Abstract. *The Integral Transform Method is employed in the solution of thermally developing flow of Phan-Thien-Tanner (PTT) fluids in ducts. In the analysis of the problem, it is considered a thermal boundary condition of prescribed heat flux at the duct walls and, in addition, viscous dissipation effects are also taken into account. Numerical results for Nusselt numbers are computed in the thermal entry region as functions of the Brinkman number and parameters of PTT model. Critical comparisons with previous results in the literature are performed, which show the consistency of the final results.*

Keywords. *integral transform method, Phan-Thien-Tanner (PTT) model, thermally developing duct flow.*

1. Introduction

The behavior of polymeric fluids is generally described for constitutive equations that include simultaneous characteristics of elastic solids and viscous fluids, these are the so-called viscoelastic models. Bird et al. (1987) have pointed out innumerous models to describe the fluid flow of viscoelastic fluids, among them we can cite the Phan-Thien-Tanner (PTT) model, which was derived from a network theory for polymeric fluids (Phan-Thien and Tanner, 1977; Phan-Thien, 1978).

Since then, this model has been broadly used in the simulation of flows of polymer solutions and melts (Baaijens, 1993; Quinzani et al., 1995; Azaiez et al., 1996; Baloch et al., 1996; Oliveira and Pinho, 1999). Once these fluids present high viscosities, they are commonly processed under laminar flow conditions and, in addition, when subjected to heat transfer in duct flows, the development of velocity profiles are faster than temperature ones, so that the study of thermally developing flow involving such fluids is more relevant. Therefore, the good prediction of temperature distributions in heat transfer forced convection is important for polymer processing equipment, such as in the extrusion processes.

In this context, the present work aims at developing analytical solutions for thermally developing flow of PTT fluids in ducts through the integral transform methodology (Mikhailov and Özisik, 1984; Cotta, 1993), following the philosophy of previous works (Pinho and Oliveira, 2000; Coelho et al., 2002a; 2002b).

The analytical solution of thermal entry region inside ducts involves difficulties due to the posterior solution of the auxiliary eigenvalue problem. Consequently, it is not feasible to calculate heat transfer results in regions which are very close to the inlet because a large number of eigenvalues are needed for the computation of the series expansion based on eigenfunctions. The present work to alleviate such difficulties employs the well-established Sign-Count Method (Mikhailov and Vulchanov, 1983; Mikhailov and Özisik, 1984), in order to solve the related Sturm-Liouville type eigenvalue problem, which permits to determine automatically and highly accurately as many eigenvalues and eigenfunctions as are needed. Alternatively, we also use the approach of the Generalized Integral Transform Technique (GITT), which has demonstrated to be efficient and safe as the sign-count method (Cotta, 1993; Mikhailov and Cotta, 1994).

Then, to overcome the difficulties related above and to be able to perform heat transfer calculations in regions very close to the inlet with a high degree of accuracy, the ideas in the integral transform technique in conjunction with the well-established sign-count method and GITT approach, to solve the related eigenvalue problem, are used in the present work, so that benchmark results are established for this problem. In the mathematical modeling a boundary condition of prescribed heat flux at the duct walls and, in addition, viscous dissipation effects are also taken into account. Numerical results for Nusselt numbers and dimensionless wall temperatures are computed in the thermal entry region as functions of the Brinkman number and parameters of PTT model. We emphasize the convergence behavior of the series solution and critical comparisons with previous results in the literature are performed, which show the consistency of the final results.

2. Analysis

We analyze steady-state heat transfer problem of an incompressible fluid that obeys the Phan-Thien-Tanner (PTT) model flowing in the thermal entry region of either a parallel plates channel or a circular tube. The flow is considered to be hydrodynamically developed; the duct wall is subjected to a prescribed uniform heat, the fluid enters with a uniform temperature, T_i, and viscous dissipation effects are also taken into account. Axial diffusion and wall-conjugation are neglected, in addition the physical properties are assumed to be constant.

The mathematical formulation for this general forced convection heat transfer problem in dimensionless form is written as:

$$W(R)\frac{\partial\theta(R,Z)}{\partial Z}=\frac{\partial}{\partial R}\left[R^p\frac{\partial\theta(R,Z)}{\partial R}\right]+Br\Phi(R),\quad \text{in } 0<R<1,\ Z>0 \tag{1.a}$$

subjected to the inlet and boundary conditions

$$\theta(R,0)=0,\quad 0\le R\le 1 \tag{1.b}$$

$$\frac{\partial\theta(0,Z)}{\partial R}=0;\quad \frac{\partial\theta(1,Z)}{\partial R}=-1,\quad Z>0 \tag{1.c,d}$$

The following dimensionless groups were employed in Eqs. (1) above

$$R=\frac{r}{r_w};\quad Z=\frac{z/D_h}{Re\,Pr};\quad Re=\frac{\rho\bar{u}D_h}{\eta};\quad Pr=\frac{\nu}{\alpha}=\frac{(\eta/\rho)}{\alpha};\quad \theta(R,Z)=\frac{T(r,z)-T_i}{(q''_w r_w/k)};$$
$$U(R)=\frac{u(r)}{\bar{u}};\quad W(R)=2^{2p-4}R^pU(R);\quad Br=\frac{\eta\bar{u}^2}{D_hq''_w};\quad We=\frac{\lambda\bar{u}}{r_w} \tag{2.a-i}$$

where $D_h = 2^{2-p}r_w$ is the hydraulic diameter and Re, Pr, Br and We are the apparent Reynolds and Prandtl, Brinkman and Weissenberg numbers, respectively.

