DEVELOPMENT OF A GENERAL TURBULENCE MODEL FOR HYBRID POROUS MEDIA - CLEAR FLOW DOMAINS

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Abstract*. The study of flow in porous media has gained an enormous attention lately due to its potential industrial application. Analysis and optimization of* ***Enhanced Oil Recovery*** *systems, cleaning of contaminated soil and improvements of modern fluidized bed combustion systems, are just a few examples of such engineering applications. This paper presents an overview of the major steps taken in developing a general two-equation turbulence model for application in such systems. Transport equations are written for both clear fluid and porous medium and the numerical scheme developed for treating both domains with a single set of equations is discussed.*

Keywords*: Porous Media, Turbulence Modeling, Numerical Methods*

1. INTRODUCTION

The mathematical modeling of flow in hybrid porous media-clear flow domains has a number of applications in petroleum and gas engineering. Flow in vicinity of pumping wells can become turbulent, due to large voids, affecting overall pressure losses and well performance. In addition, the study of flow in porous media has gained an enormous attention lately due to its potential industrial application. Analysis and optimization of **Enhanced Oil Recovery** systems, cleaning of contaminated soil and improvements of modern fluidized bed combustion systems, are just a few examples of such engineering applications.

In many instances, the flow through the porous bed becomes turbulent and, as such, requires appropriate mathematical

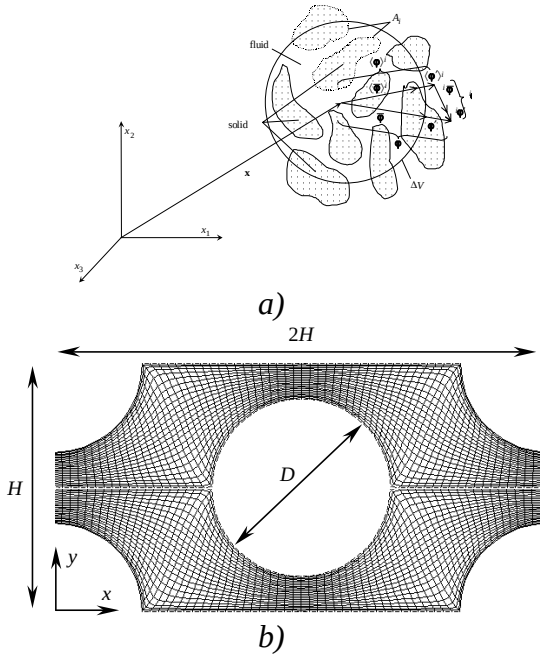

Figure 1 – *a*) Representative Elementary Volume (R.E.V.), *b)* Model of R.E.V, periodic cell and non-orthogonal grid.

tools for its reliable analysis. When the pore Reynolds number, Re_p, is less than about 150~200, classical mathematical treatment invokes the notion of a Representative Elementary Volume (R.E.V., Figure 1a) for which balance equations governing momentum, energy and mass transfer are written (Ward, 1964, Whitaker, 1969, Bear, 1972, Vafai & Tien, 1981). However, for fully turbulent flow regime (Re_p > 300), turbulence models presented in the literature follow two contradictory approaches. In the first one (Lee & Howell, 1987, Wang & Takle, 1995, Antohe & Lage, 1997), governing equations for the mean and turbulent fields are obtained by time-averaging the macroscopic equations. In the second method (Masuoka & Takatsu, 1996, Kuwahara *et al*, 1998, Kuwahara & Nakayama, 1998, Takatsu & Masuoka, 1998, Nakayama & Kuwahara, 1999), a volume-average operator is applied to the local time-averaged equation. In the literature, these two different approaches lead to different governing equations.

Motivated by the potential application involved and by the interesting controversy in the recent published literature, at the Instituto Tecnológico de Aeronáutica - ITA, in the **Computational Transport Phenomena Laboratory - LCFT/ITA**, a research effort has been carried out in the past four years in order to better understand, through adequate modeling, the flow in the turbulent regime through a porous medium.

