COMPARISON BETWEEN MODELS FOR THE HEAT TRANSFER FLUID IN LATENT HEAT STORAGE SYSTEMS

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Abstract. *The problem of solidification of a phase-change material (PCM) around a tube carrying a heat-transfer fluid (HTF) inside is solved numerically by two different methods. The first one employs the energy equation for the fluid, assumed in fully developed laminar flow. The other procedure is a simplified one which considers uniform velocity profile and performs an energy balance for the HTF using Nusselt number values obtained from a steady state correlation for either a constant wall heat flux or a constant wall temperature boundary condition. Comparisons are made in terms of the position of the phase-change front obtained by both procedures and between the calculated Nusselt number from the first method with the values of steady state correlations. The results obtained show that the accuracy of the simplified method and the decision of which steady-state values are to be used depend severely on the ratios of thermal conductivities of the HTF, solid PCM and tube wall material.*

Keywords: *Latent heat storage , Annular geometry , Heat-transfer fluid , Nusselt number*

1. INTRODUCTION

The problem of melting or solidification of a phase-change material (PCM) contained in the annular space between a tube carrying a heat transfer fluid (HTF) and an adiabatic bounding surface, has received much attention in the literature due to its application in the design of latent heat thermal energy storage units. In early papers numerical and semi-analytical solution methods were used and the coupling between convection in the HTF and conduction in the PCM was made by specifying a Biot number for the fluid. Some of those works are reviewed in Ismail & Jesus (1999).

Cao & Faghri (1991) developed what seems to be the first study in which the coupling between the conduction and convection problems was done by solving the transient momentum and energy equations for the HTF, accounting for the effects of the tube wall. They considered the fusion of PCM caused by a low Prandtl number HTF. In this paper laminar fluid flow was assumed, while in another work (Cao & Faghri, 1992), HTF flow was turbulent.

Bellecci & Conti (1993) solved the same problem as Cao & Faghri (1991) and suggested that it was not necessary to solve the fluid flow problem if a mean velocity were specified and the Nusselt number for the HTF evaluated from a steady-state correlation for constant-wall heat flux boundary condition. These statements were validated by good comparisons with Cao & Faghri (1991) results.

Zhang & Faghri (1996) solved the problem of melting of paraffin around a metallic tube using water as the HTF. The solution procedure was semi-analytical and adopted the integral method for the PCM and an expression for evaluating the Nusselt number in the fully-developed laminar flow of the HTF considering an arbitrary wall temperature variation. They showed that the calculated Nusselt numbers laid between the steady-state values for constant wall temperature (CWT) and constant-wall heat flux (CHF) boundary conditions. According to their conclusions, the simplifications proposed by Bellecci & Conti (1993) were valid only for a low Prandtl number HTF or a very long tube without practical interest. In another paper (Zhang & Faghri, 1995), they analyzed the problem using the same method but considering the turbulent fully-developed HTF flow and neglecting the tube wall thermal resistance. It was shown that, for a low Prandtl number HTF, the ratio between the HTF and PCM thermal conductivities had a strong influence in the evolution of the phase-change front and the Nusselt number could even be evaluated from a CHF correlation, if that ratio were sufficiently high. They also showed that for moderate Prandtl numbers HTF in turbulent flow the Nusselt number values could be obtained from the thermally developed flow value. At last, they concluded that those statements were not valid for laminar flow.

In this work the problem of solidification of water around a copper tube is solved numerically. Two different procedures, cited above, for treating the HTF convection problem are adopted. In the first method the energy equation is solved considering a fully developed laminar flow. The second approach considers a mean fluid velocity and performs a thermal energy balance in the tube adopting Nusselt number values obtained from either CHF or CWT steady state conditions. The results obtained for the evolution of the phase-change front are plot together as well as the calculated local Nusselt number in the first procedure is displayed with the steady state values. The thermal conductivity of the HTF was varied and comparisons show that the accuracy of the simplified method and the decision about which steady state values are to be used depend strongly on this parameter.

