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**HEAT, MOMENTUM AND MASS TRANSFER IN RAREFIED GASES BETWEEN TWO ROTATING CYLINDERS**

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**Abstract** *The mass, heat and momentum transfer through a rarefied gas confined between two coaxial rotating cylinders is investigated. The evaporation and condensation of the gas on the cylinder surfaces is assumed. The state of equilibrium is perturbed by small deviations of pressure, temperature and angular velocity on the inner cylinder. The fields of the pressure, temperature, velocity, heat flux vector, and stress tensor are calculated in the wide range of the Knudsen number. In the free molecular and transition regimes the kinetic model of the Boltzmann equation has been solved numerically by the discrete velocity method. Further, the Onsager reciprocity relations between the cross-effects are tested.*

**Key-words**: Boltzmann equation, Knudsen number, Transport Phenomena

# 1 Introduction

The transport phenomena, i.e. mass, heat and momentum transfer, through a fluid confined between two coaxial cylinder is a classical problem of mechanics of fluids, which is of a great scientific and practical interest. In our previous papers Sharipov & Kremer (1995a, 1995b, 1996a, 1996b) this type of flow was used to test the principle of material frame indifference, the validity of which is discussed by many authors. It was shown that the rotation creates an anisotropy and changes qualitatively the transport properties of the fluid. The anisotropy induces the following effects: (i) A radial temperature gradient causes both radial and tangential heat fluxes, see the papers by Sharipov & Kremer (1995a, 1995b). In other words, the thermal conductivity becomes a second-order tensor. (ii) The diagonal components of the stress tensor are not equal to zero but significantly depend on the rotation frequency [Sharipov & Kremer (1996a, 1996b)].

In our other paper [Sharipov & Kremer (1999)], a strong non-equilibrium Couette flow, i.e., when the cylinders have a large temperature difference and a large velocity difference simultaneously, was studied. Here, under the term "Couette flow" we imply all transport phenomena through a gas confined between two coaxial cylinders. From the analysis of the numerical solution based on the kinetic equations we found that the temperature difference affects the stress tensor without changing the velocity profile, while the velocity difference affects the heat flux without changing the temperature profile.

The above mentioned papers consider the heat and momentum transfer only. In order to take into account the mass transfer, one has to assume that the coaxial cylinders can absorb and emit fluid particles. One of the mechanisms of mass exchange between the gas and the cylinders is the evaporation and condensation on the cylinder walls. Gas flows with evaporation and condensation on the boundary have a very complicated solution even in systems at rest, i.e., without rotation. This type of flow was investigated by many authors on the basis of the kinetic theory and an extensive list of the corresponding publications can be found in the paper Sugimoto & Sone (1992).

The flow becomes more complex if the cylinders - that confine a gas in which processes of evaporation and condensation are taking place - rotate. A numerical solution of this problem was presented in our paper Cumin *et al.* (1998) for a wide range of the Knudsen number. It was assumed that the cylinders have the same temperature and rotate with the same velocity, but there, the gas evaporates with weakly different pressures. Some analytical results for the Couette flow with evaporation and condensation in the continuum regime can be found in the paper by Sone *et al.* (1996), while interesting results for the non-linear Couette flow with evaporation and condensation are presented in the paper by Sone *et al.* (1999) where it was pointed out that a bifurcation of the flow is possible.

The aim of the present paper is to solve a more general problem than those that were considered in our previous papers. Namely, we assume that the rotating cylinder can evaporate and condense the gas. Like the paper by Cumin *et al.* (1998), here we consider that a number of evaporated molecules on the inner cylinder is different from the number of condensed molecules. But besides this we assume that the cylinders have different angular velocities and different temperatures. So, we consider the mass, heat and momentum transfer jointly. This problem allows us to calculate many new phenomena that were not present in the previous papers. They are the so-called cross effects. What are they?