Among the steps taken, a review of different numerical techniques applied to the simulation of flow in porous media (de Lemos, 1996) and a discussion on mathematical models applied to Petroleum Engineering have been presented (Pedras & de Lemos, 1996). Later, classical flow models in porous substrates were compiled (Pedras & de Lemos, 1998a) and a preliminary proposal for a turbulence model was established (Pedras & de Lemos, 1998b). Then, a study on the different views in the literature followed, leading to the proposition of the *double-decomposition* idea (Pedras & de Lemos, 1999a) and to a subsequent development on the earlier preliminary model (Pedras & de Lemos, 1999b). The *double-decomposition* idea led to a better characterization of the flow turbulent kinetic energy (Pedras & de Lemos, 2000a) and was a step before detailed numerical solution of the flow equations. These computations - in the domain of Figure **1**b - were carried out in order to establish a working version of the model (de Lemos & Pedras, 2000a, Pedras & de Lemos, 2000b). The cell in Figure **1**b represents the R.E.V. and is characterized by *D*, the rod diameter, *H*, the height of periodic cell and, *S*= 2*H*, the length of periodic cell. Recent results for flow in hybrid domains have been documented (de Lemos & Pedras, 2000b).

The need of computing the fine flow properties in order to obtain the volume-integrated quantities has motivated the development of adequate numerical tools. As mentioned, those calculations were needed for adjusting the model and considered the high Re k-ε closure (Rocamora & de Lemos, 1998) as well as the Low Reynolds version of it (Pedras & de Lemos, 2000c, Pedras & de Lemos, 2000d). Heat transfer analysis was also the subject of additional research (Rocamora & de Lemos, 1999). One of the outcomes of this development was the ability to treat hybrid computational domains with a single mathematical tool (Rocamora & de Lemos, 2000a, Rocamora & de Lemos, 2000b).

Next, major steps taken on the material reviewed above are commented upon. First, the **Microscopic RANS Equations** are presented before **The Volume and Time-Averaging** operators are discussed. Then, the **Macroscopic k-ε equations** comprising the proposed model are presented before comments on some **Preliminary Results and Concluding Remarks** are made.

2. MICROSCOPIC RANS EQUATIONS

The Reynolds Averaged Navier-Stokes Equations describe fluid flow in a continuum medium. For steady state condition they are promptly written as:

Continuity. $\nabla \cdot \overline{\mathbf{u}} = 0$ (1)

Momentum. $\nabla \cdot (\rho\, \overline{\mathbf{u}}\,\overline{\mathbf{u}}) = -\nabla \overline{p} + \mu \nabla^2 \overline{\mathbf{u}} + \nabla \cdot (-\rho \overline{\mathbf{u}'\mathbf{u}'})$ (2)

where $\overline{\mathbf{u}}$ is the microscopic time-mean velocity vector, ρ is the fluid density, the stresses, p is the pressure, μ is the fluid dynamic viscosity and $-\rho\overline{\mathbf{u}'\mathbf{u}'}$ are the well-known Reynolds stresses. The use of the eddy-diffusivity concept for expressing the *stress–rate of strain* relationship for the Reynolds stress appearing in (2) gives,

$$-\rho\overline{\mathbf{u}'\mathbf{u}'} = \mu_t 2\overline{\mathbf{D}} - \frac{2}{3}\rho k \mathbf{I}, \tag{3}$$

where μ_t is the turbulent coefficient of exchange, $\overline{\mathbf{D}} = [\nabla\overline{\mathbf{u}} + (\nabla\overline{\mathbf{u}})^T]/2$ is the mean deformation tensor, k is the turbulent kinetic energy per unit mass and $\mathbf{I}$ is the unity tensor. Applying (3) to (2) gives further:

$$\nabla \cdot (\rho\, \overline{\mathbf{u}}\,\overline{\mathbf{u}}) = -\nabla(\overline{p} + \frac{2}{3}\rho k) + \mu \nabla^2 \overline{\mathbf{u}} + \nabla \cdot (\mu_t 2\overline{\mathbf{D}}) \tag{4}$$

The term (p + 2/3 ρ k) in (4) can be substituted by the total pressure P. In order to obtain an equivalent expression for the macroscopic Reynolds stress tensor, the volume-averaging operator with respect to ΔV is normally carried out in both equations (2) and (4).

