FLOW AND HEAT TRANSFER TO VISCOPLASTIC MUDS IN DRILLING PROCESS OF PETROLEUM WELLS

Mônica F. Naccache – naccache@mec.puc-rio.br
Paulo R. Souza Mendes – pmendes@mec.puc-rio.br
Department of Mechanical Engineering
Pontifícia Universidade Católica–Rio de Janeiro, RJ 22453-900

Abstract. *A numerical simulation of heat transfer during the laminar flow of viscoplastic materials through annular eccentric spaces was performed. This geometry is found in the drilling process of petroleum wells. The inner cylinder is adiabatic and rotates with a constant angular velocity—in order to account for rotation of the column—while at the outer wall a uniform heat flux is imposed. The conservation equations together with the Generalized Newtonian Liquid (GNL) model are solved numerically via a finite volume method, using FLUENT. In order to handle the numerical difficulties related to the Herschel-Bulkley viscosity function, the bi-viscosity model was used. The effects of angular velocity and eccentricity on the flow pattern and heat transfer coefficient are investigated.*

Keywords: *Viscoplastic muds, Heat transfer, Annular flows*

1 INTRODUCTION

Flow through annular spaces are found in processes of different industrial sectors, such as food, cosmetics and petrochemical, mainly in heat exchange devices. In these industries, viscoplastic materials are quite common.

The main mechanical characteristic a viscoplastic material is the existence of an yield stress, below which there is no flow. The drilling fluid, employed in drilling operations of petroleum wells, is desirably a viscoplastic material, because it will drag the rock chips generated by the drill at the lowest pumping power.

The performance of a well operation is typically rather dependent on the knowledge and control of the mud rheological properties, which are also strong functions of temperature. Then, flow and heat transfer information are necessary to allow reliable and successful process operations.

Heat transfer for power-law fluids through tubes is analyzed by several authors (Bird *et al.* (1987), Irvine and Karni (1987), Joshi and Bergles (1980a, 1980b), Scirocco *et al.* (1985)).

In these papers, correlations are proposed for the Nusselt number. There are also results for viscoplastic materials for the same geometry. Vradis *et al.* (1992) analyzed numerically the heat transfer problem for flows of Bingham materials, for simultaneous development of velocity and temperature profiles. An experimental and theoretical heat transfer study for Herschel-Bulkley materials was later reported by Nouar *et al.* (1994). They also investigated the entrance region, but considered fully developed velocity profiles. An interesting discussion regarding the impact of temperature-dependent rheological properties on velocity profiles and Nusselt numbers can be found in this paper.

More recently, Nouar *et al.* (1995) analyzed numerically the heat transfer of Herschel-Bulkley materials through tubes and proposed some correlations for the local Nusselt number and pressure gradient. In this work, axial diffusion of heat was neglected, while a temperature-dependent consistency index and simultaneous velocity and temperature development were considered. Soares *et al.* (1999) analyzed the entrance-region tube flow of Herschel-Bulkley materials, for constant and temperature-dependent properties, taking axial diffusion into account. Among other results, they observed that the temperature-dependent properties do not affect qualitatively pressure drop or the Nusselt number. Also, it was shown that axial diffusion is important near the entrance of the duct.

As discussed above, there is an extensive literature regarding the flow of viscoplastic materials through tubes. However, rather few heat transfer results for flows through annular spaces have been reported so far. Naimi *et al.* (1990), studied experimentally the flow of a viscoplastic material (Carbopol) through annular spaces. In their work, the inner cylinder was able to rotate, and the appearance of secondary flows due to rotation were reported. The heat transfer coefficient was also obtained as a function of the axial position and angular velocity. More recently, Soares *et al.* (1998) investigated numerically the heat transfer of viscoplastic materials flowing axially through annular concentric spaces. The case of uniform inner wall heat flux and insulated outer wall was examined. It was observed that when the velocity gradient at the wall is high the Nusselt number is larger. However, Nusselt number variations were found to be rather small in the range of rheological parameters analyzed. Nouar *et al.* (1998) analyzed numerically and experimentally the heat transfer problem during the fully developed flow of Herschel-Bulkley materials through annular concentric spaces, with a rotating inner cylinder. The influence of consistency index variation with temperature is investigated, and it was observed that its effect becomes more important as the angular velocity is increased. Also, it was shown that the iner cylinder rotation causes a decrease in the axial velocity gradient at the outer wall, and heat transfer is diminished.

