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HEAT TRANSFER IN LIQUID-LIQUID ANNULAR TWO-PHASE FLOW IN A VERTICAL DUCT

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Abstract. *Annular flow and heat transfer of immiscible liquids inside channels have several practical motivations related to pumping of very viscous fluids, heat exchange enhancement, fouling and corrosion reduction, etc. In this work, an analytical solution for the temperature field in the thermal entrance region of vertical annular undisturbed flow of two immiscible liquids is presented, taken as a limiting situation for more general stratified and/or dispersed two-phase flow modeling. The integral transform technique is used to describe the laminar flow heat transfer phenomena, under a constant heat flux boundary condition at the tube wall. To demonstrate its applicability, the theoretical model here proposed has been applied to the analysis of a situation dealing with corrosion and fouling effects reduction, by using a kerosene flow between the wall and water core inside a vertical circular duct. In this system, the heat transfer to the core is expected to be affected by the fact that kerosene has a lower thermal conductivity when compared with water, and we attempt to investigate the heat transfer rate reduction as a function of the kerosene layer relative thickness. The theoretical predictions here obtained are critically compared with previous numerical results available in the open literature.*

1. Introduction

The majority of the works about annular flow in the archival literature has given emphasis to the gas-liquid flow situation. However, there are certain liquid-liquid annular concurrent flows of considerable practical interest, which nevertheless have received much less attention. Two such systems that are common in practice are the kerosene-water flow (Hasson et al., 1974, Ziviani et al., 1991) and water-oil flow (Bentwich and Sideman, 1964, Brauner, 1991, Vanegas Prada and Bannwart, 1999.a, Vanegas Prada and Bannwart, 1999.b, Angeli and Hewitt, 2000, Bannwart, 2001). The first one, due to the contact of a less reactive fluid with the pipe wall, offers a reduction of corrosion processes and encrustation; the second one allows, due to the smaller viscosity and larger thermal conductivity of the fluid in contact with the duct wall, for the reduction in pumping power and for the enhancement of heat transfer. In fact, the transportation of a very viscous oil is in general achieved by heating the oil and thus reducing its viscosity, while providing insulation at the duct's wall. This operation, however, involves a considerable inherent cost. An alternative to this long distance oil transportation problem has been the object of experimental and theoretical analysis (Brauner, 1991, Oliemans et al., 1987, Vanegas Prada and Bannwart, 1999.a, Vanegas Prada and Bannwart, 1999.b, Bannwart, 2001), and the most promising proposal seems to be exactly the injection of a less viscous liquid (water) around the duct wall so as to establish an annular less viscous flow region. It has been demonstrated (Everage, 1973) that a complete encapsulation of the more viscous fluid makes it possible to minimize viscous dissipation as well, leading to a reduced energy loss. Vanegas Prada and Bannwart (1999.a) compared the core flow technology with the other alternatives for heavy oil production and concluded that core flow lift of heavy oil is quite viable and advantageous. An experimental apparatus for the study of core annular flows of heavy oil and water were developed by Vanegas Prada and Bannwart (1999.b). They also developed a model for upward core annular flow, from experimental measurements of pressure difference and based on a simple theoretical approach. Recently, further studies based on mass and momentum balances were developed for volume fraction and pressure drop and compared with available data for both horizontal and vertical oil-water core flow (Bannwart, 2001).

In gas-liquid systems, the laminar flow regimen in the liquid occurs within a narrow range of entrance conditions or, in other words, to within a low mass flow rate ratio between the gas and the liquid (Nogueira et al, 1990). On the other hand, for liquid-liquid systems the laminar flow regimen is observed within a large range of values of mass flow rates in both fluids (Hasson et al., 1974, Brauner, 1991). In terms of modeling, this physical observation results in a feasible analytical determination of the velocity profiles in both regions through direct integration. Nevertheless, a very limited number of previous contributions attempted to analytically solve the problem of forced convection in the annular liquid-liquid annular flow configuration, even for laminar regimen. Few exceptions are the works by Bentwich and Sideman (1964) and Leib et al. (1977). The first one considered the heat transfer problem, supposing the horizontal flow of water-oil, with a uniform temperature at the wall of the duct; the second one studied the problem for vertical flow of kerosene-water with a prescribed heat flux as boundary condition. Bentwich and Sideman (1964) concluded, in despite of the simplification made to represent the horizontal flow as annular, that a heat exchanger using the water-oil pair would be much more efficient, for a given duct length, than a heat exchanger with oil only, because of the significant heat transfer enhancement offered by the water annulus. Leib et al. (1977) demonstrated that the developed theoretical model described adequately the thermal entrance region temperature field when the flow is not disturbed, i.e., when the flow does not present waves at the interface between the liquids.

Within this context, the present contribution advances the analytical treatment of two-phase liquid-liquid annular flow and heat transfer within a vertical circular tube, by analyzing the thermal entrance region development, making use of the integral transform method (Mikhailov and Ozisik, 1984, and Cotta, 1993). From the analytical expressions for the fully developed velocity fields, the coupled energy equations for the liquid streams are solved, by considering a multiregion eigenfunction expansion (Cotta & Nogueira, 1988), following previous developments for gas-liquid annular flows (Nogueira et al, 1995, 1996). The general analytical solution so obtained is then applied to the analysis of a situation dealing with corrosion and fouling effects reduction, by using a kerosene flow between the wall and water core inside a vertical circular duct, in the laminar regimen. In this system, the heat transfer to the core is expected to be affected by the fact that kerosene has a lower thermal conductivity when compared with water, and we attempt to investigate the heat transfer rate reduction as a function of the kerosene layer relative thickness.

2. Analysis

We consider the concurrent annular vertical flow of two immiscible liquids, flowing in concentric regions within a circular tube. Steady state laminar forced convection with incompressible hydrodynamically developed flow, with constant physical properties, has also been assumed. Viscous dissipation, axial conduction and natural convection effects are neglected. The following two sections present, respectively, the flow and heat transfer problems solutions, which are decoupled in light of the above simplifying assumptions.