The low Re k-ε model. The coefficient μ_t appearing in equation (3) is here calculated through the standard low-Re k-ε model and reads $\mu_t = c_\mu\, \rho\, f_\mu\, k^2/\varepsilon$ where c_μ is a constant. Transport equations for k and ε are given by,

$$\nabla \cdot (\rho\, \overline{\mathbf{u}} k) = \nabla \cdot \left[\left(\frac{\mu_t}{\sigma_k} + \mu \right) \nabla k \right] + S_k; \qquad S_k = P_k - \rho\varepsilon; \;\; P_k = -\rho\overline{\mathbf{u}'\mathbf{u}'} : \nabla\overline{\mathbf{u}}, \tag{5}$$

$$\nabla \cdot (\rho\, \overline{\mathbf{u}}\, \varepsilon) = \nabla \cdot \left[\left(\frac{\mu_t}{\sigma_\varepsilon} + \mu \right) \nabla \varepsilon \right] + S_\varepsilon; \qquad S_\varepsilon = c_1 P_k \frac{\varepsilon}{k} - c_2\, f_\varepsilon\, \rho \frac{\varepsilon^2}{k}, \tag{6}$$

The term P_k is the production rate of k and the damping functions f_μ and f_ε are taken from Abe *et al*, 1992. The constants used read $c_\mu = 0.09$, $c_1 = 1.5$, $c_2 = 1.9$. The turbulent Pr numbers for k and ε are σ_k = 1.4 and σ_ε =1.3, respectively. In order to keep a unidirectional value for the volumetric velocity average $\overline{\mathbf{u}}_D$ (Darcy velocity), the conditions imposed at the boundaries of the periodic cell of Figure 1b were the non-slip condition at walls, symmetric profiles at symmetry lines ($y = 0$ and $y = H/2$) and periodic boundaries at $x = 0$ and $x = 2H$.

The set of equations (1)-(4)-(5)-(6) comprises the transport equations necessary for describing the flow in a clear fluid. All of these equations were discretized in the grid shown in Figure 1b used to simulate the R.E.V. of Figure 1a. The volume integrated parameters are used later for tuning the turbulence model proposed.

3. VOLUME AND TIME-AVERAGING

The use of volume and time averaging procedures for obtaining transport equations for k and ε in porous media is discussed in detail in Pedras & de Lemos, 1999a and Pedras & de Lemos, 2000a. For clarity of the material here presented, the major ideas therein are here included.

The macroscopic governing equation for flow through a porous substratum can be obtained by volume averaging the corresponding microscopic equations over a Representative Elementary Volume, ΔV. For a general fluid property, φ, the intrinsic and volumetric averages are related through the porosity ϕ as (Slattery, 1967, Whitaker, 1969, Gray & Lee, 1977),

$$\langle\varphi\rangle^i = \frac{1}{\Delta V_f}\int_{\Delta V_f}\varphi dV;\ \langle\varphi\rangle^v = \phi\langle\varphi\rangle^i;\ \phi = \frac{\Delta V_f}{\Delta V} \tag{7}$$

where ΔV_f is the volume of the fluid contained in ΔV. The property φ can then be defined as the sum of $\langle\varphi\rangle^i$ and a term related to its spatial variation within the R.E.V., ${}^i\varphi$, as

$$\varphi = \langle\varphi\rangle^i + {}^i\varphi \tag{8}$$

The spatial deviation is the difference between the real value (microscopic) and its intrinsic (fluid based average) value.

The need for considering time fluctuations occurs when turbulence effects are of concern. The microscopic time-averaged equations are obtained from the instantaneous microscopic equations. For that, the time-average value of property, φ, associated with the fluid is given as:

$$\overline{\varphi} = \frac{1}{\Delta t}\int_t^{t+\Delta t}\varphi dt \tag{9}$$

where Δt is the integration time interval. The instantaneous property φ can be defined as the sum of the time average, $\overline{\varphi}$, plus the fluctuating component, φ':

$$\varphi = \overline{\varphi} + \varphi' \tag{10}$$

From the definition of volume average (7) and time average (9) and assuming a rigid medium, one can conclude the following properties:

$$\overline{\langle\varphi\rangle^i} = \langle\overline{\varphi}\rangle^i;\ \ \langle\varphi'\rangle^i = \langle\varphi\rangle^{i\prime};\ \ {}^i\overline{\varphi} = \overline{{}^i\varphi} \tag{11}$$

The proofs of identities (11) are found in detail in Pedras & de Lemos, 1999a. In developing equations (11) the only restriction applied was the independence of ΔV in relation to time and space. If the medium is further assumed to be rigid, then ΔV_f is dependent on space but is not time-dependent (Gray & Lee, 1977).