2. MATHEMATICAL MODEL

The system on study is shown in Fig. 1 below,

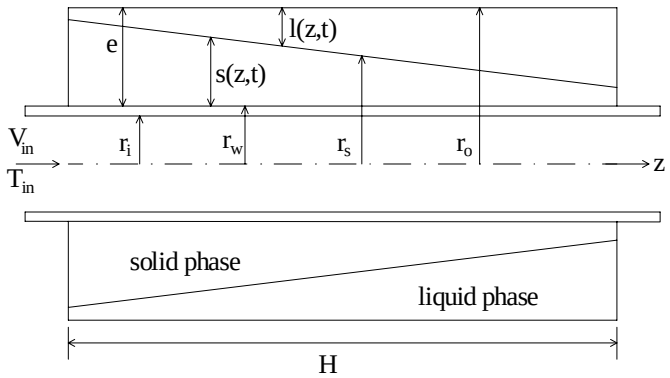

Figure 1 – Sketch of the problem.

The mathematical model is based on the following hypothesis:

- Two-dimensional and axisymmetric problem;
- Laminar, steady-state, fully-developed HTF flow. Uniform temperature, T_{in}, at inlet. The simplified method assumes a uniform mean velocity, V_{in}, at inlet.
- Constant thermophysical properties with no density differences between the solid and liquid phases;
- PCM is bounded by an adiabatic symmetry surface and the end caps of the system are considered adiabatic as well;
- Conduction is the only heat-transfer mechanism in the liquid PCM (solidification process with low initial liquid temperature, 4°C, and adiabatic external boundary conditions).

The problem is solved by two methodologies. A, so-called, complete method, where the energy equation for the HTF is used, and a simplified method which consists in performing an energy balance in the HTF using the Nusselt number values from CHF or CWT conditions. In both cases the conduction equations for the tube wall, solid and liquid PCM have to be solved. The energy conservation equations for the HTF and the tube wall are written in cylindrical axi-symmetric coordinates and then cast in dimensionless form using the following parameters,

$$
\begin{aligned}
&U = \frac{V_z}{V_{in}} \; ; \; Re = \frac{V_{in} D}{\nu_f} \quad Pr = \frac{\nu_f}{\alpha_f} \; ; \; Pe = Re\, Pr \; ; \; Nu = \frac{hD}{k_f} \\
&\theta = \frac{T - T_m}{T_m - T_{in}} \; ; \; R = \frac{r}{r_w} \; ; \; \xi = \frac{z}{r_w} \; ; \; S = \frac{s}{r_w} \\
&Fo = \frac{\alpha_s t}{r_i^{\,2}} \; ; \; Ste = \frac{c_{ps}\left(T_m - T_{in}\right)}{L} ; \; \tau = \frac{\alpha_s t}{r_w^{\,2}} Ste = Fo\left(\frac{r_i}{r_w}\right)^2 Ste
\end{aligned}
\tag{1}
$$

where, U is the dimensionless, fully-developed velocity profile, Re, Pr, Pe and Nu are the Reynolds, Prandtl, Peclet and Nusselt numbers, respectively. The tube inner diameter D is used in the definitions and the subscript f refers to the HTF properties. Dimensionless radial and axial coordinates and solid phase thickness, R, ξ and S, are defined in terms of the outside tube radius, r_w. A dimensionless temperature, θ, is defined in terms of the fluid inlet temperature T_{in} and the phase-change temperature T_m. Dimensionless time is expressed by means of a Fourier number, Fo, or by the product between this, the Stefan number, Ste, and the squared ratio of the inside and outside tube radii. The latter is grouped in the variable τ. The Stefan number is a quotient between the sensible and latent heat, L, contents of the PCM. The subscript s makes reference to the solid phase.