When we assume the difference between the number of evaporated and condensed molecules, we admit a radial mass transfer. But besides the mass transfer, this difference causes a heat transfer (even without the temperature difference) and a momentum transfer (even if the cylinders rotate with the same angular velocity). When we apply a temperature difference between the cylinders, a heat flux appears. But mass and momentum transfers also appear. And, finally, the angular velocity difference between the cylinders, besides a momentum transfer, also causes a mass and heat transfer. So, considering the three thermodynamic driving forces, i.e. the three types of equilibrium perturbations, the six cross phenomena can be calculated. A general analysis of these phenomena was carried out in the paper by Sharipov (1998), where the Onsager-Casimir reciprocity relations for them were obtained.

The solutions presented in this paper may be used to optimize the centrifugal separation of gases.

## 2 Statement of the problem

Consider a rarefied gas confined between two coaxial cylinders with radii $R_0$ and $R_1$ ($R_0 > R_1$) where the axis of the cylinders coincide with the $z$-axis. The cylinders may rotate around the $z$-axis and are assumed to be so long that the end effects can be neglected, i.e., the solution does not depend on the $z$-coordinate.

If both cylinders, which are at the same temperature $T_0$, rotate with the same angular velocity $\Omega_0$, and each of them evaporates the same number of molecules that condenses, then the temperature of the gas is constant and equal to $T_0$, the radial bulk velocity $u'_r$ of the gas is zero, and the tangential bulk velocity $u'_\varphi$ is equal to

$$u'_\varphi = \Omega_0 r', \tag{1}$$

where $r' = \sqrt{x^2+y^2}$ is the radial coordinate, i.e. the gas rotates as a solid substance. The number density $n_0(r')$ of the gas under this conditions will have the following distribution

$$n_0(r') = \frac{n_{00}(1 - R_1^2/R_0^2)(\beta\Omega_0 R_0)^2}{\exp[(\beta\Omega_0 R_0)^2] - \exp[(\beta\Omega_0 R_1)^2]} \exp[(\beta\Omega_0 r')^2] \tag{2}$$

where

$$\beta = \left(\frac{m}{2kT_0}\right)^{1/2},$$

$k$ is the Boltzmann constant, $m$ is the mass of a gas particle, and $n_{00}$ is the number density when the cylinders are at rest ($\Omega_0 = 0$). This distribution is easily obtained from the static equilibrium analysis. The equilibrium pressure distribution $P_0(r')$ is obtained from the state equation, which is valid for any rarefied gas

$$P_0(r') = n_0(r')kT_0 \tag{3}$$

In this equilibrium state there are neither heat flux nor stress tensor.

Let us consider three factors that weakly disturb this equilibrium state:

(i) The outer cylinder evaporates the particles with the equilibrium pressure $P_0(R_0)$, while the pressure of evaporated particles from the inner cylinder $P_1$ is slightly different from the equilibrium pressure at its surface, i.e.,

$$P_1 = P_0(R_1) + \Delta P, \qquad \frac{|\Delta P|}{P_0(R_1)} \ll 1. \tag{4}$$

(ii) The outer cylinder rotates with the angular velocity $\Omega_0$, while the angular velocity of the inner cylinder $\Omega_1$ slightly differs from $\Omega_0$, i.e.

$$\Omega_1 = \Omega_0 + \Delta\Omega, \qquad \beta R_1 |\Delta\Omega| \ll 1. \tag{5}$$

The smallness of $\Delta\Omega$ means that the difference between the velocities of the surfaces is smaller than the speed sound of the gas. Note, there is no restriction on the value of the angular velocity $\Omega_0$.

(iii) The outer cylinder is at the equilibrium temperature $T_0$, while the temperature of the inner cylinder $T_1$ slightly differs from $T_0$, i.e.

$$T_1 = T_0 + \Delta T, \qquad \frac{|\Delta T|}{T_0} \ll 1. \tag{6}$$

We are going to calculate the fields of the bulk velocity $\mathbf{u}(r')$, heat flux vector $\mathbf{q}(r')$ and stress tensor $\sigma_{ij}(r')$ between the cylinders.

There are three main parameters that determine the solution of the problem:

(i) The rarefaction parameter, i.e. the inverse Knudsen number, defined as

$$\delta = \frac{\sqrt{\pi}}{2} \frac{R_0}{\lambda_{00}}, \tag{7}$$

where $\lambda_{00}$ is the molecular mean free path at the density $n_{00}$ and at the temperature $T_0$. Here we will cover a wide range of this parameter.