2.1. Annular Liquid-Liquid Flow

Nogueira et al. (1990, 1995) have demonstrated that under laminar fully developed annular flow the velocity profiles in the two regions (two liquids) within a circular tube are analytically given by:

$$u_1(R) = \frac{P_2 r_2^2}{(n+1)\mu_2}\left\{\hat{\mu}\hat{P}\int_R^{\delta} R' dR' + \int_{\delta}^{1}\left[R' + \frac{(\hat{P}-1)\delta^{n+1}}{R'^n}\right]dR'\right\} \qquad 0 \le R \le \delta \tag{1.a}$$

$$u_2(R) = \frac{P_2 r_2^2}{(n+1)\mu_2}\left\{\int_R^{1}\left[R' + \frac{(\hat{P}-1)\delta^{n+1}}{R'^n}\right]dR'\right\} \qquad \delta \le R \le 1 \tag{1.b}$$

$$R = \frac{r}{r_2}, \qquad \delta = \frac{r_1}{r_2}, \qquad \hat{P} = \frac{P_1}{P_2}, \qquad \hat{\mu} = \frac{\mu_1}{\mu_2}, \qquad P_k = -\left[\frac{dp}{dz} \pm \rho_k g_\alpha\right], \qquad g_\alpha = |g| sin\alpha \tag{1.c-h}$$

where in the present application the index 1 stands for the water core, and the index 2 is related to the kerosene surrounding annulus. From integration of the above equations we obtain (n=1 for a circular tube):

$$u_1(R) = \frac{P_2 r_2^2}{(n+1)\mu_2}\left\{\frac{\hat{\mu}\hat{P}}{2}\left(\delta^2 - R^2\right) + \frac{1}{2}\left(1-\delta^2\right) - \left(\hat{P}-1\right)\delta^2 L_n \delta\right\} \qquad 0 \le R \le \delta \tag{2.a}$$

$$u_2(R) = \frac{P_2 r_2^2}{(n+1)\mu_2}\left\{\frac{1}{2}\left(1-R^2\right) - \left(\hat{P}-1\right)\delta^2 L_n \delta\right\} \qquad \delta \le R \le 1 \tag{2.b}$$

The average velocity and single-phase velocity profile, to be used in comparisons, are obtained from

$$\bar{u} = \frac{P_2 r_2^2}{(n+1)\mu_2}\left\{\frac{\delta^{n+3}}{(n+3)}\left(\hat{\mu}\hat{P}-1\right)+\frac{1}{(n+3)}+\frac{(\hat{P}-1)}{2}-\frac{(\hat{P}-1)\delta^{n+3}}{2}\right\} \tag{3}$$

$$u^*(R) = \frac{P_1 r_1^2}{4\mu_1}\left(1-R^2\right) \tag{4.a}$$

$$\bar{u}^*(R) = \frac{P_1 r_1^2}{8\mu_1} \tag{4.b}$$

where the asterisk indicates the single-phase flow situation.

2.2. Heat Transfer Problem

The general formulation of the thermal problem is established through the following coupled energy equations and their corresponding initial, interface and boundary conditions:

$$u_k(r)\frac{\partial T_k(r,z)}{\partial z} = \frac{1}{r^n}\frac{\partial}{\partial r}\left\{r^n\left[\alpha_k+\varepsilon_{hk}(r)\right]\frac{\partial T_k(r,z)}{\partial r}\right\} \qquad z>0,\ r_{k-1}\le r\le r_k,\quad k=1,2 \tag{5.a}$$

$$T_k(r,z) = f_k(r), \qquad z=0,\ r_{k-1}\le r\le r_k,\quad k=1,2 \tag{5.b}$$

$$\frac{\partial T_1(r,z)}{\partial r} = 0, \qquad z>0,\ r=0 \tag{5.c}$$

$$T_1(r,z) = T_2(r,z), \qquad z>0,\ r=r_1 \tag{5.d}$$

$$\left[k_1+\rho_1 C_1\varepsilon_{h1}(r)\right]\frac{\partial T_1(r,z)}{\partial r} = \left[k_2+\rho_2 C_2\varepsilon_{h2}(r)\right]\frac{\partial T_2(r,z)}{\partial r}, \qquad z>0,\ r=r_1 \tag{5.e}$$

$$\alpha T_2(r,z)+\beta\left[k_2+\rho_2 C_2\varepsilon_{h2}(r)\right]\frac{\partial T_2(r,z)}{\partial r} = \phi(z), \qquad z>0,\ r=r_2 \tag{5.f}$$

where α and β are constants utilized to reproduce different boundary conditions at the wall $(r=r_2)$. Therefore:

a) $\alpha=1, \beta=0 \Rightarrow \phi(z)=T_w$ - Prescribed temperature

b) $\alpha=0, \beta=1 \Rightarrow \phi(z)=q_w(z)$ - Prescribed heat flux

In addition, the model was held sufficiently general to include different geometries and the turbulent flow situation, for use in future contributions. The general dimensionless formulation of the problem is established through the following equations of energy and their interface and boundary conditions:

$$\left(\frac{l_0}{l_n}\right)^2 C\,\frac{\rho_k C_k}{\overline{\rho C}} R^n U_k(R)\frac{\partial\theta_k(R,Z)}{\partial Z} = \frac{\partial}{\partial R}\left[R^n\frac{k_k}{\bar{k}}E_{hk}(R,u_k)\frac{\partial\theta(R,Z)}{\partial R}\right] \qquad Z>0,\ R_{k-1}\le R\le R_k,\quad k=1,2 \tag{6.a}$$

$$\theta_k(R,Z) = F_k(R), \qquad Z=0,\ R_{k-1}\le R\le R_k,\quad k=1,2 \tag{6.b}$$

