FORCED CONVECTION HEAT TRANSFER PROBLEM FOR LIQUID FLOW IN TUBES WITH TEMPERATURE DEPENDENT VISCOSITY VARIATIONS

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Abstract. Forced convection heat transfer problem for liquid flow in circular tubes is studied. The solution methodology follows the hybrid analytical-numerical Generalized Integral Transform Technique (GITT). A fully developed velocity profile at tube inlet is considered in the presence of a thermally developing temperature profile. Moreover, thermophysical properties are held constant while dynamic viscosity is allowed to vary.

Keywords: Internal forced convection, Variable viscosity

1. INTRODUCTION

Diffusion problems with non-linear characteristics are generally found within the context of variable thermophysical properties. Usually, these variations are associated to intense temperature gradients within the medium such as the non-linear heat diffusion in solids. A similar situation can be found when the forced internal convection of liquids in considered. Here, the viscosity variation with respect to the fluid temperature strongly influences the velocity and temperature profiles sometimes presenting meaningful deviations from the standard constant property problem. It seems that Yang (1962) was one of the earlier researchers to acknowledge these differences when studying the laminar forced convection of liquids with temperature dependent viscosity. In his work, an analytical solution is sought through an improved integral method and consequently, these results are dependent upon the degree of the polynomial approximation for the temperature field. In order to draw comparisons with other published results, the fluid viscosity follows an inverse linear relation with the temperature field and problems associated to step changes in tube wall temperature and in wall heat flux are studied. Various results, such as the Nusselt numbers and friction factor, are presented for both the heating and cooling situations and, as expected, they indicate that both the velocity and temperature distributions are affected by the variable viscosity coefficient.

Test (1968) also considers the laminar flow heat transfer and fluid flow in forced internal convection problems. This problem is modeled with a more robust formulation than that of Yang and a finite difference scheme is employed to solve the continuity, axial direction momentum and energy equations. Moreover, data from an experimental setup is used to establish comparisons with the numerical results. In this study, the viscosity coefficient is taken to vary in an exponentially fashion with respect to the temperature. Among other findings, the research mentioned above shows that the effect of wall temperature on local

Nusselt number is not relevant when the fluid is heated but is very pronounced for the case of cooling. Also, expressions for the local Nusselt number and for the friction factor are established from the numerical simulations and compared to others obtained from purely experimental correlations.

Herwig and his collaborators investigated the effect of variable viscosity in channel flow under both the prescribed temperature (Herwig,1989) and flux (Herwig,1985,1990) conditions. A common trait to all these works is that the viscosity varies linearly with respect to the temperature and first order perturbation techniques are employed to solve the governing equations. The required zero order solution is that associated to constant viscosity and these problems are modeled either by a velocity – enthalpy or a stream function – vorticity formulation. Due to the very nature of the chosen solution procedure, these results should be interpreted with care especially in regions close to the tube inlet. In fact, the majority of the reported results center around asymptotic Nusselt numbers and friction coefficients. Nevertheless, comparisons with previously published data show that these predictions are accurate for low and moderate Prandtl numbers.

Therefore, this contribution seeks to further explore the internal forced convection problem in situations where a temperature dependent viscosity coefficient is taken into account. The formulation reported by Yang (1962) is chosen due to its simplicity and these governing equations are solved using the Generalized Integral Transform Technique (Cotta, 1993). This extremely reliable methodology has been used for other type of linear and non-linear diffusion problems (Cotta, 1998) and its application to this research is expected to clarify some aspects of the whole hydrodynamic and thermal entry regions for the heating and cooling situations.

2. ANALYSIS

Liquid flowing inside a semi-infinite tube is considered. The walls are subjected to a prescribed temperature T_w while, at the tube inlet, the temperature profile T_e is constant and the velocity profile is fully developed.