In this work, a numerical study of flow and heat transfer of viscoplastic materials through annular eccentric spaces is performed. The inner cylinder is allowed to rotate and is adiabatic. The outer cylinder is fixed and has a constant heat flux. The effect of angular velocity and of eccentricity on friction factor and heat transfer are analyzed. The governing equations are solved numerically via a finite volume method, using the FLUENT software. The material is assumed to behave as a Generalized Newtonian Liquid (GNL), with a bi-viscosity function.

2 ANALYSIS

The problem under study is the laminar, steady flow of a viscoplastic material in the entrance region of an eccentric annular space. The cross section of the annular space is shown in Fig. 1. The thermophysical and rheological properties of the flowing material are considered to be temperature independent.

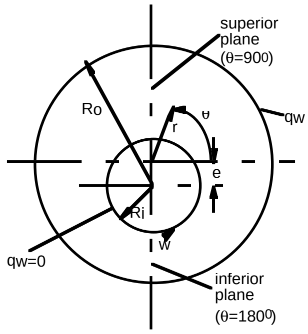

Figure 1: Cross section of the annular space

2.1 Conservation equations

For this flow, the dimensionless conservation equation of mass is given by:

$$\operatorname{div} \mathbf{v}' = 0 \tag{1}$$

The momentum conservation equation is given by:

$$\operatorname{grad} \mathbf{v}' \cdot \mathbf{v}' = -\operatorname{grad} p' + \frac{\bar{u}'}{Re} \operatorname{div}(\eta' \operatorname{grad} \mathbf{v}') \tag{2}$$

In the above equations, $\mathbf{v}' = \mathbf{v}/u_m$ is the dimensionless velocity, u_m is the inlet (or average) axial velocity, $p' = p/\frac{1}{2}\rho u_m^2$ and $Re = \rho u_m \delta/\eta_c$, where $\delta = D_h/2 = (R_o - R_i)$, D_h is the hydraulic diameter, R_o is the outer-tube inner radius and R_i is the inner-cylinder radius. $\eta' = \eta/\eta_c$ and η_c is a caracteristic viscosity ($\eta_c = \mu$, for the Newtonian fluids and $\eta_c = \eta(\dot{\gamma}_c)$ for the viscoplatic materials). $\dot{\gamma}_c$ is a characteristic deformation rate, chosen to be equal to the value of $\dot{\gamma}$ at the inner-cylinder wall for the flow of a Newtonian fluid.

The viscosity of the viscoplastic material is given by the *bi-viscosity function* (Beverly and Tanner, 1992), defined as

$$\eta = \begin{cases} \frac{\tau_0}{\dot{\gamma}} + K\dot{\gamma}^{n-1} & \text{if } \dot{\gamma} > \dot{\gamma}_{\text{small}} \\ \eta_{\text{large}}, & \text{otherwise} \end{cases} \tag{3}$$

where τ_0 is the yield stress, n the power-law index, K is the concistency index, and $\dot{\gamma} \equiv \sqrt{\frac{1}{2} \operatorname{tr} \dot{\boldsymbol{\gamma}}^2}$ the deformation rate ($\dot{\boldsymbol{\gamma}} \equiv \operatorname{grad} \mathbf{v} + \operatorname{grad} \mathbf{v}^T$ is the rate-of-deformation tensor). Beverly and Tanner (1992) recommend

$$\eta_{\text{large}} = 1000\eta_c \tag{4}$$

Therefore,

$$\dot{\gamma}_{\text{small}} = \frac{\tau_0}{1000\eta_c - K\dot{\gamma}_{\text{small}}^{n-1}} \simeq \frac{\tau_0}{1000\eta_c} \tag{5}$$

The dimensionless conservation of energy equation is given by:

$$\operatorname{grad} \Theta \cdot \mathbf{v}' = \frac{1}{Pe} \operatorname{div}(\operatorname{grad} \Theta) - \frac{2u'}{Pe} \tag{6}$$

In this equation, $\Theta \equiv (T - T_b)/(q_{\mathrm{w}}\delta/k)$ is the dimensionless temperature field, T_b is the bulk temperature, q_{w} is the heat flux at the outer wall, k is the thermal conductivity, $Pe = u_m\delta/\alpha$ is the Peclet number and $\alpha = k/\rho c$ is the thermal diffusivity.