$$\frac{\partial\theta_1(R,Z)}{\partial R} = 0, \qquad Z>0,\ R=0 \tag{6.c}$$

$$\theta_1(R,Z) = \theta_2(R,Z), \qquad Z>0,\ R=\delta \tag{6.d}$$

$$R^n\frac{k_1}{\bar{k}}E_{h1}(R,u_1)\frac{\partial\theta_1(R,Z)}{\partial R} = R^n\frac{k_2}{\bar{k}}E_{h2}(R,u_2)\frac{\partial\theta_2(R,Z)}{\partial R}, \qquad Z>0,\ R=\delta \tag{6.e}$$

$$\frac{\alpha l_0}{\bar{k}\Delta T}\left[T^* + \Delta T\theta_2(R,Z)\right] + \beta\frac{k_2}{\bar{k}}E_{h2}(R,u_2)\frac{\partial\theta_2(R,Z)}{\partial R} = \frac{\phi(z)l_0}{\bar{k}\Delta T}, \qquad Z>0,\ R=R_2 \tag{7.f}$$

The various dimensionless groups appearing above are defined as:

$$Z = \frac{\bar{\alpha}z}{\bar{u}l_n^2}, \quad \theta_k(R,Z) = \frac{T_k(r,z)-T^*}{\Delta T}, \quad F_k(R) = \frac{f_k(r)-T^*}{\Delta T}, \quad \Phi(Z) = \frac{\phi(z)l_0}{\bar{k}\Delta T}, \quad T^* = f_1(0), \quad \Delta T = \frac{\phi(z)l_n}{\bar{k}} \tag{7.a-f}$$

where

$$l_0 = r_2,\ l_n = 2r_2,\ \bar{\alpha} = \frac{\bar{k}}{\bar{\rho}\bar{C}},\ \bar{k} = \delta^{n+1}k_1 + \left(1+\delta^{n+1}\right)k_2,\ \bar{\rho} = \delta^{n+1}\rho_1 + (1+\delta^{n+1})\rho_2,\ \bar{C} = \delta^{n+1}C_1 + (1+\delta^{n+1})C_2 \tag{7.g-l}$$

and

$$E_{hk}(R,u_k) = 1 + \mathrm{Pr}_k\frac{\varepsilon_{hk}(R)}{\nu_k} = 1 + \frac{\mathrm{Pr}_k}{\mathrm{Pr}_{kt}}\left(\frac{\varepsilon_{mk}(R,u_k)}{\nu_k} + \frac{\varepsilon_{ok}(R)}{\nu_k}\right) \tag{7.m}$$

where Pr_k and Pr_{kt} are the molecular and turbulent Prandtl numbers that relate the diffusivities of heat and momentum, incorporating both effects of eddies and waves, Z is the axial coordinate, θ is the dimensionless temperature, $F(R)$ is the dimensionless inlet temperature distribution, $\Phi(Z)$ is the dimensionless prescribed boundary source function, C is the constant employed in the dimensionless velocity expression, l_0 is the reference length (= r_2), l_n is another reference length (= $2r_2$), r_0 is the channel centerline position (r_0=0), r_1 is the interface position, r_2 is the channel wall position, $E_{hk}(R)$ is the dimensionless thermal diffusivity of phase k, $E_{tk}(R)$ is the dimensionless momentum diffusivity of phase k, α_k is the thermal diffusivity of the phase k, δ is the dimensionless interface position (=r_1/r_2), ε_{mk} is the turbulent diffusivity of phase k, ε_{hk} is the eddy diffusivity of phase k, μ_k is the viscosity of phase k, $\hat{\mu}$ is the viscosities ratio (=μ_2/μ_1), ρ_k is the density of the phase k, $\phi(z)$ is the boundary condition source term, ν_k is the kinematic viscosity of phase k, τ_k is the shear stress associated to phase k, τ is the interface shear stress, and finally α and β are prescribed coefficients that recover the wall boundary conditions type, i. e., prescribed temperature, prescribed heat flux or convective type.

The solution of the dimensionless problem defined above is obtained through application of the integral transform technique (Mikhailov and Ozisik, 1984, and Cotta, 1993):

$$\theta_k(R,Z) = \sum_{i=1}^{\infty}\frac{\Psi_k(\mu_i,R)}{N_i}e^{-\beta_i^2 Z}\left[\tilde{f}_i + \left(\frac{l_n}{l_0}\right)^2\frac{1}{C}\int_0^z g_i(Z')e^{\beta_i^2 Z'}dZ'\right] \tag{8.a}$$

where N_i , f_i and $g_i(Z)$ are expressions defined by

$$N_i = N(\mu_i) = \sum_{k=1}^{2}\int_{R_{k-1}}^{R_k}W_k(R)\Psi_k^2(\mu_i,R)dR \tag{8.b}$$

$$\tilde{f}_i = \tilde{f}_i(\mu_i) = \sum_{k=1}^{2}\int_{R_{k-1}}^{R_K}W_k(R)\Psi_k(\mu_i,R)F_k(R)dR \tag{8.c}$$

$$g_i(Z) = g(\mu_i,Z) = \Phi(Z)\left[\frac{\Psi_2(\mu_i,1) - K_2(1)\Psi_2'(\mu_i,1)}{\alpha+\beta}\right] \tag{8.d}$$

where $\Psi_k(\mu_i,R) \equiv \Psi_{ki}(R)$ and μ_i are, respectively, the eigenfunctions and eigenvalues, and the weight function $W_k(R)$ is given by:

$$W_k(R) = \frac{\rho_k C_k}{\bar{\rho}\bar{C}}R^n U_k(R) \tag{9}$$

The associated eigenvalue problem is given by:

$$\frac{d}{dR}\left[K_k(R)\frac{d\Psi_{ki}(R)}{dR}\right] + \mu_i^2 W_k(R)\Psi_{ki}(R) = 0, \qquad R_{k-1} \le R \le R_k,\quad k=1,2 \tag{10.a}$$

