A NEW SKIN FRICTION EQUATION FOR TURBULENT FLOWS OF BINGHAM PLASTIC FLUIDS IN PIPES

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ABSTRACT *The purpose of this work is to analyze the problem of the turbulence modeling of non-Newtonian Bingham plastic fluids, through the introduction of a new characteristic flow velocity which is a function of the yield stress. Also, with the aid of the characteristic flow velocity and using some dimensional arguments, a new skin friction equation for non-Newtonian Bingham plastic fluids, together with a new law of the wall that use the new characteristic flow velocity are proposed. A analytical solution of the hydrodynamically developed turbulent equation of motion for flow in circular tubes is obtained. Comparisons with experimental data are made, showing good agreements.*

Key words *: Turbulence, Bingham Plastic, law of the wall*

1. INTRODUCTION

The fluid dynamics of turbulent non-newtonian fluid in ducts is of special interest in many engineering applications such as those found in the fields of chemical, food and mechanical engineering, where the non-newtonian behavior of certain materials or fluids is encountered. Among them are included polymer extrusion, manufacturing of greases and toothpaste, and the flow of drilling muds in petroleum industries. A discussion of several applications of non-newtonian fluids has been given out by Bird *et al*. (1982), and more recently, Quaresma and Macêdo (1998).

In spite of its importance, very few works can be found in the literature that analyze in detail the behavior and the characteristic parameters of turbulent flows of non-newtonian fluids. This gap in the literature is due mainly for reasons such as those pointed out in a recent work by Kostic (1994), namely, the turbulent flow of non-newtonian fluids is not quite well understood, because the classical isotropic fluids mechanics is not applicable to very complex fluids, and also because the turbulence phenomena is not well understood, even for newtonian fluids. So that many questions about these fluids remain unanswered.

The description of non-newtonian fully-developed turbulent flows in ducts for engineering ends, is basically restricted to some empirical correlations (Dodge & Metzner 1959). Recently, Malin (1997) has employed some two equation turbulence models to describe the fully-developed flow of Bingham Plastic fluids. He showed that the yield stress has little or no influence at the shear stress at the wall, even for high values of the yield stress.

In this work a new scale velocity that considers the influence of the yield stress as well as the effects of shear stress at the wall characteristic velocity is developed. This velocity is used to propose a new logarithmic law of the wall for Bingham plastic fluids. Also, a skin friction equation is developed, which can be used for many practical engineering purposes, and demonstrates clearly the influence of the yield stress in the fanning friction factor. A comparison of the new law of the wall with experimental data is made showing good agreement.

2. THE CHARACTERISTIC FLOW VELOCITY

The principal idea of this work is to define a new characteristic flow velocity. This characteristic flow velocity takes into account the effects of the yield stress of the fluid as well as the influence of the shear stress at the wall. To begin the analysis, the rheological behavior of the fluid is given by the non-Newtonian Bingham plastic model in the following form for the rz component of the viscous stress tensor:

$$\tau_{rz} = \tau_o - \mu \frac{\partial u}{\partial r}; \qquad \text{if} \quad |\tau_{rz}| \geq \tau_o$$

(1. a, b)

$$\frac{\partial u}{\partial r} = 0; \qquad \text{if} \quad |\tau_{rz}| < \tau_o$$

where, τ_{rz} is the shear stress, τ_o is the yield stress, $\partial u/\partial r$ is the velocity gradient and μ is the plastic viscosity of fluid. Making use of the eq. (1.a) in terms of the wall coordinate (i. e. $y = r_w - r$, see Figure 1) and introducing the assumption that the shear stress remains constant in the laminar sub-layer, i. e. $\tau_{yz} = \tau_w$, where τ_w denotes the shear stress at the wall, we obtain:

$$\nu \frac{\partial u}{\partial y} = \frac{\tau_w - \tau_o}{\rho} \tag{2}$$

integrating eq. (2) to the region next to the wall we have

$$u = \frac{\tau_w - \tau_o}{\rho} \frac{y}{\nu} \tag{3}$$

Applying dimensional analysis on eq. (3) a new characteristic flow velocity for the non-Newtonian Bingham plastic can be defined

$$u^* = \sqrt{\frac{\tau_w - \tau_o}{\rho}} \tag{4}$$

This equation reduces to the Newtonian case when the yield stress vanishes. Introducing eq. (4) in to eq. (3) is possible to obtain the velocity distribution in laminar sub-layer

$$\frac{u(y)}{u^*} = y^+ = \frac{u^*}{\nu} y \tag{5}$$

where y^+ is a new dimensionless wall coordinate for non-Newtonian Bingham plastic fluids.