The dimensionless fully developed velocity profile U(R) is that obtained by Oliveira and Pinho (1999) for a Phan-Thien-Tanner fluid and Φ(R) is the dimensionless viscous dissipation function, which are given in the form:

$$U(R)=K(\bar{u}_N/\bar{u})(1-R^2)[1+a(1+R^2)];\quad a=4K^2\varepsilon We^2(\bar{u}_N/\bar{u})^2 \tag{3.a,b}$$

$$\Phi(R)=2^{4-p}K^2(\bar{u}_N/\bar{u})^2(R^{2+p}+2aR^{4+p}) \tag{3.c}$$

the way to calculate the parameter $(\bar{u}_N/\bar{u})$ is also described in the work of Oliveira and Pinho (1999). The exponent p and coefficient K are related to the duct geometry, as follows:

$$\begin{cases} p=0 \text{ and } K=3/2, \text{ for parallel plates channel} \\ p=1 \text{ and } K=2, \text{ for circular tube} \end{cases} \tag{4.a,b}$$

The problem defined by Eqs. (1) can be readily solved by the classical integral transform technique (Mikhailov and Özisik, 1984; Cotta, 1993). However, in order to obtain a convergence acceleration of the final solution, the so-called splitting-up procedure is applied to this problem (Mikhailov, 1977; Mikhailov and Özisik, 1984). Then, it is proposed as a general separation into simpler problems in the form:

$$\theta(R,Z)=\theta_{av}(Z)+\theta_p(R)+\theta_h(R,Z) \tag{5}$$

where $\theta_{av}(Z)$ is the average temperature, defined as:

$$\theta_{av}(Z)=\frac{\int_0^1 W(R)\theta(R,Z)dR}{\int_0^1 W(R)dR}=2^{4-2p}(p+1)\int_0^1 W(R)\theta(R,Z)dR \tag{6}$$

and, for this case of a prescribed wall heat flux, when all boundary conditions are of the second kind, the average temperature is given *a priori* in the form:

$$\theta_{av}(Z)=2^{4-2p}(p+1)Z\left\{Br\frac{2^{3-p}K}{[1+2a(p+3)/(p+5)]}-1\right\} \tag{7}$$

In Eq. (5), $\theta_p(R)$ represents the separated solution due to the nonhomogeneous boundary condition, Eq. (1.d), and $\theta_h(R,Z)$ is the homogeneous version of problem (1), and are obtained from the following formulations:

$$\frac{d}{dR}\left[R^p \frac{d\theta_p(R)}{dR}\right] + Br\Phi(R) = W(R)\frac{d\theta_{av}(Z)}{dZ}, \quad \text{in} \quad 0<R<1 \tag{8.a}$$

with boundary conditions

$$\frac{d\theta_p(0)}{dR} = 0; \quad \frac{d\theta_p(1)}{dR} = -1; \quad \int_0^1 W(R)\theta_p(R)dR = 0 \tag{8.b-d}$$

For this case of all boundary conditions of second kind, where $\theta_{av}(R)$ is determined *a priori*, Eq. (8.d) represents an additional boundary condition necessary to determine one of the constants that appear after the integration of the problem for $\theta_p(R)$. Thus, this problem is readily integrated to furnish

$$\begin{aligned}\theta_p(R) = \theta_p(1) &+ \frac{K}{[1+2a(p+3)/(p+5)]}\left[\frac{(1+a)}{2}(1-R^2) - \frac{(p+1)}{4(p+3)}(1-R^4) - \frac{a(p+1)}{6(p+5)}(1-R^6)\right] \\ &- Br\frac{2^{4-p}K^2}{[1+2a(p+3)/(p+5)]^2}\left[\frac{(1+a)}{4}(1-R^2) - \frac{(1-R^4)}{8} - \frac{a}{12}(1-R^6)\right]\end{aligned} \tag{9.a}$$