The HTF energy equation for the complete method is defined below,

$$
\left(\frac{\alpha_s}{\alpha_f} Ste\right)\frac{\partial\theta}{\partial\tau} + U\, Pe\left(\frac{r_w}{D}\right)\frac{\partial\theta}{\partial\xi} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right) + \frac{\partial^2\theta}{\partial\xi^2}
\tag{2}
$$

In the simplified scheme only the following balance equation in terms of the bulk temperature, θ_b, is considered,

$$
\left(\frac{\alpha_s}{\alpha_f} Ste\right)\frac{\partial\theta_b}{\partial\tau} + Pe\left(\frac{r_w}{D}\right)\frac{\partial\theta_b}{\partial\xi} = \frac{\partial^2\theta_b}{\partial\xi^2} + 4\left(\frac{r_w}{D}\right)^2 Nu\left(\theta_{wi} - \theta_b\right)
\tag{3}
$$

where θ_{wi} is the inner wall temperature.

The conduction equation for the tube wall is defined as,

$$\left(\frac{\alpha_s}{\alpha_w} Ste\right)\frac{\partial \theta_w}{\partial \tau} = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial \theta_w}{\partial R}\right) + \frac{\partial^2 \theta_w}{\partial \xi^2} \tag{4}$$

The PCM is treated in a different way, as a coordinate transformation in the radial direction which immobilizes the phase-change front is used. The new radial coordinate system is defined as,

$$\eta_s = \frac{r - r_w}{s(z,t)}\,;\, \eta_l = \frac{r - r_s(z,t)}{l(z,t)} \tag{5}$$

where r_s is the radial position of the solid-liquid interface and l is the thickness of the liquid phase. Equations (5) are known as Landau transforms and map the PCM physical domain to $0 \le \eta_s \le 1$ in the solid region and $0 \le \eta_l \le 1$, for the liquid phase. The phase-change front becomes fixed at the position $\eta_s = 1\,;\, \eta_l = 0$.

Writing the conduction equation for the solid and liquid phases in terms of η_s and η_l leads to rather complex expressions in which appear pseudoconvective terms, which are multiplied by the phase-change front velocity, and pseudoanisotropic terms with mixed derivatives. For saving space reasons those equations will not be presented here and all details about the treatment of the PCM can be found in Ismail & Jesus (1999).

The boundary conditions employed for the fluid energy equation are a specified and uniform temperature at entry, axial symmetry and zero temperature derivatives at the outlet. The continuity of heat fluxes is used for both the fluid and the tube wall, which is considered adiabatic at the end caps. The boundary conditions are shown below,

$$\theta_f(R,0) = -1.0\,;\, \left.\frac{\partial \theta_f}{\partial R}\right|_{R=0} = \left.\frac{\partial \theta_f}{\partial \xi}\right|_{\xi=\frac{H}{r_w}} = 0$$

$$\left.\frac{\partial \theta_f}{\partial R}\right|_{R=R_i} = \left.\frac{k_w}{k_f}\frac{\partial \theta_w}{\partial R}\right|_{R=R_i} = Nu\left(\frac{r_w}{D}\right)(\theta_{wi} - \theta_b) \tag{6}$$

$$\left.\frac{\partial \theta_w}{\partial R}\right|_{R=R_i} = \left.\frac{k_f}{k_w}\frac{\partial \theta_f}{\partial R}\right|_{R=R_i}\,;\, \left.\frac{\partial \theta_w}{\partial R}\right|_{R=1} = \left.\frac{k_s}{k_w}\frac{\partial \theta_s}{\partial R}\right|_{R=1}\,;\, \left.\frac{\partial \theta_w}{\partial \xi}\right|_{\xi=0,\frac{H}{r_w}} = 0$$

The PCM requires, besides the equality of heat fluxes at the tube wall and the adiabatic condition at the outer surface and at the end caps, conditions concerning the phase-change process. These are a constant temperature at the solid-liquid interface and the heat balance at this surface, which accounts for the latent heat released during the process, known as Stefan condition. The boundary conditions for the PCM are the following,

$$\theta(R_S, \xi) = 0 \tag{7a}$$

$$\left.\frac{\partial \theta_w}{\partial R}\right|_{R=1} = \frac{k_s}{k_w}\left.\frac{\partial \theta_s}{\partial R}\right|_{R=1} ; \left.\frac{\partial \theta_s}{\partial R}\right|_{R=\frac{r_0}{r_w}} = 0 ; \left.\frac{\partial \theta}{\partial \xi}\right|_{\xi=0,\frac{H}{r_w}} = 0 \tag{7b}$$