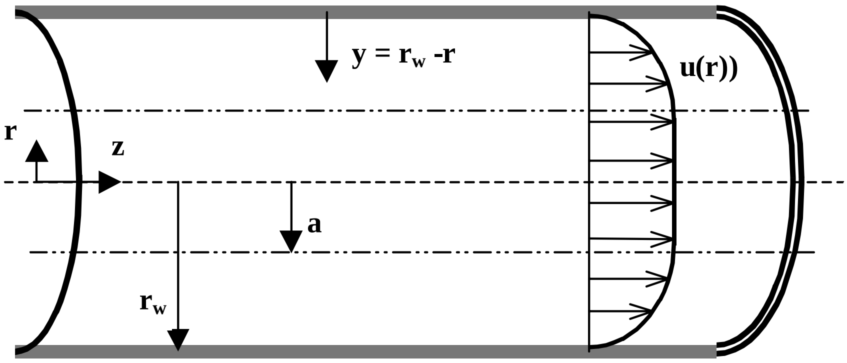


Figure 1 - Geometry and coordinate system of the problem

3. THE SKIN FRICTION EQUATION

A version of the classical law of the wall for Bingham plastic fluids can be obtained using the following approach described by Landau and Lifshtz (1987). In the turbulent region near the wall the mean velocity gradient is assumed to be a function of the characteristic velocity (defined by eq(4)) and the distance from the wall, giving

$$\frac{\partial u}{\partial y} = \frac{u^*}{\kappa y} \tag{6}$$

where κ is a constant of proportionality. Integrating eq. (6) results in:

$$u = \frac{u^*}{\kappa} \ln(y) + C(x) \tag{7}$$

C(x) is a constant of integration which can be obtained considering that in this region the characteristic length scale, is the same one deduced for the viscous sub-layer, as in the Newtonian case, and that eq (7) must reduce to the classical logarithmic law of the wall as $\tau_o \to 0$. Therefore eq (7) can be rewritten as:

$$u = \frac{u^*}{\hat{e}} \ln\left(\frac{y\,u^*}{\acute{\imath}}\right) + u^*\,5.5 \tag{8}$$

A Skin friction equation can be obtained using eq (8) in a similar way employed to deduce the Prandtl-Nikuradse relation valid for Newtonian fluids. One major difference is that the maximum velocity is now obtained at the limit of the plug flow region (*a*) and not the center of the tube anymore. It is important to remind that the flow of Binghan fluids can only occur if the Yield is exceeded, and since in this case, the shear stress is zero in the center of the tube to satisfy the symmetry condition, then in the region around the center of the tube where the shear stress is smaller then the yield stress, the substance must behave like a solid. In this region the velocity is constant and is equal to the velocity in the center of the tube. The limit of this region can be obtained, using the mean momentum equation at the point where the velocity gradient is zero, to furnish:

$$\tilde{a} = \frac{2\,Y}{\mathrm{Re}\, f_f} \tag{9}$$

In eq(9) $\tilde{a} = a/r_w$ is the dimensionless limit of the plug flow region, $\tilde{O} = \hat{o}_0\ D/\grave{\imath}\ V$ is the dimensionless yield stress, Re is the Reynolds number and f_f represents the fanning friction factor. A relation between the maximum velocity and the mean velocity can be obtained using the mean velocity definition and eq (8) to give:

$$U = V - \frac{u^*}{\kappa 2}(1-\tilde{a})(3+\tilde{a}) \tag{10}$$

in the above equation *V* is the mean velocity and *U* is the maximum velocity. The value of *U* can also be calculated using eq (8) at $y = r_w - a$ as follows:

$$U = \frac{u^*}{\hat{e}} \ln\left(\frac{(r_w - a)\,u^*}{\acute{\imath}}\right) + u^*\ 5.5 \tag{11}$$

The eq (10) and eq(11) can combined to furnish the following relation for the friction factor

$$\frac{1}{\sqrt{f_f - \frac{2Y}{\mathrm{Re}}}} = 4.073 \log\left[(1-\tilde{a})\left[\mathrm{Re}^2\frac{f_f}{8} - \mathrm{Re}\frac{Y}{4}\right]^{\frac{1}{2}}\right] + \frac{5.5}{\sqrt{2}} - \frac{(1-\tilde{a})(3+\tilde{a})}{3.28} \tag{12}$$

The equations set (9) and (12) can now be solved numerically for the fanning friction factor. It is important to note that the eq(12) contains the Prandtl-Nikuradse equation as a particular case when *Y*=0.