$$\begin{aligned}\theta_p(1) = Br\frac{2^{4-p}K^3}{24[1+2a(p+3)/(p+5)]^3}\Bigg\{ & a^2\left[4 - \frac{6(p+1)}{(p+3)} - \frac{4(p+1)}{(p+5)} + \frac{8(p+1)}{(p+7)} - \frac{2(p+1)}{(p+11)}\right] \\ & + a\left[7 - \frac{16(p+1)}{(p+3)} + \frac{6(p+1)}{(p+5)} + \frac{8(p+1)}{(p+7)} - \frac{5(p+1)}{(p+9)}\right] \\ & + \left[3 - \frac{9(p+1)}{(p+3)} + \frac{9(p+1)}{(p+5)} - \frac{3(p+1)}{(p+7)}\right]\Bigg\} \\ - \frac{K^2}{[1+2a(p+3)/(p+5)]^2}\Bigg\{ & a^2\left[\frac{1}{2} - \frac{(p+1)}{2(p+3)} - \frac{2(p+1)}{3(p+5)} + \frac{(p+1)}{2(p+7)} + \frac{(p+1)^2}{6(p+5)^2} + \frac{(p+1)^2}{6(p+5)(p+7)} \right. \\ & \left. - \frac{(p+1)^2}{6(p+5)(p+11)}\right] + \\ & a\left[1 - \frac{7(p+1)}{4(p+3)} - \frac{(p+1)}{6(p+5)} + \frac{(p+1)}{2(p+7)} + \frac{2(p+1)^2}{3(p+3)(p+5)} - \frac{(p+1)^2}{4(p+3)(p+9)} \right. \\ & \left. + \frac{(p+1)^2}{6(p+5)(p+7)} - \frac{(p+1)^2}{6(p+5)(p+9)}\right] + \\ & \left[\frac{1}{2} - \frac{5(p+1)}{4(p+3)} + \frac{(p+1)}{2(p+5)} + \frac{(p+1)^2}{4(p+3)^2} \right. \\ & \left. + \frac{(p+1)^2}{4(p+3)(p+5)} - \frac{(p+1)^2}{4(p+3)(p+7)}\right]\Bigg\}\end{aligned} \tag{9.b}$$

and, the general homogeneous problem is given by:

$$W(R)\frac{\partial\theta_h(R,Z)}{\partial Z}=\frac{\partial}{\partial R}\left[R^p\frac{\partial\theta_h(R,Z)}{\partial R}\right],\quad \text{in}\quad 0<R<1,\;\; Z>0 \tag{10.a}$$

$$\theta_h(R,0)=-\theta_p(R),\quad 0\le R\le 1 \tag{10.b}$$

$$\frac{\partial\theta_h(0,Z)}{\partial R}=0\,;\quad \frac{\partial\theta_h(1,Z)}{\partial R}=0\,,\quad Z>0 \tag{10.c,d}$$

The homogeneous problem given by Eqs. (10) can also be solved by the classical integral transform technique (Mikhailov and Özisik, 1984; Cotta, 1993). Then, following the procedures of this technique, the general appropriate eigenvalue problem needed for its solution is taken as

$$\frac{d}{dR}\left[R^p\frac{d\psi_i(R)}{dR}\right]+\mu_i^2W(R)\psi_i(R)=0\,,\quad \text{in}\quad 0<R<1 \tag{11.a}$$

$$\frac{d\psi_i(0)}{dR}=0\,;\quad \frac{d\psi_i(1)}{dR}=0 \tag{11.b,c}$$

where $\psi_i(R)$ and μ_i are, respectively, the eigenfunctions and eigenvalues. The problem defined by Eqs. (11) is solved by the so-called Sign-Count Method (Mikhailov and Vulchanov, 1983) and Generalized Integral Transform Technique (Cotta, 1993; Mikhailov and Cotta, 1994), which offer safe and automatic computations of as many eigenvalues and eigenfunctions as it is desired, with controlled accuracy. The eigenvalue problem above allows for the development of the following integral transform pair:

$$\bar{\theta}_{hi}(Z)=\int_0^1 W(R)\,\psi_i(R)\,\theta_h(R,Z)\,dR\,,\qquad \text{transform} \tag{12.a}$$

$$\theta_h(R,Z)=\sum_{i=1}^{\infty}\frac{1}{N_i}\psi_i(R)\,\bar{\theta}_{hi}(Z)\,,\qquad \text{inversion} \tag{12.b}$$

where N_i is the normalization integral given by:

$$N_i=\int_0^1 W(R)\psi_i^2(R)\,dR \tag{13}$$

Taking the integral transform of the system given by Eqs. (10), equations above are operated with $\int_0^1 \psi_i(R)dR$, and the following ordinary differential equation for the transformed potential, $\bar{\theta}_{h,i}(Z)$, is obtained:

$$\frac{d\bar{\theta}_{h,i}(Z)}{dZ}+\mu_i^2\,\bar{\theta}_{h,i}(Z)=0 \tag{14.a}$$

with the transformed inlet condition given by

$$\bar{\theta}_{h,i}(0)=\bar{f}_i=-\int_0^1 W(R)\psi_i(R)\theta_p(R)dR \tag{14.b}$$

The solution for the transformed potential given by Eqs. (14) is readily obtained in the form

$$\bar{\theta}_{h,i}(Z)=\bar{f}_i\exp(-\mu_i^2Z) \tag{15}$$

Therefore, introducing Eq. (15) into the inversion formula (12.b), the solution for $\theta_h(R,Z)$ is determined as follows

$$\theta_h(R,Z)=\sum_{i=1}^{\infty}\frac{\bar{f}_i}{N_i}\psi_i(R)\exp(-\mu_i^2Z) \tag{16}$$