With these ideas in mind, a proposition for volume and time averaged transport equation for k and ε equations can be made.

4. MACROSCOPIC K-ε EQUATIONS

4.1 Model Equation for $\langle k\rangle^i$.

In the work of Pedras & de Lemos, 1999a, it was shown that the order of application of both time and volume average operators is immaterial regarding the final momentum equation obtained. This is also true for the continuity and energy equations. For clear fluids (ϕ = 1), the turbulent kinetic energy defined as $k = \overline{\mathbf{u}'\cdot\mathbf{u}'}/2$ is used by most turbulence models. However, how to determine *k* in a porous medium is still an open question. Depending on the order of application of the average operators, the final governing equation for the flow turbulent kinetic energy will refer to different quantities.

The starting point for deriving an equation for *k* is the microscopic velocity fluctuation $\mathbf{u}'$. Its transport equation reads:

$$\rho\left\{\frac{\partial\mathbf{u}'}{\partial t}+\nabla\cdot[\overline{\mathbf{u}}\,\mathbf{u}'+\mathbf{u}'\overline{\mathbf{u}}+\mathbf{u}'\mathbf{u}'-\overline{\mathbf{u}'\mathbf{u}'}]\right\}=-\nabla p'+\mu\nabla^2\mathbf{u}'. \tag{12}$$

Now, volume-averaging (12) will give,

$$\begin{aligned}\rho\frac{\partial}{\partial t}(\phi\langle\mathbf{u}'\rangle^i)+\rho\nabla\cdot\{\phi[&\langle\overline{\mathbf{u}}\rangle^i\langle\mathbf{u}'\rangle^i+\langle\mathbf{u}'\rangle^i\langle\overline{\mathbf{u}}\rangle^i+\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i+\\ &\langle{}^i\overline{\mathbf{u}}\,{}^i\mathbf{u}'\rangle^i+\langle{}^i\mathbf{u}'\,{}^i\overline{\mathbf{u}}\rangle^i+\langle{}^i\mathbf{u}'\,{}^i\mathbf{u}'\rangle^i-\overline{\langle\mathbf{u}'\rangle^i\langle\mathbf{u}'\rangle^i}-\langle\overline{{}^i\mathbf{u}'\,{}^i\mathbf{u}'}\rangle^i]\}=\\ &-\nabla(\phi\langle p'\rangle^i)+\mu\nabla^2(\phi\langle\mathbf{u}'\rangle^i)+\mathbf{R}-\overline{\mathbf{R}},\end{aligned} \tag{13}$$

where $\mathbf{R}-\overline{\mathbf{R}}=\frac{\mu}{\Delta V}\int_{A_i}\mathbf{n}\cdot(\nabla\mathbf{u}')dS-\frac{1}{\Delta V}\int_{A_i}\mathbf{n}\,p'dS$.

From this point on there are two distinct approaches to determine a transport equation associated with the flow turbulence kinetic energy. One can start with equation (13), take the scalar product of it by $\langle\mathbf{u}'\rangle^i$ and apply the time-average operator. In this method, one uses the volume operator **first** followed by time-averaging, leading to $k_m=\overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}/2$ where k_m can be seen as the macroscopic turbulent kinetic energy based on $\langle\overline{\mathbf{u}}\rangle^i$, the intrinsic velocity vector.