4. RESULTS AND DISCUSSION

In figure 2 the Fanning friction factor is presented for various yield stress numbers as a function of the Reynolds number. It is show that the friction factor increases as the dimensionless yield stress increase. This behavior is already expected since the flow of Bingham plastic fluids can only be established if the shear stress at the wall is larger than the yield stress. For many fluids the value of the yield stress can vary from 0.9Pa to 4.4Pa. It is important to note the dimensionless yield stress is not a function of the fluid alone, but depends on the diameter of the tube and the mean flow velocity.

The figure 2 clearly shows that the proposed relation reproduces the Karman–Nikuradse equation when the dimensionless yield stress approaches to zero. This fact shows that the eq(8) represents a generalization of the classical law of the wall and indicates how is the influence of yield stress in the turbulent Binghan fluid flow

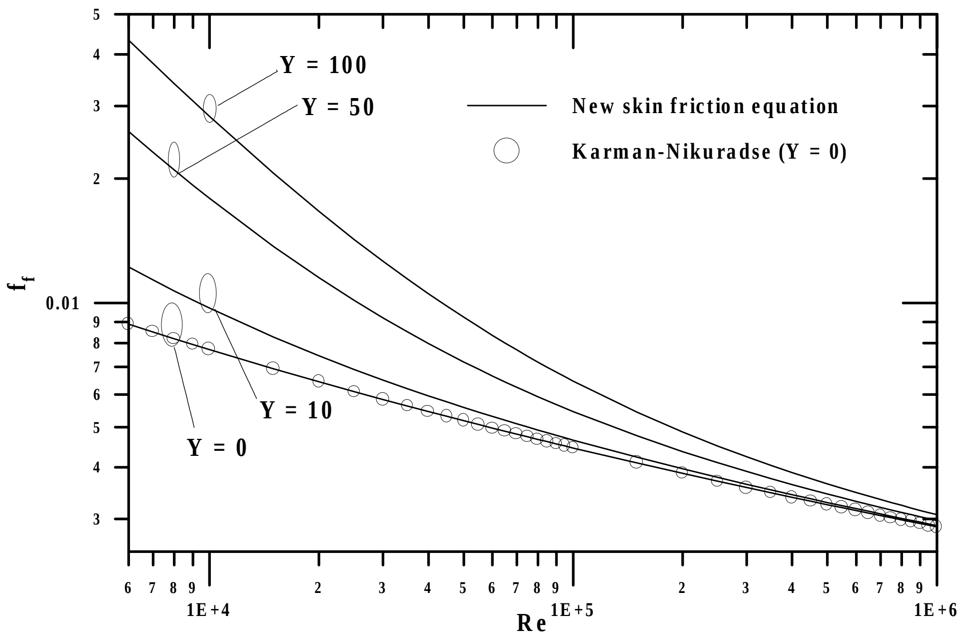


Figure 2 - Fanning friction factor behavior

In figures 3 and 4, a comparison of the law of the wall developed here (eq. 8) with the experimental data and the numerical results of Malin (1997) is made. The new law of the wall developed here is, in all the presented cases, at least as good as the numerical predictions, reproducing fairly well the experimental results. It is important to remark that the present formulation is simpler and requires less computational time than the turbulence models used to describe the flow numerically.