$$\left[1+\left(\frac{\partial S}{\partial \xi}\right)^2\right]\left(\frac{1}{S}\left.\frac{\partial \theta_s}{\partial \eta_s}\right|_{\eta_s=1} - \frac{k_l}{k_s}\left.\frac{\partial \theta_l}{\partial \eta_l}\right|_{\eta_l=0}\right) = \frac{\partial S}{\partial \tau} \tag{7c}$$

Equation (7c) is the Stefan equation and although it is a boundary condition it must be solved for the dimensionless phase-change front thickness $S(\xi,\tau)$.

The system of equations shown above is written in discrete form by the finite volume method. The energy equations are solved for the HTF, tube wall and solid and liquid PCM, using the coupling conditions. The local Nusselt number value is calculated by one of Eqs. (6). The Stefan condition is used to obtain the phase-change front position, and its time derivatives are treated explicitly, while in the energy equations the implicit method is used. Details about the PCM discrete equations and the treatment of the pseudo-convective and pseudo-anisotropic terms can be found in Ismail & Jesus (1999).

It is found accurate enough results using a grid in the radial direction with 20 control volumes in the HTF, 5 in the tube wall and 15 in each phase of the PCM. In the axial directions 100 control volumes were employed for all regions. In the simplified method there is only one control volume in the HTF and the local Nusselt number is obtained from the values for a CWT or CHF boundary condition.

3. RESULTS AND DISCUSSION

The proposed problem is solved for the solidification of water (PCM), around a copper tube with a refrigerant fluid flowing inside (HTF). The HTF was assumed to have a fixed density, ρ_f, and specific heat, c_{pf}, but the thermal conductivity, k_f, was varied. The adopted values of ρ_f and c_{pf} are typical of secondary refrigerant fluids found in ice-storage systems, as ethanol or ethylene-glycol. The HTF thermal conductivity has three possible values, a lower one, an intermediate, the same of ethanol, and a higher one. The solution is performed firstly by the complete method, after by the simplified scheme with CHF Nusselt number values and then by the simplified method adopting CWT Nusselt number values. The Peclet number is held fixed in all cases.

The geometric parameters chosen are the same used by Zhang & Faghri (1996) and the computational program was validated by comparing with their results. In their work the PCM was paraffin initially at the phase-change temperature and the HTF was water in laminar fully-developed flow. The tube wall was made of a high thermal conductivity material. The results chosen for comparison are based in a numerical study of the same authors used in (Zhang & Faghri, 1996) to validate their semi-analytical procedure. The geometric parameters adopted are described below and Fig. 2 shows the phase-change front evolution predicted by them and the complete method of this work.

$$\frac{r_w}{D} = 0.575 ; \frac{H}{D} = 50 ; E = \frac{r_o - r_w}{r_w} = 1.304 \tag{8}$$

Present results are at maximum 5% higher than Zhang & Faghri (1996) due to the different methods for treating the phase-change and convection processes.

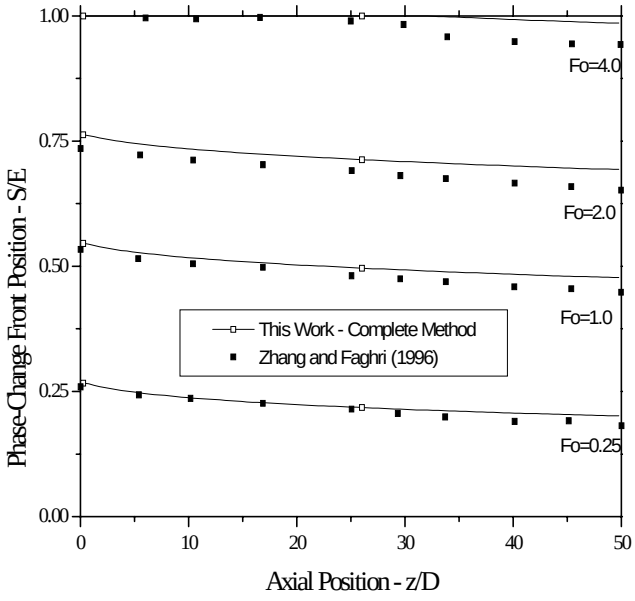

Figure 2 – Comparison with Zhang & Faghri (1996) results.