(ii) The dimensionless angular velocity $\omega$, defined as

$$\omega = \beta \Omega_0 R_0. \tag{8}$$

This parameter has the physical meaning of the Mach number based on the surface velocity of the outer cylinder. Here, we will cover the range of $\omega$ from 0 to 1. Nevertheless, applying the method used here it is possible to extend the range of $\omega$.

(iii) The ratio of the radii of the cylinders, defined as

$$a = R_1/R_0. \tag{9}$$

Since the solution does not change qualitatively with the variation of the ratio $a$, here we restrict ourselves by the value $a = 0.5$.

For the problem in question it is convenient to introduce the three small parameters for the linearization:

$$X_P = \frac{\Delta P}{P_0(R_1)}, \qquad X_\Omega = \beta R_1 \Delta\Omega, \qquad X_T = \frac{\Delta T}{T_0}. \tag{10}$$

Due to the constraints (4), (5) and (6) we have that $|X_\alpha| \ll 1$ ($\alpha = P, \Omega, T$). Since the forces are small, the pressure $P$, the radial $u'_r$ and tangential $u'_\varphi$ component of the bulk velocity, the temperature $T$, the heat flux vector $\mathbf{q}'$ and the viscous stress tensor $\sigma'_{ij}$ can be written as

$$P(r') = P_0(r') \left[ 1 + \sum_\alpha \upsilon^{(\alpha)}(r) X_\alpha \right], \tag{11}$$

$$u'_r(r') = \beta^{-1} \left[ \sum_\alpha u_r^{(\alpha)}(r) X_\alpha \right], \tag{12}$$

$$u'_\varphi(r') = \Omega_0 r' + \beta^{-1} \left[ \sum_\alpha u_\varphi^{(\alpha)}(r) X_\alpha \right], \tag{13}$$

$$T(r') = T_0 \left[ 1 + \sum_\alpha \tau^{(\alpha)}(r) X_\alpha \right], \tag{14}$$

$$q'_i(r') = P_{00} \beta^{-1} \left[ \sum_\alpha q_i^{(\alpha)}(r) X_\alpha \right], \qquad i = r, \varphi, \tag{15}$$

$$\sigma'_{ij}(r') = 2 P_{00} \left[ \sum_\alpha \sigma_{ij}^{(\alpha)}(r) X_\alpha \right], \qquad i, j = r, \varphi, \tag{16}$$

where $v^{(\alpha)}$, $u_r^{(\alpha)}$, $u_\varphi^{(\alpha)}$, $\tau^{(\alpha)}$, $q_i^{(\alpha)}$, $\sigma_{ij}^{(\alpha)}$ are dimensionless quantities. Moreover, the dimensionless radial coordinate $r = r'/R_0$ has been introduced.

In the work Sharipov (1998) it was shown that (Onsager-Casimir relations)

$$2\sigma_{r\varphi}^{(P)}(a) = -n_0(a)u_r^{(\Omega)}(a), \qquad q_r^{(P)}(a) = n_0(a)u_r^{(T)}(a), \qquad 2\sigma_{r\varphi}^{(T)}(a) = -q_r^{(\Omega)}(a), \tag{17}$$

where the dimensionless equilibrium density $n_0(r)$ has been introduced

$$n_0(r) = \frac{n_0'(r')}{n_{00}}. \tag{18}$$

Note, that the relations (17) are valid only at the inner cylinder surface, i.e. at $r = a$.

The physical meaning of the coeficients for the cross effects satisfying the relations (17) is the following:

The coefficient $\sigma_{r\varphi}^{(P)}$ describes the momentum transfer between the cylinders caused by the pressure difference $\Delta P$. This coefficient is coupled with the radial mass flux $n_0 u_r^{(\Omega)}$ caused by the angular velocity difference $\Delta\Omega$. Since the stress tensor $\sigma_{r\varphi}^{P}$ and the radial velocity $u_r^{(\Omega)}$ are odd functions of the angular velocity $\omega$, we conclude that in the state of rest ($\omega = 0$) $\sigma_{r\varphi}^{(P)} = 0$ and $u_r^{(\Omega)} = 0$, i.e., the phenomenon exists in rotating systems only.