On the other hand, if one starts out with equation (12) and one takes the scalar product of it by $\mathbf{u}'$ before time-averaging, one ends up, after volume averaging, with an equation for $\langle k\rangle^i=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2$, where $\langle k\rangle^i$ is the intrinsic turbulence kinetic energy. Now, using the double decomposition idea suggested by Pedras & de Lemos, 1999a, one can clarify the connection between these two quantities as being,

$$\begin{aligned}\langle k\rangle^i&=\langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i/2=\overline{\langle\mathbf{u}'\rangle^i\cdot\langle\mathbf{u}'\rangle^i}/2+\langle\overline{{}^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2\\ &=k_m+\langle\overline{{}^i\mathbf{u}'\cdot{}^i\mathbf{u}'}\rangle^i/2\end{aligned} \tag{14}$$

The last term on the right of (14) is the extra turbulent kinetic energy associated with the solid structure. As seen, models based on k_m do not fully account for all of the turbulent kinetic energy associated with the flow.

An equation for $\langle k\rangle^i$ is obtained by applying the volume average operator (7) to the transport equation for k (equation (5)). Making use of the Dupuit-Forchheimer relationship, $\overline{\mathbf{u}}_D = \phi \ \langle\overline{\mathbf{u}}\rangle^i$, a model for it can be proposed as (de Lemos & Pedras, 2000a),

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle k\rangle^i)+\nabla\cdot(\overline{\mathbf{u}}_D\langle k\rangle^i)\right]=\nabla\cdot\left[(\mu+\frac{\mu_{t_\phi}}{\sigma_k})\nabla(\phi\langle k\rangle^i)\right] - \rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\overline{\mathbf{u}}_D + c_k\rho\phi\frac{\langle k\rangle^i|\overline{\mathbf{u}}_D|}{\sqrt{K}}-\rho\phi\langle\varepsilon\rangle^i . \tag{15}$$

where c_k and σ_k are constants, K is the medium permeability and $\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ is given by

$$-\rho\phi\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i = \mu_{t_\phi}\,2\langle\overline{\mathbf{D}}\rangle^{\mathrm{v}} - \frac{2}{3}\phi\rho\langle k\rangle^i\mathbf{I}, \tag{16}$$

Eq. (16) is similar to the eddy-diffusivity for microscopic flow embodied in equation (3) and the coefficient. Note, however, that the coefficient $\mu_{t\phi}$ appearing in (16) and known as the macroscopic coefficient of exchange, is not necessarily the same coefficient appearing for clear fluid flow used in (3). Also, the introduced constant c_k needs to be determined for closure of the mathematical problem. The methodology established for finding it is discussed in Pedras & de Lemos, 2000c, and consists mainly in comparing the volume average values with the model results for the periodic cell of Figure 1b.

4.2 Model Equation for $\langle\varepsilon\rangle^i$.

Making use of the Dupuit-Forchheimer relationship, $\overline{\mathbf{u}}_D = \phi \ \langle\overline{\mathbf{u}}\rangle^i$, a transport equation for $\langle\varepsilon\rangle^i$ can be proposed as,

$$\rho\left[\frac{\partial}{\partial t}(\phi\langle\varepsilon\rangle^i)+\nabla\cdot(\overline{\mathbf{u}}_D\langle\varepsilon\rangle^i)\right]=\nabla\cdot\left[(\mu+\frac{\mu_{t_\phi}}{\sigma_\varepsilon})\nabla(\phi\langle\varepsilon\rangle^i)\right] + c_{1\varepsilon}(-\rho\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i:\nabla\overline{\mathbf{u}}_D)\frac{\langle\varepsilon\rangle^i}{\langle k\rangle^i}+c_{2\varepsilon}\,\rho\phi\left\{c_k\frac{\langle\varepsilon\rangle^i|\overline{\mathbf{u}}_D|}{\sqrt{K}}-\frac{\langle\varepsilon\rangle^{i\,2}}{\langle k\rangle^i}\right\}, \tag{17}$$

where, σ_k , $c_{1\varepsilon}$ and $c_{2\varepsilon}$ are constants. As with the case of $\langle k\rangle^i$, the overall dissipation rate of $\langle\varepsilon\rangle^i$, the last term on the right of (17), contains an additional factor that is dependent on the porous substrate. This additional term vanishes for the limiting case of clear fluid ($\phi\to 1 \Rightarrow K\to\infty$). In addition, for macroscopic fully developed uni-dimensional flow in isotropic and homogeneous media, the production rate of $\langle\varepsilon\rangle^i$ will be solely due to spatial deviations within the R.E.V. and will be totally dissipated within the same domain. These ideas are used in de Lemos & Pedras, 2000a when determining a numerical value for the introduced constant c_k .