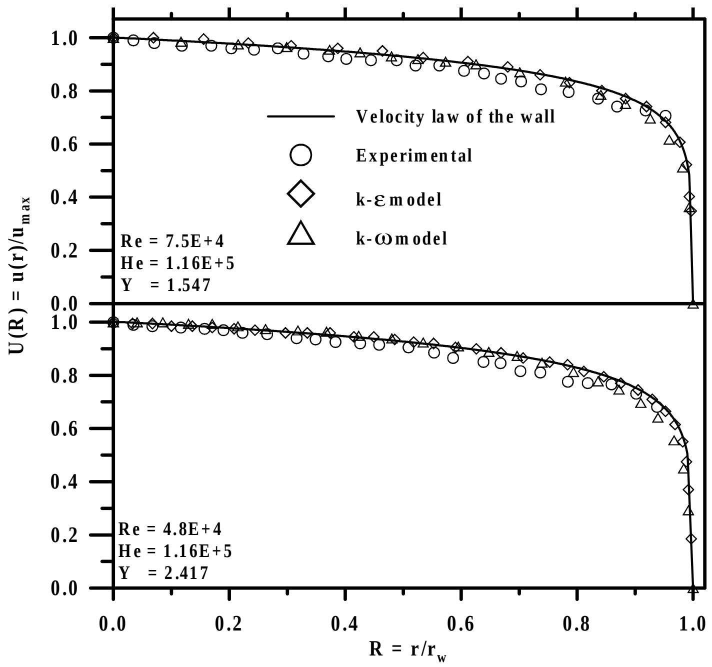
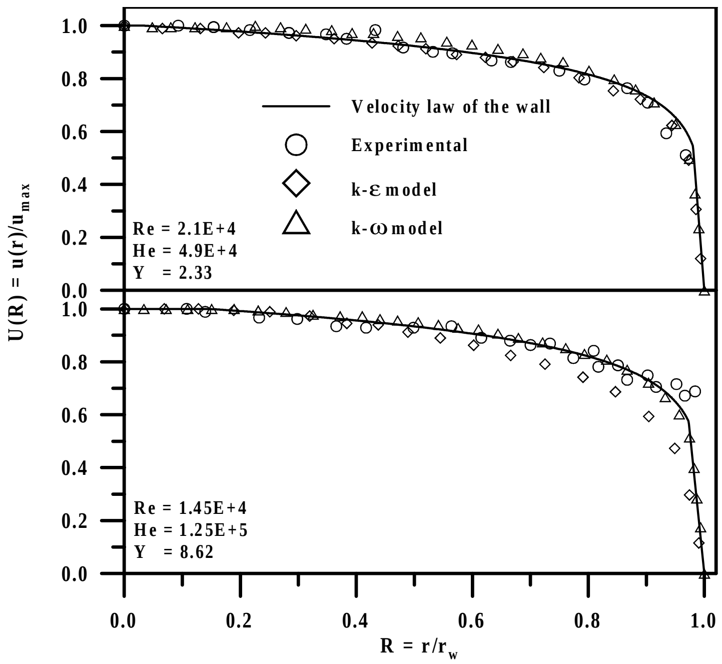


Figure 3 - Comparison of the present formulation with some experimental and numerical results



Figures 4- Comparison of the present formulation with some experimental and numerical results

These results indicate that the turbulent flow of non Newtonian Bingham plastic fluids have a similar behavior of Newtonian fluids, in the sense that a logarithmic law of the wall still describes the flow in the turbulent region near the wall. The great difference between the turbulent flow of these two fluids, is that for the Bingham plastic fluid, the characteristic velocity has a significant influence of the yield stress.

The approach used here can be extended for other type of non-Newtonian fluids. In each case a characteristic near wall velocity should be derived and therefore, a law of the wall and a skin friction equation can be obtained.

5. CONCLUSIONS

The problem of the turbulence modeling for non-Newtonian Bingham plastic fluids has been analyzed. A new scale velocity that takes into account the effect of the yield stress as well as the influence of the shear stress at the wall was developed. This scale velocity was used to propose a alternative law of the wall and also, a new skin friction equation for Bingham plastic fluids It was shown that the proposed velocity law of the wall reproduces well the experimental data and, is at least as good as more sophisticated turbulence models, although a more extensive comparison with experimental data should be made, to confirm this tendency. The comparison with the experimental data, also indicates that the turbulent profile of Binghan fluids near the wall have a logarithmic behavior, and that the influence of the yield stress occurs through the characteristic velocity. The implications of this fact are important, suggesting that the major difference between the turbulent flow of Newtonian fluids and non-Nentonian Bingham fluids are concentrated in the viscous sub-layer, and that no additional modifications from the Newtonian fluid should be introduced in the turbulence models to describe the flow far from solid boundaries.

The influence of the yield stress in the shear stress at the wall was analyzed too, showing that the dimensionless yield stress has a significant effect in the shear stress at the wall.

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