The coefficient $q_r^{(P)}$ describes the radial heat flux caused by the pressure difference $\Delta P$. It is related to the radial mass flux $n_0 u_r^{(T)}$ caused by the temperature difference $\Delta T$. These phenomena exist in the state of rest too.

The coefficient $\sigma_{r\varphi}^{(T)}$ describes the momentum transfer between the cylinders caused by the temperature difference $\Delta T$. This coefficient is coupled with the radial heat flux $q_r^{(\Omega)}$ caused by the velocity difference $\Delta\Omega$. Since $\sigma_{r\varphi}^{T}$ and $q_r^{(\Omega)}$ are odd functions of $\omega$, these two phenomena exist only in rotating systems. It should be noted that the analogous phenomena exist in polyatomic gases in the presence of magnetic field. Scott *et al.* (1967) showed that there is a torque between two cylinders confining a polyatomic gas if they have different temperatures, i.e. there exist a momentum transfer $\sigma_{r\varphi}^{(T)}$. This effect is coupled with the heat flux between the cylinders if one of them rotates Sharipov (1999). The only difference is that in the phenomena considered here the rotation creates the anisotropy, while in the polyatomic gases the magnetic field plays the same role.

Below, the Onsager-Casimir reciprocity relations will be used as a criterion for the numerical accuracy.

## 3 Kinetic Equation

To solve the problem in question the linearized Boltzmann equation should be applied. This equation provides reliable numerical data but it requires great computational efforts. Since the models of the kinetic theory allow us to reduce essentially the computational efforts, they still continue to be a good tool for practical calculations. The question that remains is: What model equation one should apply to obtain reliable numerical results?

It was shown in the review by Sharipov and Seleznev (1998) that the S-model [Shakhov (1968)] is the most suitable for non-isothermal gas flows because it provides reliable numerical results in the whole range of the gas rarefaction $\delta$.

For a stationary gas flow in cylindrical coordinates $(r', \varphi, z')$ the S-model for a distribution function that does not depend on the coordinates $z'$ and $\varphi$ reads

$$v_r \frac{\partial f}{\partial r'} + \frac{v_\varphi^2}{r'}\frac{\partial f}{\partial v_r} - \frac{v_r v_\varphi}{r'}\frac{\partial f}{\partial v_\varphi} = \frac{P}{\mu}\left\{ f^M \left[1 + \frac{8\beta^4}{15nm}\mathbf{q}' \cdot \mathbf{V}\left(\beta^2 V^2 - \frac{5}{2}\right)\right] - f(r', \mathbf{v})\right\}, \tag{19}$$

where $f^M$ is the local Maxwellian distribution (Maxwell Velocity Distribution)

$$f^M(\mathbf{r}',\mathbf{v}) = n(\mathbf{r}')\left(\frac{m}{2\pi kT(\mathbf{r}')}\right)^{3/2}\exp\left[-\frac{m\mathbf{V}^2}{2kT(\mathbf{r}')}\right], \qquad \mathbf{V}=\mathbf{v}-\mathbf{u}'(\mathbf{r}'), \tag{20}$$

$\mathbf{v}=(v_r, v_\varphi, v_z)$ is the molecular velocity in cylindrical coordinates. The shear viscosity $\mu$ is related to the mean free path $\lambda_{00}$ through

$$\mu = n_{00}\lambda_{00}\left(\frac{2mkT}{\pi}\right)^{1/2}, \tag{21}$$

that corresponds to the molecular model of rigid spheres.

The number density $n$, the bulk fluid velocity $\mathbf{u}'$, the temperature $T$, the heat flux vector $\mathbf{q}'$ and the viscous stress tensor $\sigma'_{ij}$ are defined in terms of the distribution function as

$$n = \int f d\mathbf{v}, \tag{22}$$

$$\mathbf{u}' = \frac{1}{n}\int \mathbf{v} f d\mathbf{v}, \tag{23}$$

$$T = \frac{m}{3nk}\int \mathbf{V}^2 f d\mathbf{v}, \tag{24}$$

$$\mathbf{q}' = \int \frac{1}{2} m\mathbf{V}\,\mathbf{V}^2 f d\mathbf{v}, \tag{25}$$

$$\sigma'_{ij} = P\delta_{ij} - m\int V_i V_j f\, d\mathbf{v}, \qquad P = nkT. \tag{26}$$