The velocity boundary conditions are the usual no-slip and impermeability conditions at walls. The inner cylinder is allowed to rotate with an angular velocity w (rad/s). The thermal boundary conditions are uniform heat flux ($q_{\mathrm{w}} =$ constant) at the outer wall and adiabatic ($q_{\mathrm{w}} = 0$) inner wall.

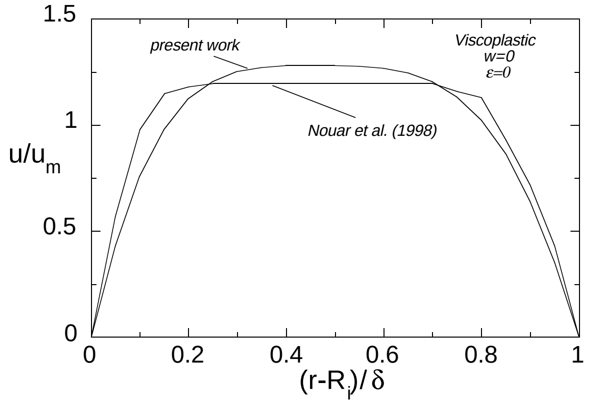

Figure 2: Comparison of the velocity profile with experimental results (Nouar *et al.*, 1998) for a viscoplastic material

2.2 Numerical solution

The governing equations presented above have been discretized via the finite volume method described by Patankar (1980), using a central-difference scheme. The numerical results were obtained using the FLUENT (Fluent Inc.) software.

In order to validate the numerical solution, some tests have been performed. The numerically evaluated value of the product $fRe^* = -(8(dp/dx)\delta^2)/(\eta_c u_m)$ for concentric cylinders, Newtonian fluid and $\delta/R_o = 0.5$, was compared with the ones found in the literature, namely $fRe^* = 95.2$. For a mesh with 21 nodal points (20 control volumes) in the tangential and radial directions, and 17 nodal points (16 control volumes) in the axial direction, the error obtained was equal to 0.75%. The fRe results for a viscoplastic material were compared to the ones obtained with a finer mesh in the axial direction (21$\times$ 21$\times$ 31). For concentric cylinders and zero angular velocity, the deviation in fRe was equal to 1.6%. For $\epsilon = e/(R_o - R_i) = 0.8$ and angular velocity of the inner cylinder w =13.8 rd/s, the deviation in fRe was equal to 2.2%. The axial velocity profile was compared for the viscoplastic material, with $w = 0$ and $\epsilon = 0$ and a 21$\times$ 41$\times$ 11 mesh. It was observed that the fRe value remains unchanged after this mesh refinement.

The fully developed axial profile for a viscoplastic material with $n = 0.4$, $K = 1537$ Pa.s^n and $\tau_0 = 41$ Pa was compared with the experimental results obtained by Nouar et al. (1998). The comparison is shown in Fig. 2. It can be seen that the agreement is good, the difference between the maximum velocities being equal to 5.8%.

3 RESULTS

The effect of angular velocity and eccentricity on the flow pattern and heat transfer are now presented and discussed.

Figures 3–7 show the developed axial velocity profile for $\epsilon = 0$, 0.4 and 0.8, for the Newtonian case with fixed inner cylinder and for the viscoplastic material with three different angular velocities of the inner cylinder, namely $w = 0$, 1 and 13.8 rad/s. It can be observed that the qualitative behavior is almost independent of the eccentricity. When the inner cylinder is fixed, there always exists a plug flow region in the middle of the annular space, for the viscoplastic

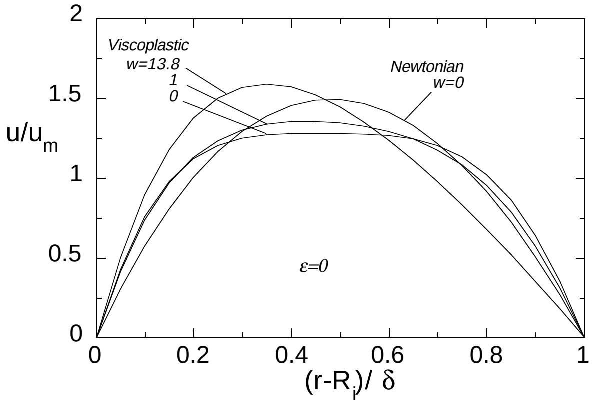

Figure 3: Axial velocity profile for the Newtonian and viscoplastic materials, $\epsilon = 0$.