$$\frac{d\Psi_{1i}(R)}{dR} = 0, \qquad R = 0 \qquad (10.b)$$

$$\Psi_{1i}(R) = \Psi_{2i}(R), \qquad K_1(R)\frac{d\Psi_{1i}(R)}{dR} = K_2(R)\frac{d\Psi_{2i}(R)}{dR}, \qquad R = \delta \qquad (10.c,d)$$

$$\alpha^* \Psi_{2i}(R) + \beta K_2(R)\frac{d\Psi_{2i}(R)}{dR} = 0, \qquad R = 1 \qquad (10.e)$$

where

$$K_k(R) = R^n \frac{k_k}{\bar{k}} E_{hk}(R) \qquad (10.f)$$

Due to a possible slow convergence behavior of the related infinite series defined through eq. (8.a), especially for the case of a prescribed heat flux boundary condition (α =0, β =1), it is convenient to rewrite the solution of our problem through an analytical "Splitting-up" procedure (Mikhailov and Özisik, 1984), in the form:

$$\theta_k(R,Z) = \bar{\theta}(Z) + \theta_{kZ}(R,Z) + \theta_{k0}(R) \qquad (11)$$

where the dimensionless average temperature, $\bar{\theta}(Z)$, is defined as

$$\bar{\theta}(Z) = \frac{\frac{2Z}{C} + \sum_{k=1}^{2}\int_{R_{k-1}}^{R_k} W_k(R)F_k(R)dR}{\sum_{k=1}^{2}\int_{R_{k-1}}^{R_k} W_k(R)dR} \qquad (12)$$

The solution of the homogeneous problem within the entrance region, $\theta_{kZ}(R,Z)$, is obtained as

$$\theta_{kZ}(R,Z) = \sum_{i=1}^{\infty}\frac{\Psi_{ki}(Z)}{N_i}e^{-\beta_i^2 Z}\left[\bar{f}_i - \frac{\bar{g}_i(Z)}{\mu_i^2}\right] \qquad (13)$$

where

$$\mu_i^2 = \left[\frac{l_o}{l_n}\right]^2 C\beta_i^2 \qquad (14)$$

Finally, the component $\theta_{ko}(R)$ is analytically determined to yield:

$$\theta_{ko} = \Phi(Z)\left[H_k''(R) - \frac{\sum_{k=1}^{2}\int_{R_{k-1}}^{R_k} W_k(R)H_k''(R)dR}{\sum_{k=1}^{2}\int_{R_{k-1}}^{R_k} W_k(R)dR}\right] \qquad (15.a)$$

where

$$H_1'(R) = \int_0^R \frac{H_1(R')}{K_1(R')}dR' \qquad H_2'(R) = H_1'(R) + \int_0^R \frac{H_1(\delta') + H_2(R')}{K_2(R')}dR' \qquad (15.b, c)$$

.

$$H_k = \frac{\int_{R_{k-1}}^{R_k} W_k(R')dR'}{\sum_{k=1}^{2}\int_{R_{k-1}}^{R_k} W_k(R)dR}, \qquad k = 1,2 \qquad (15.d,e)$$

From the temperature distribution, eq. (11), the heat transfer coefficient at the channel wall can be evaluated, for instance, in the case of a prescribed wall heat flux condition, as:

$$h(z) = \frac{-k_2 \left.\frac{\partial T_2}{\partial r}\right|_{r=r_2}}{\bar{T}(z) - T_2(r_2,z)} = \frac{\phi(z)}{T_2(r_2,z) - \bar{T}(z)} \qquad (16.a)$$

or, yet, the equivalent dimensionless form in terms of the Nusselt number

$$Nu(Z)=\frac{h(z)\,l_n}{K_1}=\left[\frac{l_n}{l_o}\right]\left[\frac{\overline{K}}{K_1}\right]\frac{\Phi(Z)}{\theta_2(1,Z)-\overline{\theta}(Z)} \tag{16.b}$$

that can be expressed by

$$Nu(Z)=\left[\frac{l_n}{l_o}\right]\left[\frac{\overline{K}}{K_1}\right]\frac{\Phi(Z)}{\sum_{i=1}^{\infty}\frac{\psi_{2i}(1)}{N_i}e^{-\beta_i^2 Z}\left[\tilde{f}_i-\frac{g_i(Z)}{\mu_i^2}\right]+\theta_{20}(R_2)} \tag{16.c}$$

For thermally developed flow ($Z\rightarrow\infty$) we then have

$$Nu(Z)=\left[\frac{l_n}{l_o}\right]\left[\frac{\overline{K}}{K_1}\right]\frac{\Phi(Z)}{\theta_{20}(R_2)}\,,\ Z\rightarrow\infty \tag{16.d}$$

For the particular application here considered, we have $\beta_k=0$ (no mass transfer at the interface). Moreover, turbulence and waves effects are suppressed from the simplified equations for laminar flow, and $E_{hk}(R)=1$. Also, the eigenvalue problem is solved through the computational procedure described by Cotta and Nogueira (1988).

The parameter C, that appears in eq. (6.a), is here defined to allow for comparisons with the single-phase flow results:

$$C=\frac{u^*_{max}}{\overline{u}^*} \tag{17.a}$$

where u^*_{max} is obtained from eq. (4.a) for R=0, i.e.:

$$u^*_{max}=\frac{P_1 r_1^2}{4\mu_1} \tag{17.b}$$

and $\overline{u}^*$ has already been defined by eq. (4.b). Then,

$$C = 2.0 \tag{17.c}$$

For a consistent analysis and comparisons between single-phase and two-phase flows it is necessary to employ the following dimensionless distance:

$$Z^*=\frac{\alpha_1 z}{\overline{u}^* l_n^2} \tag{18}$$

which can be represented as a function of the original Z by

$$Z^*=\frac{\alpha_1}{\overline{\alpha}}\frac{\overline{u}}{\overline{u}^*}Z \tag{19}$$

This definition becomes necessary since Z is a function of the interface position and does not represent, for distinct experiments, the same physical position in relation to the entrance of the duct; so, with Z^* we can perform a consistent comparison between single- and two-phase systems.