We assume a complete condensation of the gas on the cylinder surfaces, i.e. the surfaces condense all incident molecules. The distribution function of molecules evaporated from the inner cylinder ($r'=R_1$ and $v_r>0$) is given by

$$f = \frac{P_1}{kT_1}\left(\frac{m}{2\pi kT_1}\right)^{3/2}\exp\left\{-\frac{m\left[v_r^2+\left(v_\varphi-\left(\Omega_0+\Delta\Omega\right)R_1\right)^2+v_z^2\right]}{2kT_1}\right\}, \tag{27}$$

while the distribution function of the molecules evaporated from the outer cylinder ($r' = R_0$ and $v_r < 0$) reads

$$f = \frac{P_0}{kT_0}\left(\frac{m}{2\pi kT_0}\right)^{3/2}\exp\left\{-\frac{m\left[v_r^2+\left(v_\varphi-\Omega_0 R_0\right)^2+v_z^2\right]}{2kT_0}\right\}. \tag{28}$$

The kinetic equation (19) was linearized and then solved by the discrete velocity method. For more detail one is referred to Cumin *et al.* (1998). The calculations were carried out for the radius ratio $a$=0.5 in the range of $\delta$ from 0.01 to 50 for five values of $\omega$ = 0, 0.25, 0.5, 0.75 and 1.

The numerical error depends on both parameters $\delta$ and $\omega$. For a fixed value of $\delta$ the largest error was observed at $\omega = 1$. That is why the accuracy estimation was made only for this value, since for the smaller value of $\omega$ the numerical error is smaller. Three criteria for numerical accuracy were used:

Table 1: Velocity, heat flux vector and stress tensor caused by the pressure difference at the mid-point ($r = 0.75$) vs $\delta$ and $\omega$.

| $\delta$ | $\omega$ | $u_r^{(P)}$ | $u_\varphi^{(P)}$ | $q_r^{(P)}$ | $q_\varphi^{(P)}$ | $\sigma_{r\varphi}^{(P)}$ | $\sigma_{\varphi\varphi}^{(P)}$ |
|---|---|---|---|---|---|---|---|
| 0.01 | 0 | 0.1881 | 0 | -0.0939 | 0 | 0 | -0.0373 |
| | 0.5 | 0.1739 | -0.0541 | -0.0720 | 0.0479 | 0.0356 | -0.0475 |
| | 1 | 0.1376 | -0.0806 | -0.0236 | 0.0450 | 0.0526 | -0.0624 |
| 1. | 0 | 0.1934 | 0 | -0.0861 | 0 | 0 | -0.0578 |
| | 0.5 | 0.1757 | -0.0788 | -0.0648 | 0.0521 | 0.0333 | -0.0683 |
| | 1 | 0.1342 | -0.1127 | -0.0198 | 0.0500 | 0.0483 | -0.0816 |
| 10. | 0 | 0.2061 | 0 | -0.0438 | 0 | 0 | -0.1271 |
| | 0.5 | 0.1706 | -0.1894 | -0.0318 | 0.0225 | 0.0204 | -0.1296 |
| | 1 | 0.1088 | -0.2311 | -0.0089 | 0.0232 | 0.0559 | -0.1253 |
| 20. | 0 | 0.2078 | 0 | -0.0277 | 0 | 0 | -0.1446 |
| | 0.5 | 0.1606 | -0.2721 | -0.0190 | 0.0121 | 0.0157 | -0.1480 |
| | 1 | 0.0924 | -0.3008 | -0.0050 | 0.0119 | 0.0189 | -0.1426 |
| 50. | 0 | 0.2081 | 0 | -0.0130 | 0 | 0 | -0.1573 |
| | 0.5 | 0.1355 | -0.4585 | -0.0071 | 0.0041 | 0.0112 | -0.1686 |
| | 1 | 0.0647 | -0.4173 | -0.0011 | 0.0035 | 0.0106 | -0.1671 |

(i) Test calculations were carried out for $\delta = 0.1$; 1; 10; 20 and 40 with doubling of every grid parameter.