5. PRELIMINARY RESULTS AND CONCLUDING REMARKS

In summary, this research has shown that both average operators for time and volume commute when obtaining the macroscopic governing equation for momentum. The order of

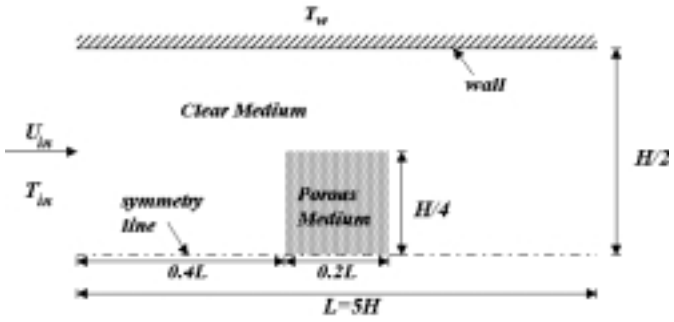

Figure 2 - Problem geometry and dimensions.

application of the operators is immaterial and the equations end up having the same form, However, when obtaining macroscopic transport equation for the turbulent kinetic energy, the order of application of averages will imply in a different quantity being transported.

An example of the computational work done so far is presented below with the help of Figure **2**. This preliminary case consisted in computing laminar flow in a heated channel past a porous obstacle. The data used is detailed in Table 1 where all dimensions can be found. Figure **3** shows results for the velocity and temperature fields. The tick mark values appearing in Figure **3** refer to the schematic of Figure **2** and corresponding data in Table 1. The vector plots show, at the top and right edge of the obstacle, an increasing mass flow rate as the permeability increases. Ultimately, for a higher K the temperature pattern, as expected, resembles the one for clear flow in channels (Rocamora & de Lemos, 2000b).

Another example of computational work done to complete the proposed model (equations (15)-(17)) is reproduced in Figure **4**. Therein the flow in a channel filled with a porous material crosses a cavity in the middle of the channel. The cavity itself has a different porosity than the surrounding medium. For high cavity porosity and permeability, the flow is pushed inside the cavity where is encounter less resistance. As the cavity porosity decreases, less fluid penetrates through it, up to the limiting case of a solid obstacle. Interesting to note is the absence of a weak region behind the solid obstacle (Figure **4***c*) when compared to flow of clear fluid past the same obstruction. The flow resistance due to the porous structure quickly flattens the velocity profile and boundary layers are confined to a narrow region of the flow. The turbulence kinetic energy rapidly increases within the channel if one considers the inlet condition assumed ($\langle k \rangle^i = \langle \varepsilon \rangle^i = 0$ at inlet). The level of $\langle k \rangle^i$ decreases close to the cavity boundaries and within it. This decrease is a consequence of a higher porosity and permeability within the cavity that, in turn, decreases the extra production term responsible for $\langle k \rangle^i$ generation within the porous domain.

These and other results are used to tune the proposed model which, ultimately, aims at a better understanding of turbulent flow through porous structures.

Table 1 - Parameter values for the geometry of Figure 2

L(m)	0.10
H(m)	0.02
T_{in} (^{0}C)	50.0
T_w (^{0}C)	100.0
Fluid	air
Pr	0.7
μ(Ns/m^2)	1.8 10^{-5}