3. Results and discussion

The present work is concerned with the analytical solution for thermally developing forced convection in the laminar regimen of a liquid-liquid two-phase annular flow. The situation considered for analysis deals with a water-kerosene two-phase flow previously studied by Hasson et al. (1974) and Leib et al. (1977). The specific physical cases here computed are summarized in Table 1, as extracted from the flow data in Leib et al.(1977) and calculated to match the definitions here considered. Some minor differences may be observed, from the equivalent table in the original reference, due to some uncertainties on the fluids physical properties. Figures 1.a-c illustrate the velocity profiles for the cases analyzed, each for a fixed volumetric flow rate of water and variable flow rate of the kerosene. These local distributions are important in the interpretation of the heat transfer results to be shown in what follows.

Table 1 – Main data and flow parameters for the cases analyzed, as obtained from Leib et al. (1977)

EXP	Q_1 (l/min)	Q_2 (l/min)	$-dp/dz$ (N/m^3)	δ	$\bar{u}_1$ (m/s)	$\bar{u}_2$ (m/s)	$\overline{\mathrm{Re}}$
A2	2.03	2.45	9508	0.693	0.896	1.001	8029
B2	2.03	2.05	9552	0.723	0.823	0.913	7412
C2	2.03	1.50	9617	0.765	0.735	0.769	6515
A3	2.70	2.45	9592	0.705	1.151	1.036	9320
B3	2.70	2.05	9628	0.732	1.070	0.936	8727
C3	2.70	1.50	9678	0.776	0.951	0.801	7841
A4	3.45	2.45	9676	0.717	1.426	1.068	10785
B4	3.45	2.05	9704	0.744	1.325	0.972	10168
C4	3.45	1.50	9741	0.785	1.185	0.833	9263
B5	2.70	2.05	9581	0.756	1.017	0.997	8372

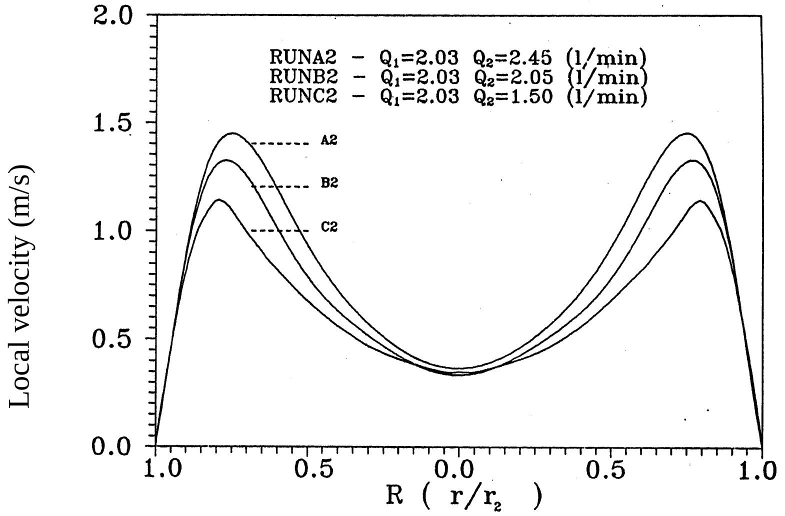

Figure 1.a – Fully developed velocity field in vertical annular liquid-liquid flow of water-kerosene. Flow rate and experimental data as provided by Leib et al. (1977) and Hasson et al. (1974).

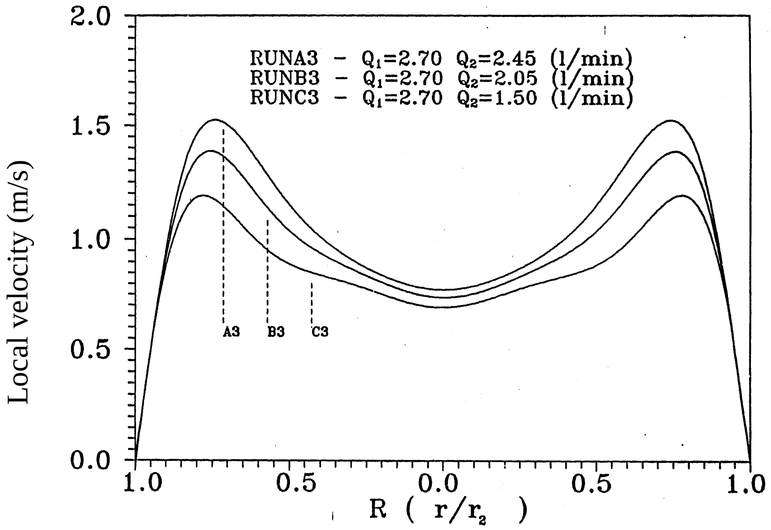

Figure 1.b – Fully developed velocity field in vertical annular liquid-liquid flow of water-kerosene. Flow rate and experimental data as provided by Leib et al. (1977) and Hasson et al. (1974).