material. Inside this region, the extra-stress tensor intensity, τ, ($\tau \equiv \sqrt{\frac{1}{2}\,\mathrm{tr}\,\boldsymbol{\tau}^2}$) is lower than the yield stress,τ_0, and the material moves as a solid, i.e., with a uniform velocity.

Increasing the angular velocity of the inner cylinder, the shear rate increases, and so does the stress. This causes a reduction in size of the plug-flow region. In some cases, the plug flow region disappears. Comparing Fig. 4 with Fig. 5 and Fig. 6 with Fig. 7, it can also be noted that where the gap is largest (i.e., at $\theta = 90^0$), the axial velocity is largest. However, the opposite trend is observed for the tangential velocity profile. This fact can be observed with the aid of Figs. 10 and 11, where it is shown that the maximum tangential velocities occur at the smallest gap. Another effect of the angular velocity is to distort the axial velocity profile.

In order to get a better visualization of the flow pattern, tridimensional views of the developed velocity vectors are shown in Figs. 12–15. It can be noted that, as the angular velocity is increased, the tangential velocity dominates the flow.

The product fRe is shown in Fig. 16 as a function of the eccentricity. It is noted that fRe decreases with eccentricity, because most of the flow occurs at the larger gap region, where there is less flow resistance than for the case of concentric cylinders. It can be observed also that fRe decreases with the angular velocity, because the deformation rate, $\dot{\gamma}$, increases as the angular velocity is increased, and hence, the viscosity decreases (see eq.(5)).

Finally, the influence of eccentricity and angular velocity on the Nusselt number ($Nu = 2q_w\delta/k(T_w - T_b)$) for the fully-developed flow region is presented in Fig. 17. Comparing the results for Newtonian and viscoplastic materials, for $w = 0$ and $\epsilon = 0$, it is observed that Nu is larger for the viscoplastic material, because of the larger velocity gradients that occur near the walls. For the viscoplastic material, it is observed that heat transfer decreases with the eccentricity for zero angular velocity. However, the opposite trend occurs when there is tangential flow. This behavior can be explained by the colder fluid that is convected into the small gap region by the tangential flow.

4 CONCLUSIONS

This paper reports a preliminary study of the heat transfer problem during the flow of viscoplastic materials through annular eccentric spaces, with a rotating inner cylinder. The case of

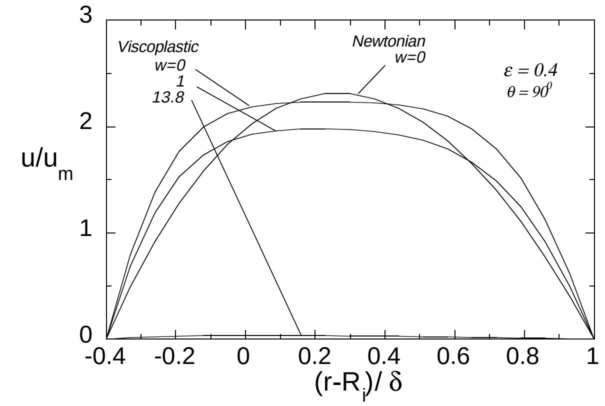

Figure 4: Axial velocity profile for the Newtonian and viscoplastic materials, $\epsilon = 0.4, \theta = 90^0$.

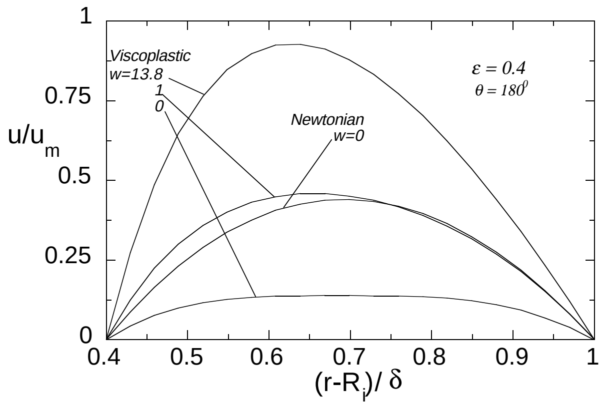

Figure 5: Axial velocity profile for the Newtonian and viscoplastic materials, $\epsilon = 0.4, \theta = 180^0$.