Table 1 shows the thermophysical properties adopted for the HTF in each case.

Table 1 – Properties of the HTF

	T_{in} [^{o}C]	k_f [W/m K]	ρ_f [kg/m^3]	c_{pf} [J/kg K]	Pe
Case 1	-15	0.05	1000	3000	40000
Case 2	-15	0.25	1000	3000	40000
Case 3	-15	2.5	1000	3000	40000

Figures 3 and 4 show, respectively, the phase-change front evolution and Nusselt number in the first case considered, a low thermal conductivity HTF. It can be noticed that the calculated Nusselt number values are close to the CWT condition and that the simplified method can predict accurately the phase-change front position using the CWT Nusselt number values. It may be observed that the solid-liquid interface presents a steep decay and this is a consequence of the exponential decrease of the heat transfer flux that occurs in convection problems in tubes under CWT boundary conditions.

Figures 5 and 6 show the results for the second case, an intermediate fluid thermal conductivity value, equal to the real ethanol thermal conductivity. The Nusselt number values shown in Fig. 5 laid approximately middle way between CWT and CHF values. The simplified method produces high errors when trying to capture the phase-change front position along the solidification process. The solid-liquid interface continues to have a high two dimensional profile.

Figures 7 and 8 show the same results for the third case considered, a k_f = 2.5 W/mK value, greater than the solid PCM thermal conductivity, k_s=2 W/mK. A different behavior of both the phase-change front evolution and the Nusselt number can be observed. The solid-liquid interface is nearly uniform along the axial direction which is characteristic of a constant heat flux along the tube. The Nusselt number values are now closer to the CHF condition steady state values and the simplified method performs better when using these Nusselt number values.

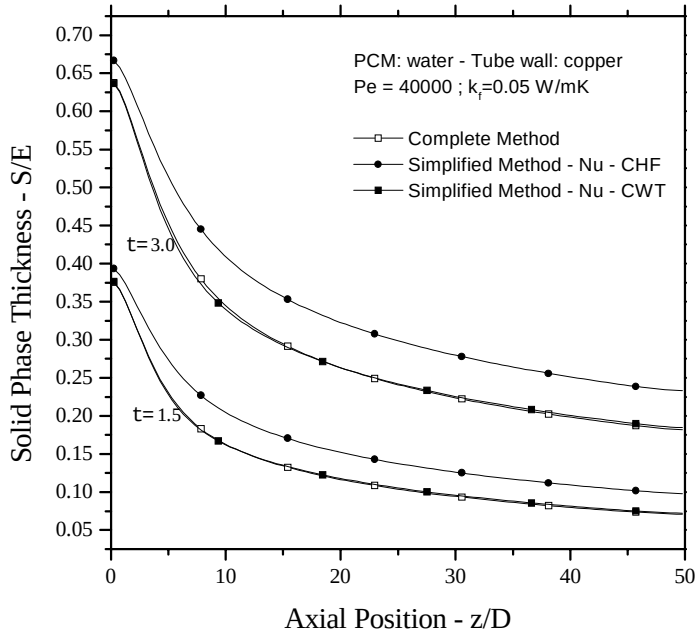

Figure 3 – Phase change front evolution – Case 1.