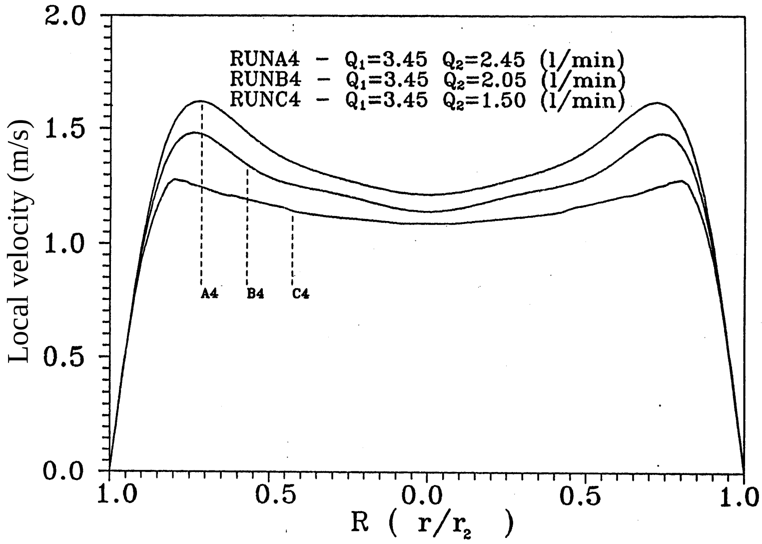

Figure 1.c – Fully developed velocity field in vertical annular liquid-liquid flow of water-kerosene. Flow rate and experimental data as provided by Leib et al. (1977) and Hasson et al. (1974).

Before attempting to employ the developed solution for the physical analysis of the problem, a thorough validation of the constructed computer code was conducted. The numerical results obtained through application of the integral transform technique are first critically compared against benchmark results available in the literature for the single-phase situation (Cotta & Ozisik, 1986). In fact, the validation is performed through comparison for the single fluid solution, when the present problem was approximately solved for a relative interface position of $\delta = 0.9999$ in a circular tube. Table 2 then presents results for the local Nusselt number along the thermal entry region, for a wide range of the longitudinal coordinate, initially for the single fluid situation, as represented by the second and third columns of this tabulation. The numerical results have then demonstrated that the present model is consistent and precise, while tending to the single fluid benchmark solution as the relative interface position approaches the wall. A set of reference results is also presented in this table for a few other cases considered by Leib et al. (1977), according to their data in Table 1.

Table 2 – Validation of local Nusselt number results with single phase flow benchmark results and present numerical results for physical situations considered by Leib et al. (1977).

Z^*	Single Phase Flow Cotta & Ozisik (1986)	Present-ITT $\delta = 0.9999$	A2	B2	C2	B3	B5
0.000001	129. 21	128. 23	42. 71	43. 96	45. 58	42. 22	10. 994
0.000002	102. 36	101 .90	33. 85	34. 86	36. 14	33. 49	10. 313
0.000003	89.309	88.94	29.54	30.42	31.54	29.22	9.883
0.000005	75. 191	74. 92	24. 87	25. 62	26. 55	24. 61	9. 310
0.000010	59. 510	59. 33	19. 69	20. 28	21 .02	19. 48	8. 490
0.000020	47. 077	46. 96	15. 58	16. 05	16. 63	15. 41	7. 637
0.000030	41 .037	40. 95	13. 58	13. 99	14. 49	13. 43	7. 132
0.000050	34. 511	34. 45	11 .42	11 .76	12. 18	11 .29	6. 498
0.000100	27. 276	27. 23	9. 02	9. 29	9. 62	8. 91	5. 656
0.000200	21.558	21.53	7.12	7.34	7.59	7.03	4.856
0.000300	18. 790	18. 77	6. 20	6. 39	6. 60	6. 12	4. 414
0.000500	15. 813	15. 80	5. 21	5. 36	5. 54	5. 13	3. 890
0.001000	12.538	12.53	4.11	4.23	4.36	4.04	3.245
0.002000	9.9863	9.980	3.24	3.33	3.50	3.18	2.695
0.003000	8.7724	8.768	2.80	2.90	2.99	2.76	2.426
0.005000	7.4937	7.490	2.38	2.44	2.51	2.31	2.133
0.0l0000	6.1481	6.146	1.89	1.94	2.01	1.84	1.805
0.020000	5.1984	5.197	1.55	1.61	1.71	1.53	1.589
0.030000	4.8157	4.814	1.42	1.49	1.62	1.43	1.521
0.050000	4.5139	4.513	1.34	1.43	1.58	1.38	1.486
0.100000	4.3748	4.373	1.32	1.41	1.57	1.36	1.479
0.200000	4.3637	4.362	1.32	1.41	1.57	1.36	1.479

We also perform a covalidation of the present model, Table 3, through a comparison of the present results against two other solutions presented by Leib et al. (1977). In that work, the authors obtained asymptotic solutions for both the entry region, close to the tube inlet, and for large longitudinal distances. The matching of these two solutions, in an approximate sense, offered a third solution and the recommended model in their work. Theoretical and experimental results were then obtained for a tube of 1.0 cm diameter and 1.0 m length, for an applied wall heat flux of 17,440 W/m^2. Therefore, table 3 below presents three columns of results, the first one related to the asymptotic entry region solution in Leib et al.(1977), the second for their matched solution, and the third corresponds to the present fully converged integral transform results. Clearly, the asymptotic solution for the entry region may degenerate for some of the situations considered, but the overall agreement with the matched solution is very good.

Table 3 – Comparison of local Nusselt results obtained through the Integral Transform Technique and two approximate solutions proposed by Leib et al.(1977).

Experimental	z = 0.25 m			z = 0.75 m			z = 0. 50 m		
	Entry*	Matching*	I.T.T.	Entry*	Matching*	I.T.T.	Entry*	Matching*	I.T.T.
A2 f_1=28.4 f_2=28.4	NA	29.43	30.88	22.67	21.09	21.97	10.66	10.64	10.60
B2 f_1=28.4 f_2=28.4	NA	29.82	30.96	23.22	21.56	22.23	11.76	11.76	11.68
C2 f_1=28.4 f_2=28.4	NA	30.20	30.96	24.19	22.33	22.73	13.75	13.75	13.64
A3 f_1=27.7 f_2=27.7	NA	29.79	31.42	22.96	21.41	22.38	10.85	10.85	10.82
B3 f_1=27.7 f_2=27.7	NA	30.01	31.36	23.44	21.79	22.55	11.90	11.82	11.84
C3 f_1=27.7 f_2=27.7	NA	30.77	31.53	24.72	22.87	23.23	14.14	14.14	14.07
A4 f_1=27.7 f_2=27.7	NA	30.25	31.96	23.38	21.82	22.79	11.15	11.10	11.10
B4 f_1=27.7 f_2=27.7	NA	30.53	31.95	23.86	22.22	23.02	12.24	12.22	12.20
C4 f_1=27.7 f_2=27.7	NA	31.43	32.11	25.29	23.44	23.74	14.61	14.59	14.53
B5 f_1=28.5 f_2=51.9	NA	30.49	30.32	25.48	21.79	21.70	13.23	13.21	13.04