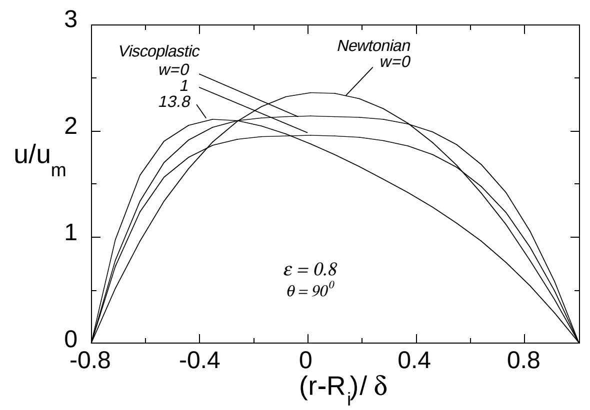

Figure 6: Axial velocity profile for the Newtonian and viscoplastic materials, $\epsilon = 0.8, \theta = 90^0$.

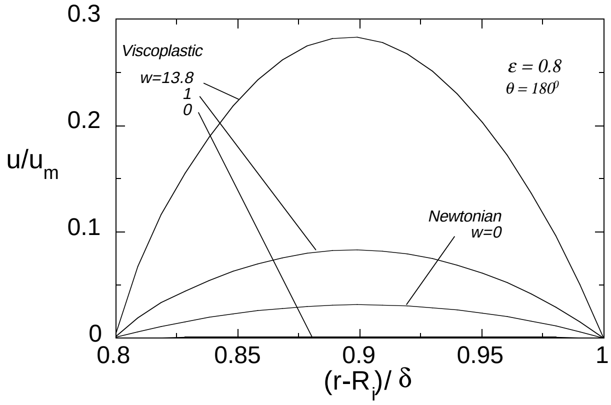

Figure 7: Axial velocity profile for the Newtonian and viscoplastic materials, $\epsilon = 0.8, \theta = 180^0$.

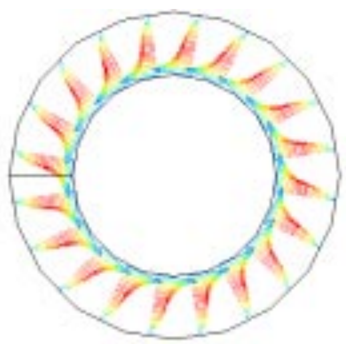

Figure 8: tangential velocity vectors for the viscoplastic material, $\epsilon = 0$ and $w = 1$ rd/s.

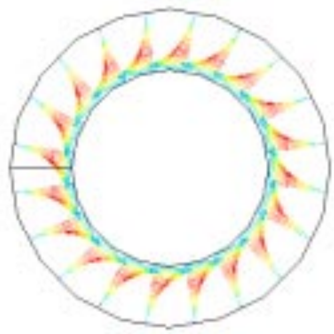

Figure 9: tangential velocity vectors for the viscoplastic material, $\epsilon = 0$ and $w = 13.8$ rd/s.

Figure 10: tangential velocity vectors for the viscoplastic material, $\epsilon = 0.8$ and $w = 1$ rd/s.

Figure 11: tangential velocity vectors for the viscoplastic material, $\epsilon = 0.8$ and $w = 13.8$ rd/s.

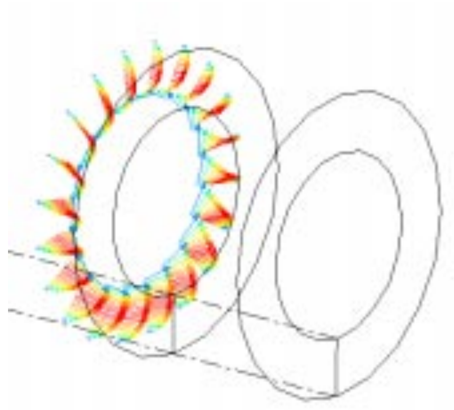

Figure 12: Velocity vectors for the viscoplastic material, $\epsilon = 0$ and $w = 1$ rd/s.