Thus, Eq. (16) in conjunction with Eqs. (9) for $\theta_p(R)$, complete the solution for the potential $\theta(R,Z)$ defined in Eq. (5). This solution is written as:

$$\begin{aligned}\theta(R,Z) = \theta_{av}(Z) + \theta_p(1) + \frac{K}{[1+2a(p+3)/(p+5)]}\left[\frac{(1+a)}{2}(1-R^2) - \frac{(p+1)}{4(p+3)}(1-R^4) - \frac{a(p+1)}{6(p+5)}(1-R^6)\right] \\ - Br\frac{2^{4-p}K^2}{[1+2a(p+3)/(p+5)]^2}\left[\frac{(1+a)}{4}(1-R^2) - \frac{(1-R^4)}{8} - \frac{a}{12}(1-R^6)\right] + \\ + \sum_{i=1}^{\infty}\frac{\bar{f}_i}{N_i}\psi_i(R)\exp(-\mu_i^2 Z)\end{aligned} \tag{17}$$

The local Nusselt number is defined as:

$$Nu(Z) = \frac{h(z)D_h}{k} = \frac{-2^{2-p}\dfrac{\partial\theta(1,Z)}{\partial R}}{\theta_{av}(Z) - \theta(1,Z)} = \frac{2^{2-p}}{\theta_{av}(Z) - \theta(1,Z)} \tag{18}$$

The wall temperature $\theta(1,Z)$ is obtained from Eq. (17) as:

$$\theta(1,Z) = \theta_{av}(Z) + \theta_p(1) + \sum_{i=1}^{\infty}\frac{\bar{f}_i}{N_i}\psi_i(1)\exp(-\mu_i^2 Z) \tag{19}$$

The local Nusselt number is now completed by substituting Eq. (19) into Eq. (18), resulting

$$Nu(Z) = \frac{-2^{2-p}}{\theta_p(1) - \displaystyle\sum_{i=1}^{\infty}\frac{\psi_i^2(1)}{N_i\mu_i^2}\exp(-\mu_i^2 Z)} \tag{20}$$

where, $\theta_p(1)$ is given by Eq. (9.b). The asymptotic Nusselt number, Nu_∞, is determined by making $Z \to \infty$ in Eq. (20), so that

$$Nu_\infty = \frac{-2^{2-p}}{\theta_p(1)} \tag{21}$$

To complete the solution is necessary to evaluate the eigenvalues, μ_i, the eigenfunctions ψ_i and the normalization integral N_i of the eigenvalue problem (11). Here, for instance, we have used both the sign-count method established in references (Mikhailov and Vulchanov, 1983; Mikhailov and Özisik, 1984) and the generalized integral transform technique (Cotta, 1993; Mikhailov and Cotta, 1994) to determine the eigenvalues and another related eigenquantities necessary to compute the average temperature and the local Nusselt numbers from Eq. (20).

3. Results and discussion

Numerical results for Nusselt numbers were produced for different values of the product εWe^2 (product of the extensional parameter of PTT model-Weissenberg number to square) and Brinkman numbers (Br < 0 that corresponds to a fluid heating) in the thermal entry region of both a parallel plates channel and a circular tube. The computational code was developed in FORTRAN 90/95 programming language and implemented on a PENTIUM-IV 1.3 GHz computer.

First, the eigenvalues and another related eigenquantities were obtained by the two approaches cited above, and are in perfect agreement. Due to space limitations they are not listed here. Then, the average temperature, $\theta_{av}(Z)$, and the local Nusselt numbers, Nu(Z), were calculated.

The numerical code was validated for the case of $\varepsilon We^2 = 0$ and Br = 0 (Newtonian situation without viscous dissipation) against those results presented by Quaresma and Macêdo (1998), which have also employed the integral transform methodology to forced convection heat transfer problem in channels involving Herschel-Bulkley fluids. Table (1) shows this comparison, emphasizing the convergence behavior of the Nusselt numbers with different truncation orders N, and an excellent agreement is verified, which provides a direct validation of the numerical code here developed.

Table 1. Comparison and convergence behavior of the local Nusselt number in the thermal entry region for the case of $\varepsilon We^2 = 0$ and Br = 0 (Newtonian situation without viscous dissipation).