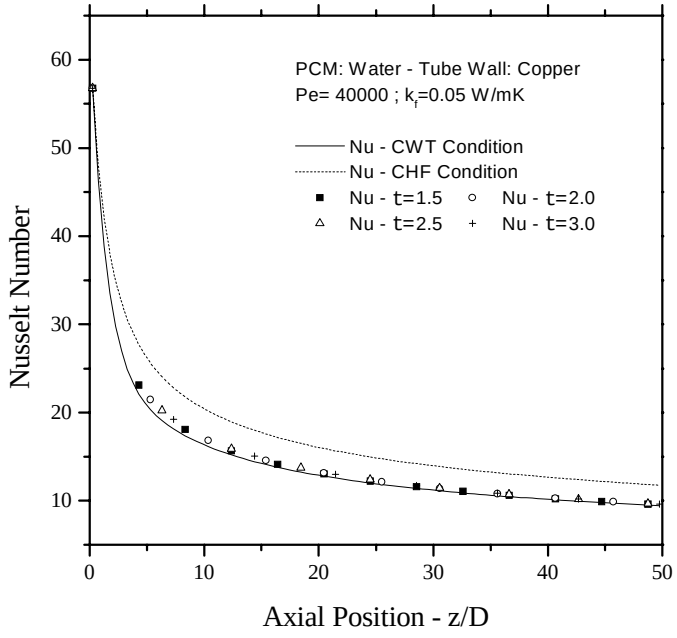

Figure 4 – Nusselt number evaluation – Case 1.

The performance of the simplified scheme compared to the complete method for different HTF thermal conductivity values can be analyzed in terms of the external thermal resistance imposed to the HTF.

As the phase change front is an isothermal surface, neglecting axial conduction effects and assuming a quasi-steady state condition, the external thermal resistance and can be written as,

$$h_{ext} = \frac{1}{\frac{r_i}{k_w} ln\left(\frac{r_w}{r_i}\right) + \frac{r_w}{k_s} ln\left(\frac{r_s}{r_w}\right)} \tag{9}$$

and a Biot number can be defined,

$$Bi = \frac{h_{ext} r_i}{k_f} = \frac{1}{\frac{k_f}{k_w} ln\left(\frac{r_w}{r_i}\right) + \frac{k_f}{k_s}\left(\frac{r_w}{r_i}\right) ln\left(\frac{r_s}{r_w}\right)} \tag{10}$$

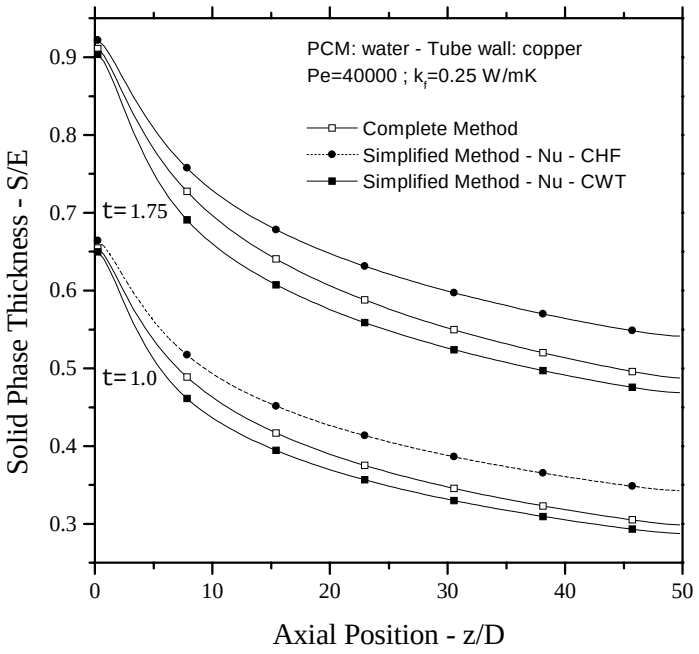

Figure 5 – Phase change front evolution – Case 2.