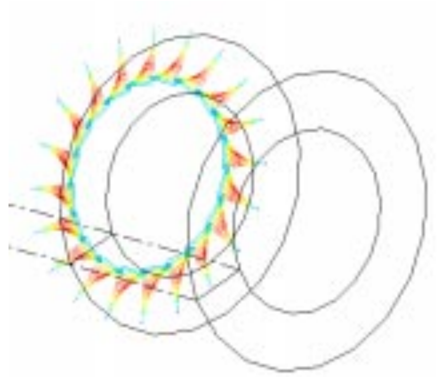

Figure 13: Velocity vectors for the viscoplastic material, $\epsilon = 0$ and $w = 13.8$ rd/s.

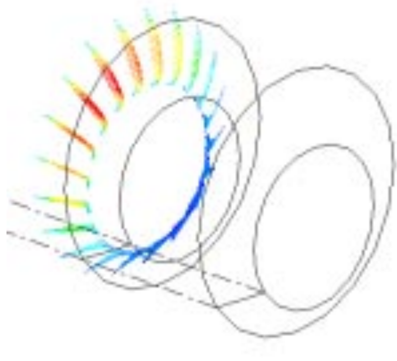

Figure 14: Velocity vectors for the viscoplastic material, $\epsilon = 0.8$ and $w = 1$ rd/s.

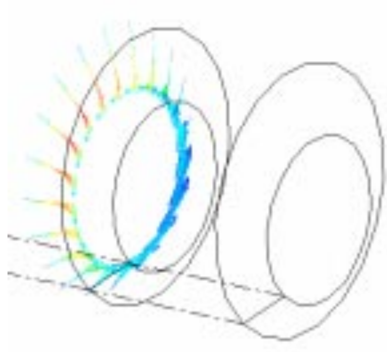

Figure 15: Velocity vectors for the viscoplastic material, $\epsilon = 0.8$ and $w = 13.8$ rd/s.

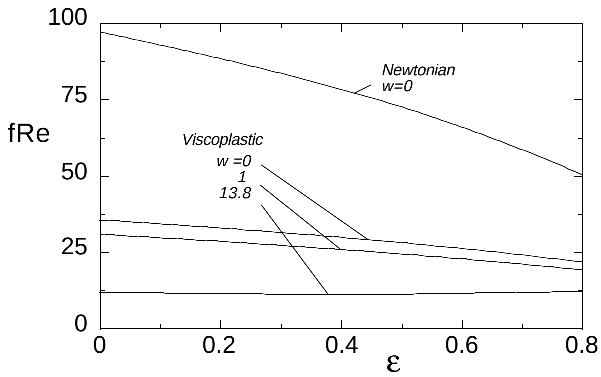

Figure 16: fRe variation with ϵ and w.

uniform outer wall heat flux and insulated inner wall was investigated.

The governing equations were solved numerically via a finite-volume technique, using the software FLUENT. Velocity profiles, friction factor and Nusselt numbers are presented as functions of the relevant parameters.

For the viscoplastic material, it was observed that the product fRe decreases with the eccentricity as well as with the angular velocity. On the other hand, the Nusselt number decreases with the eccentricity for zero angular velocity, but this trend is reversed when there is tangential flow.

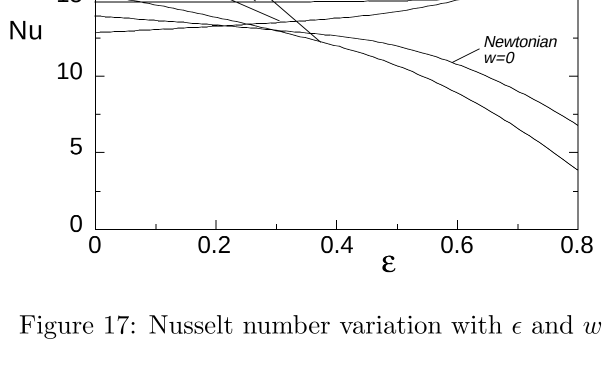

Figure 17: Nusselt number variation with ϵ and w.

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