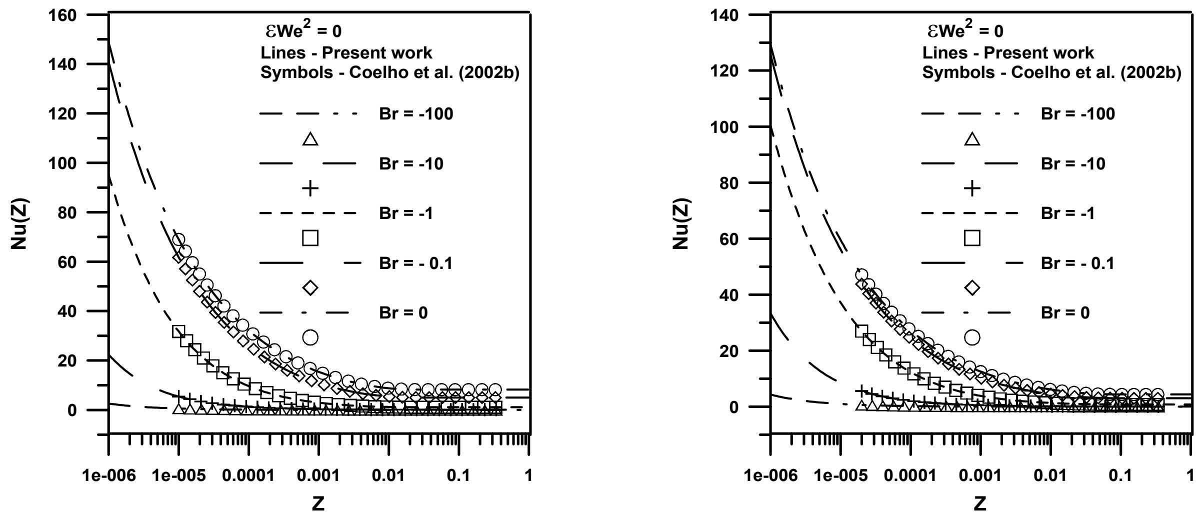
Z	Nu (Z)								
	parallel plates channel				circular tube				
	N = 100	N = 200	N = 400	Quaresma and Macêdo (1998)	N = 100	N = 200	N = 400	N = 600	Quaresma and Macêdo (1998)
$1x10^{-6}$	146.82	146.77	146.77	146.78	104.35	125.75	129.19	129.20	129.18
$5x10^{-6}$	86.953	86.954	86.954	86.955	74.037	75.188	75.190	75.190	75.180
$1x10^{-5}$	69.011	69.011	69.011	69.011	59.423	59.510	59.510	59.510	59.504
$5x10^{-5}$	40.419	40.419	40.419	40.420	34.511	34.511	34.511	34.511	34.508
$1x10^{-4}$	32.156	32.156	32.156	32.156	27.276	27.276	27.276	27.276	27.274
$5x10^{-4}$	19.112	19.112	19.112	19.113	15.813	15.813	15.813	15.813	15.812
$1x10^{-3}$	15.427	15.427	15.427	15.427	12.538	12.538	12.538	12.538	12.538
$5x10^{-3}$	9.9878	9.9878	9.9878	9.9878	7.4937	7.4937	7.4937	7.4937	7.4936
$1x10^{-2}$	8.8031	8.8031	8.8031	8.8031	6.1481	6.1481	6.1481	6.1481	6.1481
$5x10^{-2}$	8.2355	8.2355	8.2355	8.2355	4.5139	4.5139	4.5139	4.5139	4.5138
$1x10^{-1}$	8.2353	8.2353	8.2353	8.2353	4.3748	4.3748	4.3748	4.3748	4.3748
$5x10^{-1}$	8.2353	8.2353	8.2353	8.2353	4.3636	4.3636	4.3636	4.3636	4.3636
1.0	8.2353	8.2353	8.2353	8.2353	4.3636	4.3636	4.3636	4.3636	4.3636

In Figs. (1) to (6) are presented comparisons of the present results for the local Nusselt numbers with those of Coelho et al. (2002b) in the thermal entry region of both ducts analyzed by varying the governing parameters, i.e., the product εWe^2 and Brinkman numbers. The present results for Nu(Z) were plotted in the range $10^{-6} \leq Z \leq 1$, while comparisons with results of Coelho et al. (2002b) were done in the range $10^{-5} \leq Z \leq 0.4$, where one can see an excellent agreement among the results in all dimensionless positions analyzed, this way once again validating the numerical code developed here.

From these sets of figures, it can be verified that the effect of Brinkman number in both duct geometric configurations analyzed is more pronounced for values of Br < -1. For example, with Br = - 100, there is a high internal generation of heat in the fluid, so that the heat flux supplied at the wall duct is of the same magnitude order as that generated in the fluid and, consequently, the local Nusselt number along the thermal entry region assumes lower values near zero.