*Leib et al. (1977) – Entry (Asymptotic solution for entry region) & Matching (Matched solution between the two asymptotic estimates)

Figure 2 illustrates, comparatively, the local Nusselt number axial distributions for two of the cases considered by Lieb et al. (1977), and a limiting result representing the single-flow analysis. From Fig. 2 below and also from Table 2 above, it can be observed that the kerosene layer offers a significant resistance to heat transfer. The two-phase flow situations, however, do not present a significant deviation between them, motivating the analysis that follows.

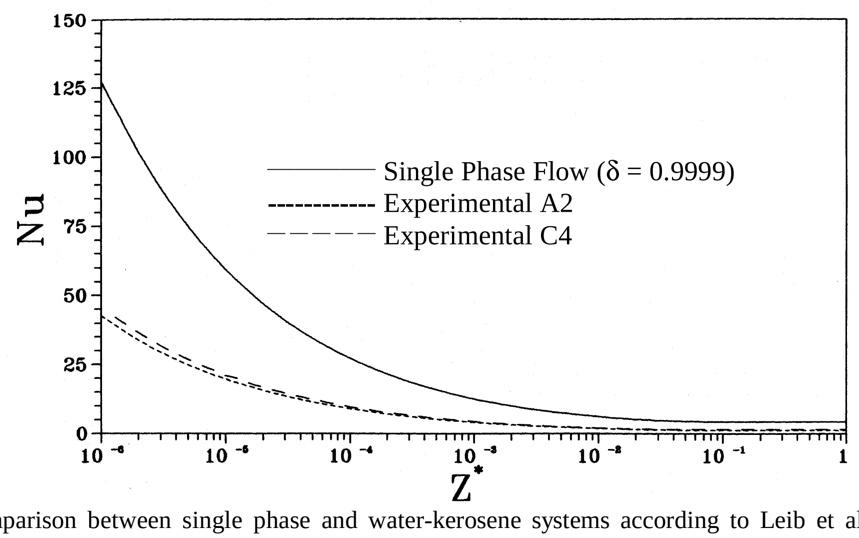

Figure 2 – Comparison between single phase and water-kerosene systems according to Leib et al. (1977) for heat exchange in the thermal entry region and considering constant heat flux in the wall tube.

The comparative results, shown in Table 2 and fig.2 above, lead to different conclusions than those obtained by Leib et al. (1977), in certain aspects. Their analysis predicted a relatively large entry length for the two-phase flow system, at least one order of magnitude greater than that encountered in single-phase flows. In fact, the results here obtained demonstrate that the temperature field becomes developed at a smaller duct length, for the kerosene-water systems analyzed, when compared to a single-phase flow considering equal entry temperature and equivalent volumetric flow rate.

Figures 3.a-c show the effect of flow rate and kerosene annulus relative thickness on the heat transfer behavior of the two-phase system. It can be noticed that the heat transfer coefficient is more pronounced for the larger values of kerosene flow rate in the region very close to the tube inlet, as expected; however, with decreasing flow rate of kerosene and, consequently, smaller values of the kerosene thickness, bulk temperatures more rapidly approach the wall temperature, which results in larger heat transfer coefficient for a thermally developed regime with a prescribed wall heat flux. Thus, it may be concluded that the heat transfer reduction effect may be minimized through an adequate choice of the flow rates ratio. In fact, this effect can be investigated, for a given water flow rate, with the help of figs.1; there is a minimum kerosene flow rate for a fixed water flow rate, for which the annular regime remains achievable and flow-reversal does not occur, as previously discussed by Hasson et al. (1974).

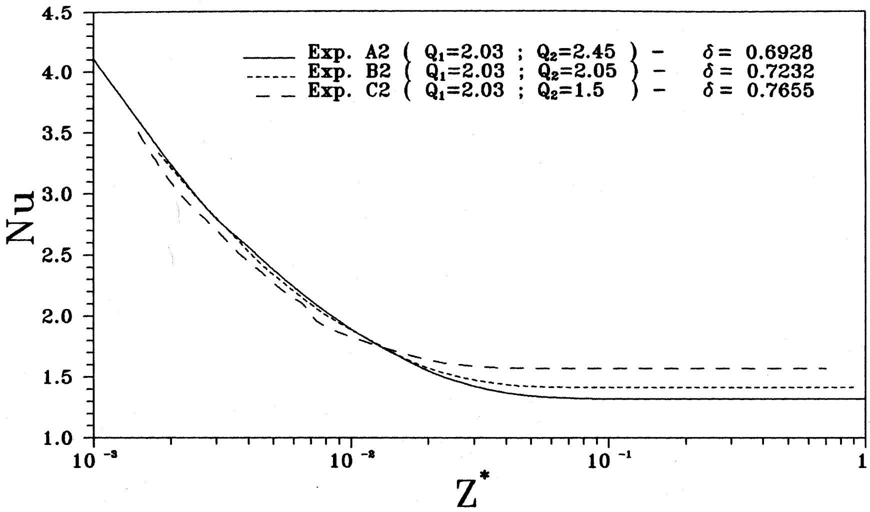

Figure 3.a – Effect of the flow rate and kerosene annulus relative thickness on the heat exchange in annular water-kerosene flow with constant wall heat flux (cases considered by Lieb et al., 1977).