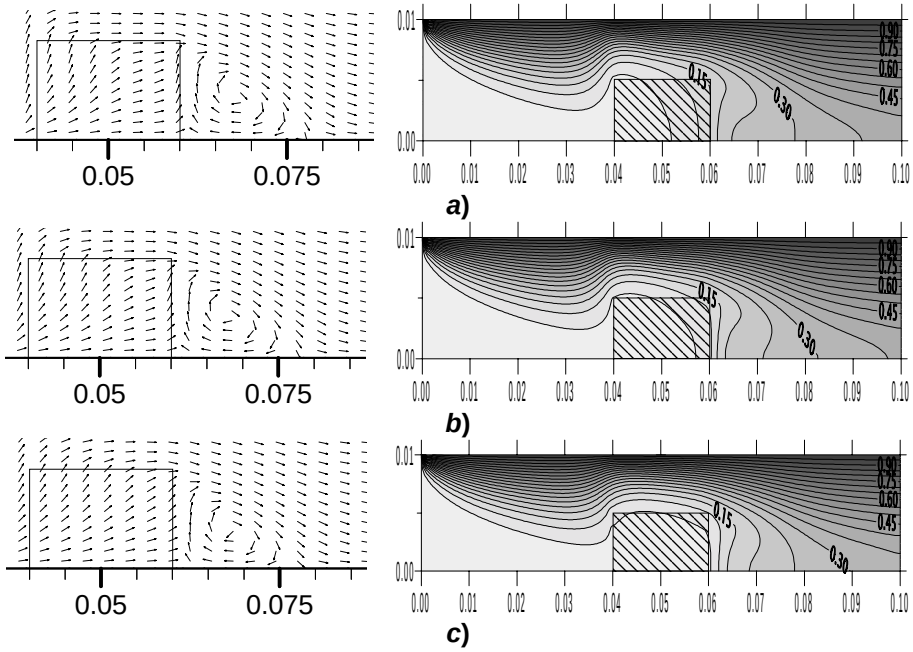

Figure 3 – Taken from Rocamora & de Lemos, 2000b. Detailed velocity field and temperature contours for flow past a porous obstacle: *a*) K=2.0 10^{-8}m^2; *b*) K=4.0 10^{-8}m^2; *c*) K=8.0 10^{-8}m^2. Refer to Fig. 2 and Table 1 for tick mark labels.

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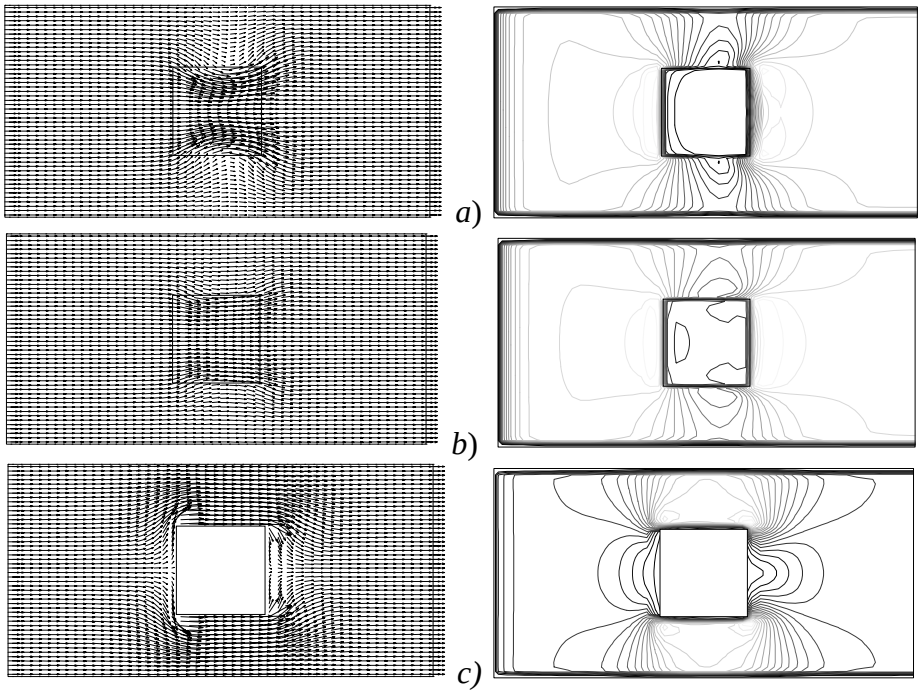

Figure 4: Taken from de Lemos & Pedras, 2000b. Two-dimensional maps for $\overline{\mathbf{u}}_D$ (top) and $\langle k \rangle^i$ (bottom). Turbulent flow: $\mathrm{Re}_K = 3.5 \times 10^3$, K_2=2.34x10^{-4} and ϕ_1 =0.40): *a*) ϕ_2 = 1.0, *b*) ϕ_2 = 0.8 and *c*) solid obstacle.

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