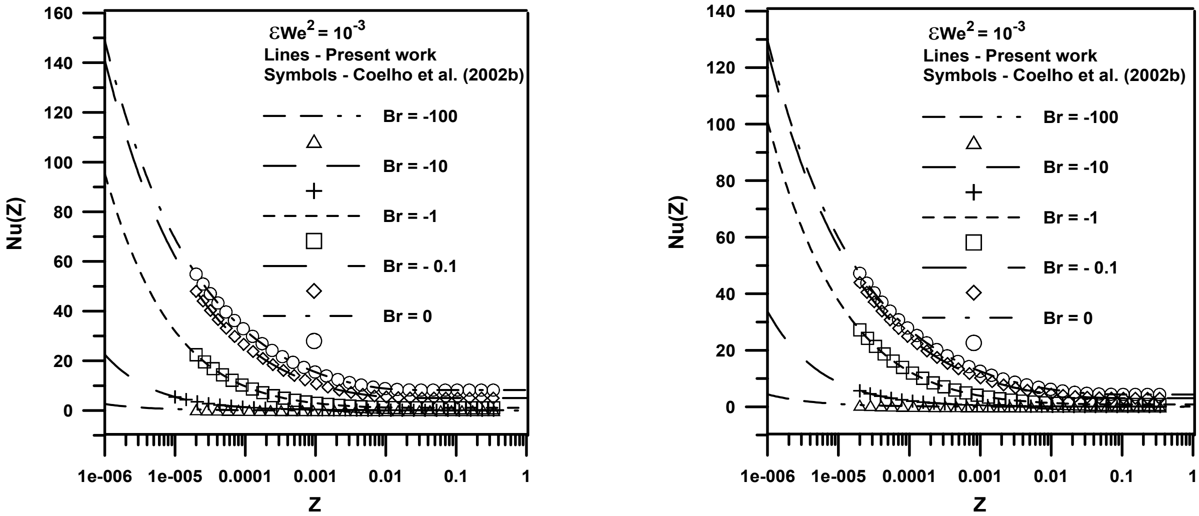
The influence of parameter εWe^2 is evident for values near unity. In a general way the effect of this parameter is to increase the Nusselt number due to a more pronounced shear-thinning effect in the fluid as verified by Coelho et al. (2002a). It is important to note that the situation of $\varepsilon We^2 = 0$ represents the case with absence of both extensional and elastic fluid properties, i.e., the Newtonian case is reproduced as was observed in Table (1).

Finally, it can be noticed that the results are systematically larger for a parallel plates channel than for a circular tube due to higher exchange heat areas presented by this flat duct.



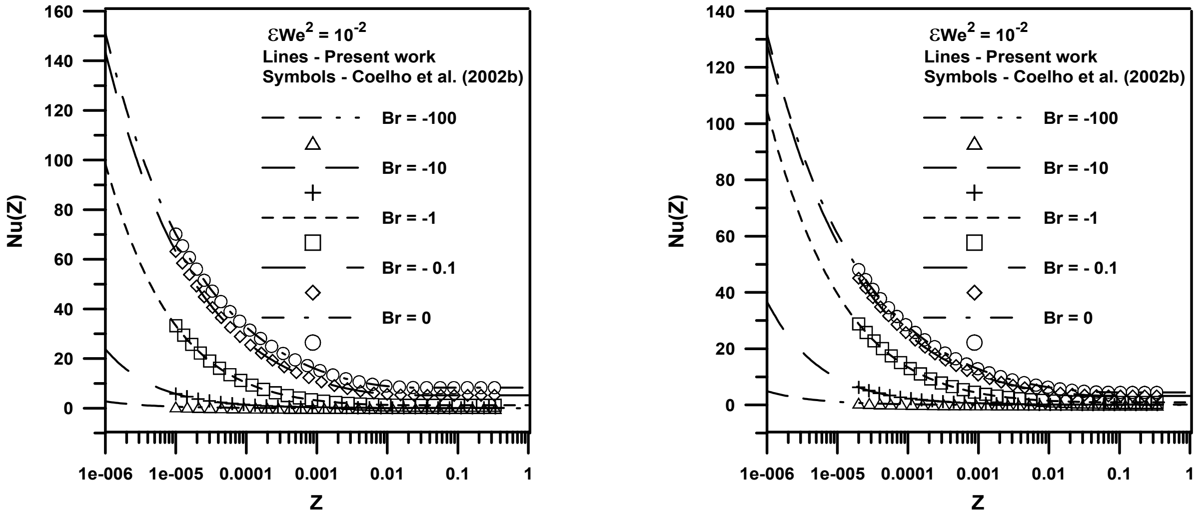
(a) (b)

Figure 1. Comparison of local Nusselt numbers for the product $\varepsilon We^2 = 0$. (a) parallel plates channel; (b) circular tube.



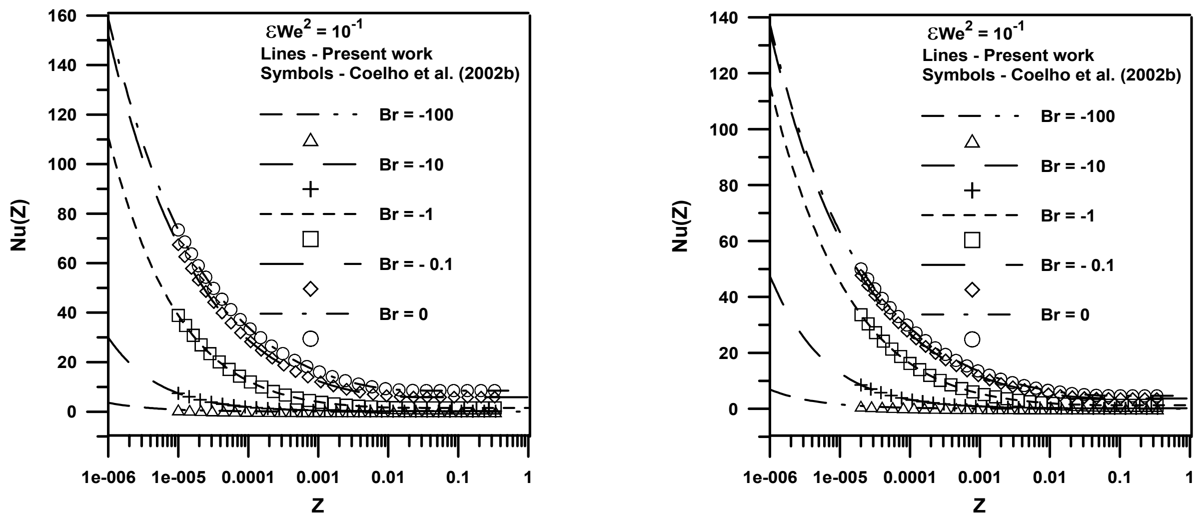
(a) (b)