(ii) The conservation laws of mass, energy and momentum, which are expressed by the equalities

$$n_0 u_r^{(\alpha)} r = \text{const}, \tag{29}$$

$$\left[q_r^{(\alpha)} - n_0 u_r^{(\alpha)} (\omega r)^2\right] r = \text{const}, \tag{30}$$

$$\left(\sigma_{r\varphi}^{(\alpha)} - \omega n_0 u_r^{(\alpha)} r\right)^2 = \text{const}, \tag{31}$$

respectively, were verified.

(iii) The relations (17) were verified.

An analysis of the test calculation, the conservation laws and the reciprocity relations showed that the numerical error does not exceed 1%.

# 4 Numerical Results

## 4.1 Flow caused by the pressure difference

In Table 1 the values of the velocity, heat flux and stress tensor caused by the pressure difference at the mid-point ($r$=0.75) of the cylinders are presented. One can conclude that: i) The radial velocity $u_r^{(P)}$ and the radial heat flux $q_r^{(P)}$ are affected by the rotation significantly; ii) The tangential velocity $u_\varphi^{(P)}$ is negative, i.e. the gas rotates with a smaller angular velocity than the cylinders. This is a consequence of the Coriolis force action; iii) The tangential heat flux $q_\varphi^{(P)}$ and the stress tensor $\sigma_{r\varphi}^{(P)}$ decrease for large values of $\delta$; iv) The stress tensor $\sigma_{\varphi\varphi}^{(P)}$ increases with increasing $\delta$. For the state of rest $\omega$=0 we have that $u_\varphi^{(P)}$, $q_\varphi^{(P)}$ and $\sigma_{r\varphi}^{(P)}$ are zero.

Table 2: Velocity, heat flux vector and stress tensor caused by the velocity difference at the mid-point ($r = 0.75$) vs $\delta$ and $\omega$.

| $\delta$ | $\omega$ | $u_r^{(\Omega)}$ | $u_\varphi^{(\Omega)}$ | $q_r^{(\Omega)}$ | $q_\varphi^{(\Omega)}$ | $\sigma_{r\varphi}^{(\Omega)}$ | $\sigma_{\varphi\varphi}^{(\Omega)}$ |
|---|---|---|---|---|---|---|---|
| 0.01 | 0 | 0 | 0.1112 | 0 | 0 | -0.1253 | 0 |
| | 0.5 | 0.0001 | 0.1107 | -0.00007 | 0.0010 | -0.1139 | 0.0528 |
| | 1 | 0.0001 | 0.1031 | -0.00004 | 0.0127 | -0.0841 | 0.0854 |
| 1. | 0 | 0 | 0.1414 | 0 | 0 | -0.1179 | 0 |
| | 0.5 | 0.0076 | 0.1351 | -0.0058 | 0.0027 | -0.1059 | 0.0556 |
| | 1 | 0.0099 | 0.1179 | -0.0034 | 0.0136 | -0.0763 | 0.0866 |
| 10. | 0 | 0 | 0.2772 | 0 | 0 | -0.0681 | 0 |
| | 0.5 | 0.0446 | 0.2153 | -0.0104 | 0.0055 | -0.0592 | 0.0182 |
| | 1 | 0.0472 | 0.1249 | -0.0333 | 0.0170 | -0.0419 | 0.0291 |
| 20. | 0 | 0 | 0.3220 | 0 | 0 | -0.0443 | 0 |
| | 0.5 | 0.0532 | 0.2230 | -0.0070 | 0.0046 | -0.0377 | -0.0018 |
| | 1 | 0.0543 | 0.1024 | -0.0040 | 0.0107 | -0.0272 | -0.0049 |
| 50. | 0 | 0 | 0.3213 | 0 | 0.0003 | -0.0223 | 0 |
| | 0.5 | 0.0519 | 0.1774 | -0.00284 | 0.00182 | -0.0166 | -0.01961 |
| | 1 | 0.0478 | 0.0342 | -0.0009 | 0.0035 | -0.0118 | -0.0419 |