2.1 Problem formulation

After considering the above simplifications as well as the formulation proposed by Yang (1962), the governing equations, in dimensional form, are given by:

Momentum equation in the axial direction:

$$0 = -\frac{dp(x)}{dx} + \frac{1}{r}\frac{\partial}{\partial r}\left(r.\mu^*(T).\frac{\partial u(x,r)}{\partial r} \right) \tag{1}$$

Energy equation:

$$u(x,r)\frac{\partial T(x,r)}{\partial x} = \frac{\alpha}{r}.\frac{\partial}{\partial r}\left(r\frac{\partial T(x,r)}{\partial r} \right) \tag{2}$$

The initial and boundary conditions are:

$$u(0,r) = -\frac{1}{4}.\frac{\left.\frac{dp}{dx}\right|_{x=0}}{\mu^*(T_e)}(a^2 - r^2); \qquad T(0,r) = T_e \qquad 0 < r < a \tag{3}$$

$$\frac{\partial u}{\partial r}(x,0)=0; \quad \frac{\partial T}{\partial r}(x,0)=0 \qquad x>0$$

$$u(x,a)=0; \qquad T(x,a)=T_w \qquad x>0 \tag{4}$$

By taking into account the problem symmetry and by integrating the momentum equation (1) twice with respect to the radial variable *r*, and by utilizing the average velocity formulation (Shah & London, 1978), the following expression for the axial velocity in dimensionless form is obtained

$$U(X,R)=\frac{1}{2}.\left[\int_R^1 \frac{R'}{\mu(\theta)}dR' \Big/ \int_0^1 R.\left(\int_R^1 \frac{R'}{\mu(\theta)}dR'\right)dR\right] \tag{5}$$

where various dimensionless variables are defined as:

$$R=\frac{r}{a}; \quad X=\frac{\alpha.x}{a^2.u_M}; \quad U(X,R)=\frac{u(x,r)}{u_M};$$

$$\theta(X,R)=\frac{T(x,r)-T_w}{T_e-T_w}; \quad \mu(\theta)=\frac{\mu^*(T)}{\mu_w^*} \tag{6}$$

As far as viscosity dependence on temperature is concerned, the following expression (Yang, 1962) is adopted :

$$\mu(\theta)=\frac{1}{1+\gamma.\theta(X,R)} \tag{7}$$

Velocity profile (5) is rewritten in terms of the above expression as:

$$U(X,R)=\frac{1}{2}\left[\int_R^1 R'(1+\gamma.\theta).dR' \Big/ \int_0^1 R.\left(\int_R^1 R'(1+\gamma.\theta).dR'\right)dR\right] \tag{8}$$

where γ is a viscosity parameter which depends on the fluid, its value being negative for heating and positive for cooling.

Now, governing equations (1) and (2), in dimensionless form, are reduced to a single equation for the temperature potential $\theta(X,R)$:

$$\frac{1}{2}\left[\int_R^1 R'(1+\gamma.\theta).dR' \Big/ \int_0^1 R.\left(\int_R^1 R'(1+\gamma.\theta).dR'\right)dR\right].\frac{\partial\theta(X,R)}{\partial X}=\frac{1}{R}.\frac{\partial}{\partial R}\left(R.\frac{\partial\theta(X,R)}{\partial R}\right) \tag{9}$$

Subjected to the following initial and boundary conditions:

$$\theta(0,R)=1 \qquad 0<R<1 \tag{10}$$

$$\frac{\partial\theta}{\partial R}(X,0)=0 \quad ;\theta(X,1)=0 \qquad X>0 \tag{11}$$

2.2 Solution through integral transform (GITT)

Definition of an Auxiliary Eigenvalue Problem

Following the ideas in the Generalized Integral Transform Technique, an auxiliary eigenvalue problem must be chosen in such a manner to contain the maximum amount of information on the diffusive operator from the original problem, as well as on the boundary conditions. Therefore, the following Sturm-Liouville problem is chosen:

$$\frac{d}{dR}\left(R\frac{d\psi_i(R)}{dR}\right)+R.\beta_i^2.\psi_i(R)=0,\ 0<R<1 \tag{12}$$

$$\left.\frac{d\psi_i}{dR}\right|_{R=0}=0,\ \psi_i(1)=0 \tag{13}$$

Solution to the above problem furnishes the following eigenvalues, eigenfunctions and norm, in terms of the zero order Bessel function $J_0(R)$ as:

Eigenvalues β_i: positive roots of $J_o(\beta_i)=0$ i = 1,2,3,....