Figure 2. Comparison of local Nusselt numbers for the product $\varepsilon We^2 = 10^{-3}$. (a) parallel plates channel; (b) circular tube.



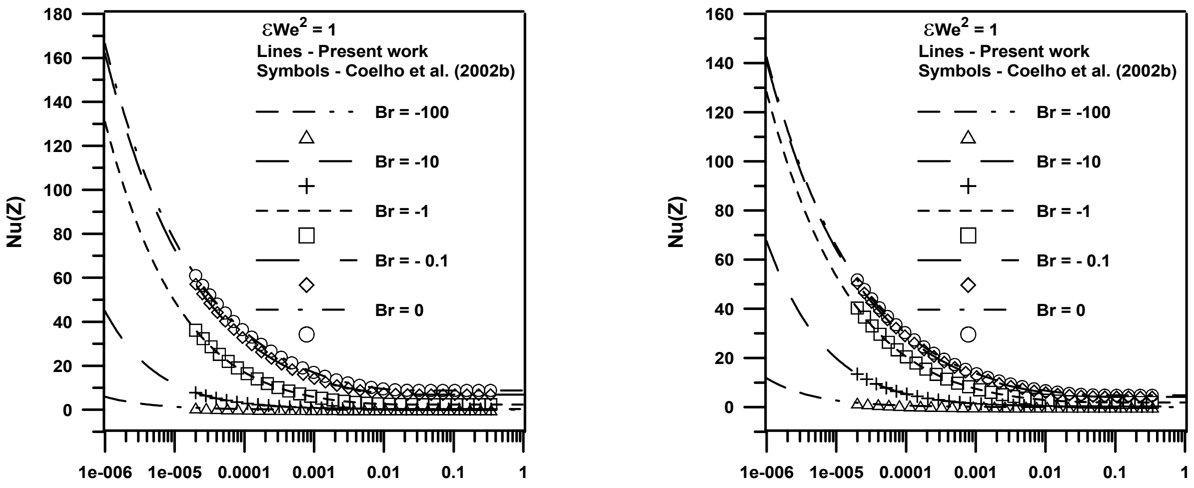
(a) (b)

Figure 3. Comparison of local Nusselt numbers for the product $\varepsilon We^2 = 10^{-2}$. (a) parallel plates channel; (b) circular tube.



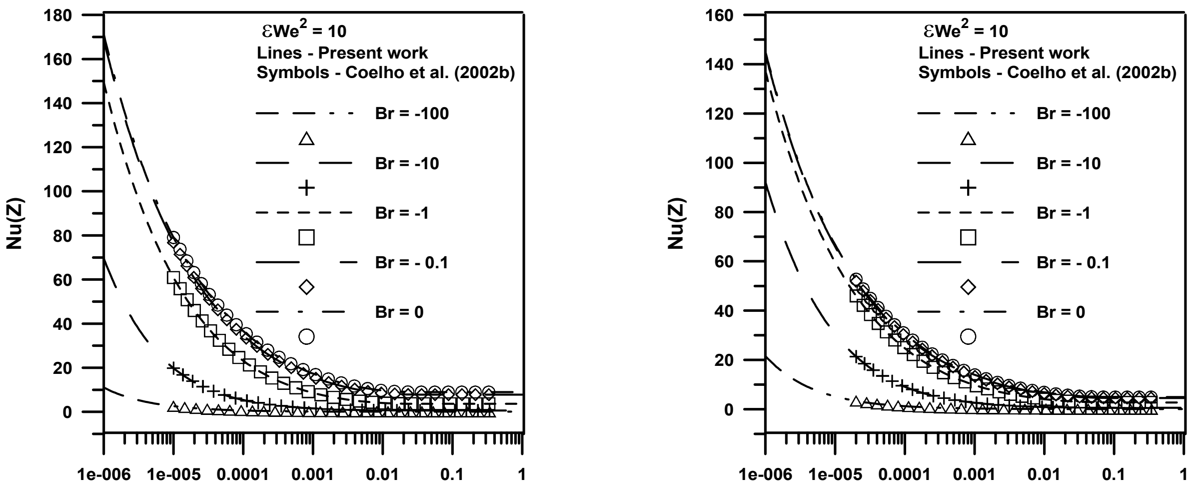
(a) (b)

Figure 4. Comparison of local Nusselt numbers for the product $\varepsilon We^2 = 10^{-1}$. (a) parallel plates channel; (b) circular tube.