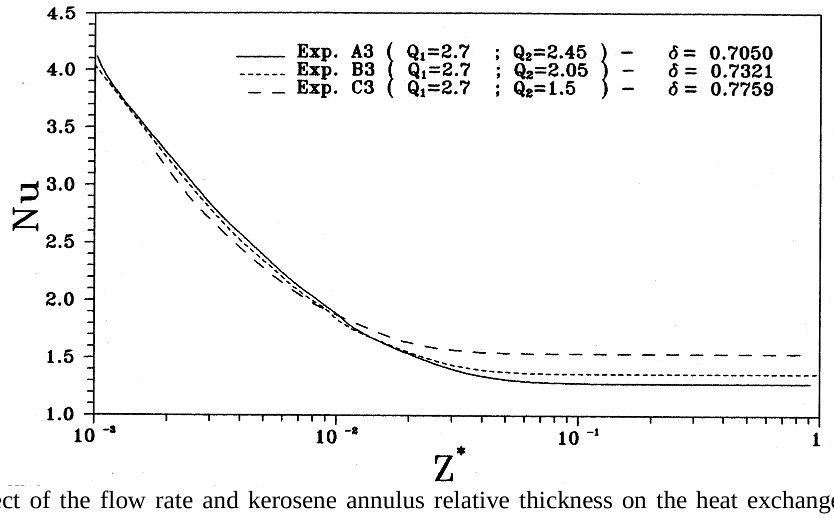

Figure 3.b – Effect of the flow rate and kerosene annulus relative thickness on the heat exchange in annular water-kerosene flow with constant wall heat flux (cases considered by Lieb et al., 1977).

As shown in figure 4, a critical comparison with the experimental data of Leib et al. (1977) makes it clear that the theoretical model underestimates the heat transfer coefficient. This fact, according to Leib et al. (1977), can be partially justified by the waves that were experimentally observed at the two fluid streams interface. Another fact to be considered, not actually discussed in previous analyses (Leib et al., 1977), is that the imposed conditions at the present flow examples, yield higher Reynolds numbers than those expected for transition in single fluid flow in circular tube, according with the values shown in table 1. When the effective Reynolds number decreases, the differences between experimental and theoretical results for the heat transfer coefficient also decrease. The approach advanced in this work, however, can not be extended to confirm this observation. Nevertheless, Leib et al.(1977) briefly reported that the experimental data obtained are near the transition region.

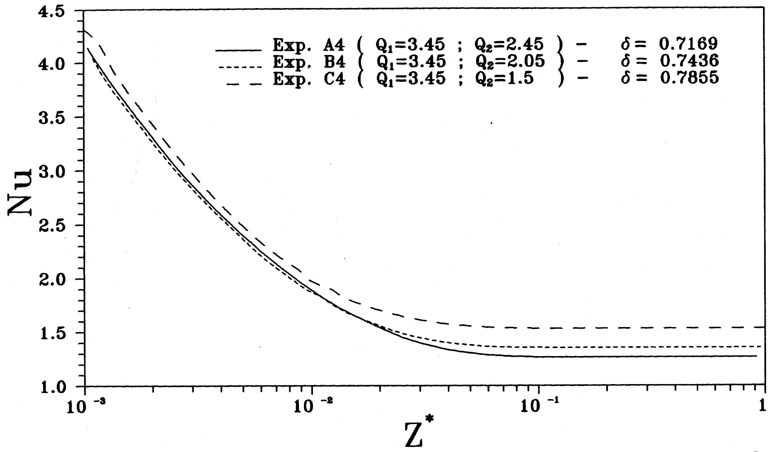

Figure 3.c – Effect of the flow rate and kerosene annulus relative thickness on the heat exchange in annular water-kerosene flow with constant wall heat flux (cases considered by Lieb et al., 1977).

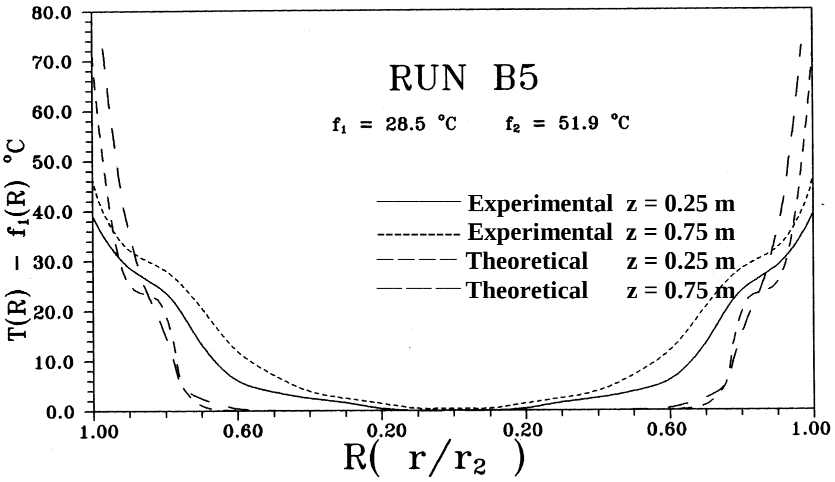

Figure 4 – Comparison between experimental results (Leib et al., 1977) and present theoretical predictions for temperature profile in the thermal entry region.

4. Conclusions

The present work advances a theoretical model in the representation of thermally developing laminar forced convection in liquid-liquid annular flow, utilizing the integral transform technique. It can be conclude that the present integral transform solutions are more adequate in representing such temperature fields than the previous models reported in the open literature. The approach proposed here has also been extended to more complex two-phase flow situations than in fact that investigated in the present work, such as in an annular gas-liquid flow (Nogueira, 1993). The analytical solutions so obtained find usefulness as limiting situations for physical analysis, as well as benchmark results for the verification of more general models and/or numerical codes. Nevertheless, critical comparisons with few experimental results available indicate that some modeling extensions might be feasible under the same analytical framework, in order to improve the model adherence to the experimental findings.

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