## 4.2 Flow caused by the velocity difference

In Table 2 the results on the flow fields caused by the velocity difference are presented. An inspection of the results shows that the tangential velocity $u_\varphi^{(\Omega)}$ is weakly affected by the rotation for small values of $\delta$, while for large values of $\delta$ the velocity $u_\varphi^{(\Omega)}$ depends significantly on the angular velocity. Note that $\sigma_{\varphi\varphi}^{(\Omega)}$ changes its sign with increasing $\delta$. The fields $u_r^{(\Omega)}$, $q_r^{(\Omega)}$, $q_\varphi^{(\Omega)}$ and $\sigma_{\varphi\varphi}^{(\Omega)}$ are zero for $\omega$=0.

It would be interesting to compare these results with that obtained by Sharipov & Kremer (1996a), where the same problem was solved with the only difference that there was no evaporation and condensation. Comparing the values of the stress tensor given in Table 2 with that presented in Table I of the paper by Sharipov & Kremer (1996a) one can see that at the small values of the angular velocity the evaporation and condensation does not affect the non-diagonal term of the tensor $\sigma_{r\varphi}$, while at the large values of $\omega$ this term essentially differs from its value without evaporation. The diagonal term $\sigma_{\varphi\varphi}$ is also strongly affected by the evaporation at the large angular velocity $\omega$.

## 4.3 Flow caused by the temperature difference

In Table 3 the results on the flow fields caused by the temperature difference are given. From this table we conclude that $q_r^{(T)}$ and $u_r^{(T)}$ decrease with increasing $\omega$ and $\delta$, while $\sigma_{\varphi\varphi}^{(T)}$ increases with $\omega$. The tangential velocity $u_\varphi^{(T)}$ is positive, i.e. the gas rotates more rapidly than the cylinders. It is observed that for $\omega$=0 the values of the quantity $u_\varphi^{(T)}$, $q_\varphi^{(T)}$ and $\sigma_{r\varphi}^{(T)}$ are zero.

These results should be compared with that obtained in the paper by Sharipov & Kremer (1995b), where the same situation was considered without the evaporation and condensation. Comparing the values of the heat flux vector $q_r$ and $q_\varphi$ given in Table 3

Table 3: Velocity, heat flux vector and stress tensor caused by the temperature difference at the mid-point ($r = 0.75$) vs $\delta$ and $\omega$.

| $\delta$ | $\omega$ | $u_r^{(T)}$ | $u_\varphi^{(T)}$ | $q_r^{(T)}$ | $q_\varphi^{(T)}$ | $\sigma_{r\varphi}^{(T)}$ | $\sigma_{\varphi\varphi}^{(T)}$ |
|---|---|---|---|---|---|---|---|
| 0.01 | 0 | -0.0939 | 0 | 0.4227 | 0 | 0 | 0.0001 |
| | 0.5 | -0.0869 | 0.0530 | 0.3777 | -0.1262 | -0.0178 | 0.0134 |
| | 1 | -0.0682 | 0.0743 | 0.2642 | -0.1521 | -0.0263 | 0.0340 |
| 1. | 0 | -0.0861 | 0 | 0.3817 | 0 | 0 | 0.0083 |
| | 0.5 | -0.0796 | 0.0652 | 0.3456 | -0.1175 | -0.0142 | 0.0186 |
| | 1 | -0.0635 | 0.0915 | 0.2500 | -0.1497 | -0.0222 | 0.0357 |
| 10. | 0 | -0.0438 | 0 | 0.1989 | 0 | 0 | 0.0064 |
| | 0.5 | -0.0457 | 0.0613 | 0.1924 | -0.0299 | -0.0048 | 0.0083 |
| | 1 | -0.0437 | 0.1106 | 0.1651 | -0.0534 | -0.0101 | 0.0147 |
| 20. | 0 | -0.0278 | 0 | 0.1288 | 0 | 0 | 0.0026 |
| | 0.5 | -0.0320 | 0.0591 | 0.1269 | -0.0115 | -0.0028 | 0.0047 |
| | 1 | -0.0344 | 0.1204 | 0.1160 | -0.0225 | -0.0066 | 0.0114 |
| 50. | 0 | -0.0129 | 0 | 0.0613 | 0 | 0 | 0.0006 |
| | 0.5 | -0.0177 | 0.0649 | 0.0612 | -0.0024 | -0.0013 | 0.0037 |
| | 1 | -0.0213 | 0.1442 | 0.0590 | -0.0051 | -0.0033 | 0.0137 |

with that given in Table 3 of the paper by Sharipov & Kremer (1995b) one can see that the radial heat flux $q_r$ increases up to 10%, while the values of the tangential heat flux $q_\varphi$ increase up to 25% of their corresponding values without evaporation and condensation.