(a) (b)

Figure 5. Comparison of local Nusselt numbers for the product $\varepsilon We^2 = 1$. (a) parallel plates channel; (b) circular tube.



(a) (b)

Figure 6. Comparison of local Nusselt numbers for the product $\varepsilon We^2 = 10$. (a) parallel plates channel; (b) circular tube.

4. Conclusions

The problem of the laminar convection heat transfer to a PTT fluid in the thermal entry region of both a parallel-plates channel and a circular tube, for the case of prescribed wall heat flux, has been analyzed, with excellent computational performance, through the integral transform methodology in conjunction with the sign-count method and GITT approach for the solution of the related eigenvalue problem. Benchmark results for the local Nusselt number were tabulated and graphically established with different values of the product εWe^2 and Brinkman numbers, which demonstrated to have significant influence in the final results, for values near unity and –1, respectively.

5. References

Azaiez, J., Guénette, R. and Aït-Kadi, A., 1996, "Numerical Simulations of Viscoelastic flows through a Planar Contraction", Journal of Non-Newtonian Fluid Mechanics , Vol. 62, pp. 253-277.

Baaijens, F.P.T., 1993, "Numerical Analysis of Start-up Planar and Axisymmetric Contraction Flows Using Multi-Mode Differential Constitutive Models", Journal of Non-Newtonian Fluid Mechanics, Vol. 48, pp. 147-180.

Baloch, A., Townsend, P. and Webster, M.F., 1996, "On Vortex Development in Viscoelastic Expansion and Contraction Flows", Journal of Non-Newtonian Fluid Mechanics, Vol. 65, pp. 133-149.

Bird, R.B., Armstrong, R.C. and Hassager, O., 1987, "Dynamics of Polymeric Liquids", Vol. 1, 2nd Ed., John Wiley, New York, USA.

Coelho, P.M., Pinho, F.T. and Oliveira, P.J., 2002a, "Fully Developed Forced Convection of the Phan-Thien-Tanner Fluid in Ducts with a Constant Wall Temperature", International Journal of Heat and Mass Transfer, Vol. 45, pp. 1413-1423.

Coelho, P.M., Pinho, F.T. and Oliveira, P.J., 2002b, "Thermal Entry Flow for a Viscoelastic Fluid: The Graetz Problem for the PTT Model", International Journal of Heat and Mass Transfer, submitted.

Cotta, R.M., 1993, "Integral Transforms in Computational Heat and Fluid Flow", CRC Press, Boca Raton, USA.

Mikhailov, M.D., 1977, "Splitting Up of Heat-Conduction Problems", Letters in Heat and Mass Transfer, Vol. 4, pp. 163-166.

Mikhailov, M.D. and Vulchanov, N.L., 1983, "Computational Procedure for Sturm-Liouville Problems", Journal of Computational Physics, Vol. 50, pp. 323-336.

Mikhailov, M.D. and Özisik, M.N., 1984, "Unified Analysis and Solutions of Heat and Mass Diffusion", John Wiley, New York, USA.

Mikhailov, M.D. and Cotta, R.M., 1994, "Integral Transform Method for Eigenvalue Problems", Communications in Numerical Methods in Engineering, Vol. 10, pp. 827-835.

Oliveira, P.J. and Pinho, F.T., 1999, "Analytical Solutions for Fully Developed Channel and Pipe Flow of Phan-Thien-Tanner Fluids", Journal of Fluids Mechanics, Vol. 387, pp. 271-280.

Phan-Thien, N. and Tanner, R.I., 1977, "A New Constitutive Equation Derived from Network Theory", Journal of Non-Newtonian Fluid Mechanics, Vol. 2, pp. 353-365.

Phan-Thien, N., 1978, "A Nonlinear Network Viscoelastic Model", Journal of Rheology, Vol. 22, pp. 259-283.

Pinho, F.T. and Oliveira, P.J., 2000, "Analysis of Forced Convection in Pipes and Channels with the Simplified Phan-Thien-Tanner Fluid", International Journal of Heat and Mass Transfer, Vol. 43, pp. 2273-2287.

Quaresma, J.N.N. and Macêdo, E.N., 1998, "Integral Transform Solution for the Forced Convection of Herschel-Bulkley Fluids in Circular Tubes and Parallel-Plates Ducts", Brazilian Journal of Chemical Engineering, Vol. 15, pp. 77-89.

Quinzani, L., Armstrong, R.C. and Brown, R.A., 1995, "Use of Coupled Birefringence and LDV Studies of Flow through a Planar Contraction to Test Constitutive Equations for Concentrated Polymer Solutions", Journal of Rheology, Vol. 39, pp. 1201-1228.