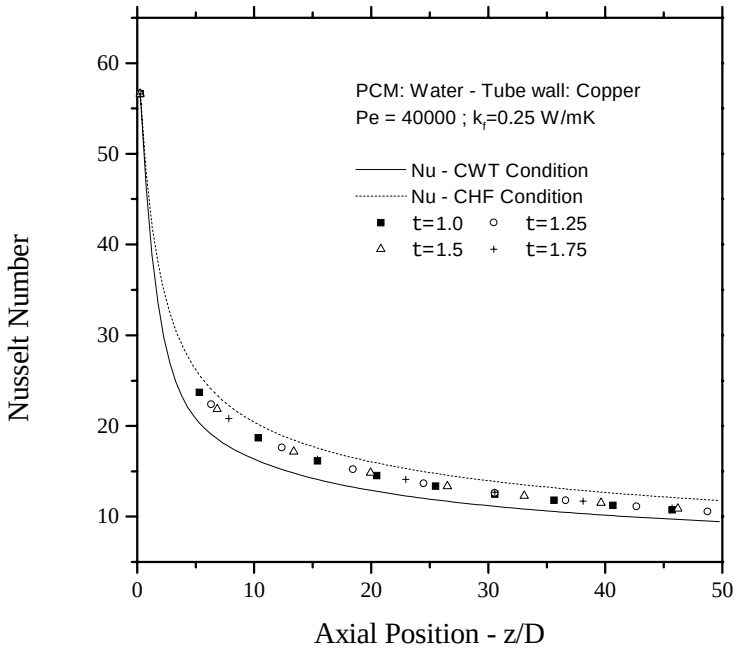

Figure 6 – Nusselt number evaluation – Case 2.

Sparrow & Patankar (1977) showed that for a thermally developed flow inside a tube immersed in an external isothermal fluid, the CWT and CHF conditions are limiting cases

related to a Biot number, as defined in Eq. (10). When this number tends to infinity the Nusselt number will approach the CWT value and as it goes to zero the Nusselt number is closer to the CHF value.

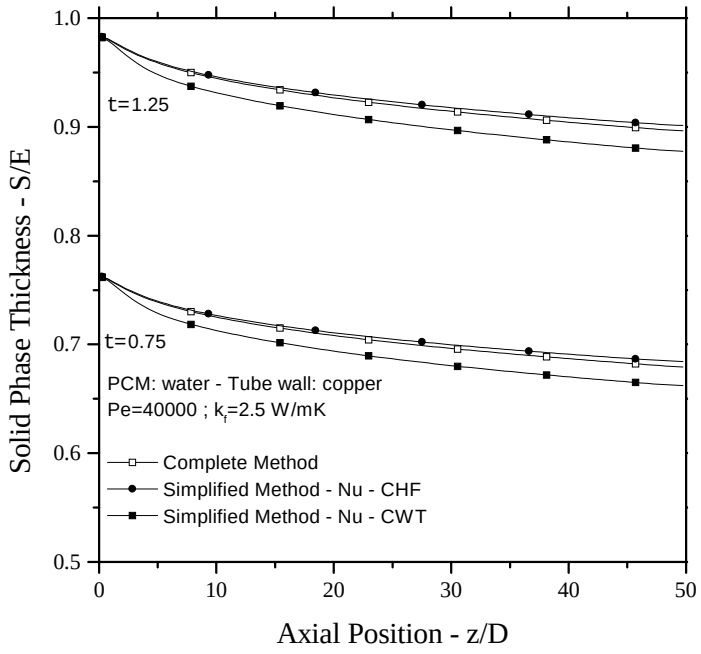

Figure 7 – Phase change front evolution – Case 3.