# 5 Conclusions

The mass, heat and momentum transfer through a rarefied gas confined between two rotating cylinders has been calculated numerically in a wide range of the gas rarefaction and of the angular velocity. Three types of the equilibrium perturbation have been considered: the difference between the pressures of evaporated and condensed particles on the inner cylinder, the temperature difference between the cylinders and the difference of the angular velocities of the cylinders. Numerical calculations based on the S model of the Boltzmann equation have been carried out. The fields of the bulk velocity, heat flux vector, shear stress, pressure and temperature have been calculated. It has been showed that the numerical results satisfy the reciprocity relations within the numerical error. Some of the present results have been compared with that obtained earlier without the consideration of the evaporation and condensation. The range of the parameters, where the evaporation and condensation affect the flowfield, has been indicated.

**REFERENCES**

Cumin, L. M. G., Sharipov, F. and Kremer, G. M., 1998, Rarefied gas flow between two cylinders caused by the evaporation and condensation on their surfaces, Phys. Fluids, vol. 10, p. 3203.

Scott, G. G., Sturner, H. W. and Williamson, R. M., 1967, Gas torque effect for molecular combinations of hydrogen and deuterium, Phys. Lett., vol. 25A, p. 573.

Shakhov, E. M., 1968, Generalization of the Krook Kinetic Equation, Fluid Dynamics, vol. 3, p. 95.

Sharipov, F. and Kremer, G. M., 1995a, On the frame dependence of constitutive equations. I. Heat transfer through a rarefied gas between two rotating cylinders, Continuum Mech. Thermodyn., vol. 7, p. 57 .

Sharipov, F. and Kremer, G. M., 1995b, Heat Conduction through a Rarefied Gas between two Rotating Cylinders at Small Temperature Difference, ZAMP, vol. 46, p. 680.

Sharipov, F. and Kremer, G. M., 1996a, Linear Couette Flow between two Rotating Cylinders, Eur. J. Mech., B/Fluids, vol. 15, p. 493.

Sharipov, F. and Kremer, G. M., 1996b, Nonlinear Couette flow between two rotating cylinders, Transport Theory Stat. Phys., vol. 25, p. 217.

Sharipov, F. and Kremer, G. M., 1999, Non-isothermal Couette flow of a rarefied gas between two rotating cylinders, Eur. J. Mech. B/Fluids, vol. 18, p. 121.

Sharipov, F., 1998, Onsager-Casimir Reciprocity Relations for Open Gaseous Systems at the Arbitrary Rarefaction. IV. Rotating Systems, Physica A, vol. 260, p. 499.

Sharipov, F., 1999, Onsager-Casimir reciprocity relation for gyrothermal effect with polyatomic gases, Phys. Rev. E, vol. 59, p. 5128.

Sharipov, F. and Seleznev V., 1998, Data on Internal Rarefied Gas Flows, J. Phys. Chem. Ref. Data, vol. 27, p. 657.

Sone, Y., Takata, S. and Sugimoto, H., 1996, The behavior of a gas in the continuum limit in the light of kinetic theory: The case of cylindrical Couette flows with evaporation and condensation, Phys. Fluids, vol. 8, p. 3403.

Sone, Y., Sugimoto, H. and Aoki, K., 1999, Cylindrical Couette flows of a rarefied gas with evaporation and condensation: Reversal and bifurcation of flows, Phys. Fluids, vol. 11, p. 476.

Sugimoto, H. and Sone, Y., 1992, Numerical analysis of steady flows of a gas evaporating from its cylindrical condensed phase on the basis of kinetic theory, Phys. Fluids A, vol. 4, p. 419.