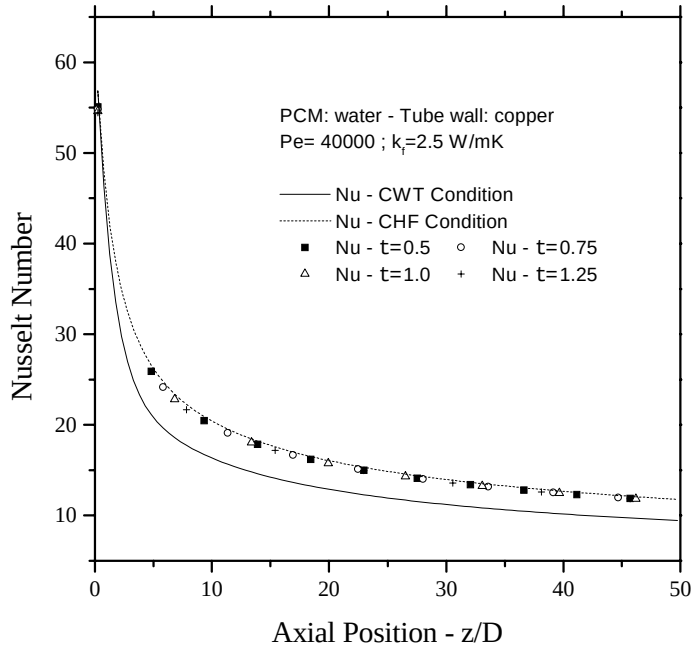

Figure 8 – Nusselt number evaluation – Case 3.

Applying the same argument the Biot number can be calculated for each case considered using Eq. (10). In the beginning of the process, when the phase-change front thickness is small, it will achieve a high value, characterizing a CWT condition, but decreasing as the phase-change front moves. The velocity of decay will depend on the wall thickness and the ratios between thermal conductivities of the HTF and the tube wall material and the HTF and

the solid PCM. A similar conclusion was stated by Zhang & Faghri (1996) for a turbulent and low Prandtl number HTF.

Table 2 shows the minimum values of the Biot number for each case considered. It corresponds to a phase-change front thickness equal to the annular space.

Table 2 – Calculated Biot numbers

Case	1	2	3
Bi	39	7.8	0.78

Using the results tabulated in Sparrow and Patankar (1977) with the values of Table 2, it becomes clear that case 1 is closer to a CWT condition, case 2 has an intermediate position and case 3 has a nearly CHF boundary condition. The results above show that the simplified treatment of the HTF in the design of latent heat thermal energy storage units can lead to erroneous results unless care is taken to know whether the system under consideration is closer to a CHF or a CWT boundary condition, or none of these.

4. CONCLUSIONS

The problem of solidification of PCM around a tube carrying a cold, laminar HTF inside was solved by a complete and a simplified method, which adopted steady state Nusselt number values for either a constant wall temperature or a constant wall heat flux condition. The results obtained show that the accuracy of the simplified scheme is related to the thermal resistance imposed to the HTF during the process, depending specially on the ratios between the thermal conductivities of the HTF, solid PCM and tube wall material.

REFERENCES

Bellecci, C. & Conti, M., 1993, Phase Change Thermal Storage: Transient Behavior Analysis of a Solar Receiver/Storage Module Using the Enthalpy Method, Int. Journal of Heat and Mass Transfer, vol. 36, n. 8, pp. 2157-2163.

Cao, Y. & Faghri, A., 1991, Performance Characteristics of a Thermal Energy Storage Module: a Transient PCM/Forced Convection Conjugate Analysis, Int. Journal of Heat and Mass Transfer, vol. 34, n. 1, pp. 93-101.

Cao, Y. and Faghri, A., 1992, A Study of Thermal Energy Storage Systems with Conjugate Turbulent Forced Convection, Journal of Heat Transfer, vol. 114, pp. 1019-1027.

Ismail, K.A.R. & Jesus, A.B., 1999, Modeling and Solution of the Solidification Problem of PCM around a Cold Cylinder, Numerical Heat Transfer – Part A: Applications, vol. 36, n.1, pp. 95-114.

Sparrow, E.M. & Patankar, S.V., 1977, Relationships among Boundary Conditions and Nusselt Numbers for Thermally Developed Duct Flows, Journal of Heat Transfer, vol. 99, pp. 483-485.

Zhang, Y. & Faghri, A., 1995, Analysis of Thermal Energy Storage System with Conjugate Turbulent Forced Convection,. Journal of Thermophysics and Heat Transfer, vol. 9, n. 4, pp. 722-726.

Zhang, Y. & Faghri, A., 1996, Semi-Analytical Solution of Thermal Energy Storage System with Conjugate Laminar Forced Convection, Int. Journal of Heat and Mass Transfer, vol. 39, n. 4, pp. 717-724.