Eigenfunctions: $\psi_i(R)=J_o(\beta_i R)$,i = 1,2,3,.... (14)

Norm: $N_i=\frac{J_1^2(\beta_i)}{2}$ i = 1,2,3,....

The Transform – Inversion pair

By using the orthogonality property for a regular Sturm-Liouville problem (Boyce & Diprima, 1992), it is a simple matter to show that the transform-inversion pair for the problem here considered, Mikhailov and Ozisik (1993), is:

$$\bar{\theta}_i(X)=\frac{1}{N_i^{1/2}}\int_0^1 R.\psi_i(R).\theta(X,R).dR \quad \text{(Transform)} \tag{15}$$

$$\theta(X,R)=\sum_{i=1}^{\infty}\frac{1}{N_i^{1/2}}.\psi_i(R).\bar{\theta}_i(X) \quad \text{(Inversion)} \tag{16}$$

System of Ordinary Differential Equations

The main objective in the methodology here adopted consists in the integral transformation of the original problem. In other words, the system described by Eq. (9) – (11) shall be rewritten in terms of the transformed temperature potentials. As a result, a system of ordinary differential equations will be obtained for the determination of the unknown transformed potentials. In order to accomplish this task, several mathematical operations need to be performed and will be briefly discussed below:

Equation (9) is integrated with respect to the radial coordinate in the interval ranging from 0 to R and, again, from R to 1, with the aid of boundary conditions (11) , resulting in:

$$\theta(X,R) = -\int_R^1 \frac{1}{R``}\left(\int_0^{R``} R'.U(X,R).\frac{\partial\theta(X,R)}{\partial X}.dR'\right).dR`` \tag{17}$$

Following, Eq. (17) is substituted in Eq. (15), to yield:

$$\bar{\theta}_i(X) = \frac{1}{N_i^{1/2}}\int_0^1 R\psi_i(R).\left[-\int_R^1 \frac{1}{R``}.\left(\int_0^{R``} R'.U(X,R)\frac{\partial\theta(X,R)}{\partial X}.dR'\right)dR``\right]dR \tag{18}$$

Then, Eq. (8) is substituted into Eq. (18) and integration by parts are performed, yielding:

$$\beta_i^2\,\bar{\theta}_i(X) =$$
$$-\frac{1}{2.N_i^{1/2}}\int_0^1 R\psi_i(R)\left[\int_R^1 R'.[1+\gamma.\theta(X,R)].dR' \Big/ \int_0^1 R'''\left(\int_{R'''}^1 R'.[1+\gamma.\theta(X,R)].dR'\right)dR'''\right]\frac{\partial\theta(X,R)}{\partial X}dR \tag{19}$$

Next, Inversion formulae, Eq. (16) is substituted into Eq. (19) followed by several algebraic manipulations to finally yield:

$$-2.\beta_i^2\,\bar{\theta}_i(X) =$$
$$\sum_{j=1}^{\infty}\frac{d\bar{\theta}_j(X)}{dX}\frac{1}{(N_i.N_j)^{1/2}}\int_0^1 R\left[\frac{1-R^2}{2}+\gamma\sum_{k=1}^{\infty}F_k(R).\bar{\theta}_k(X)\Big/\frac{1}{8}+\gamma\sum_{k=1}^{\infty}M_k.\bar{\theta}_k(X)\right]\psi_i(R).\psi_j(R).dR \tag{20}$$

where,

$$F_k(R) = \frac{1}{N_k^{1/2}}.\int_R^1 R'\psi_k(R').dR' \tag{21}$$

$$M_k = \int_0^1 R.F_k(R).dR \tag{22}$$

At this point, well established numerical integration schemes, such as the DIVPAG from IMSL subroutines library, are employed to solve a truncated version of the denumerable system represented by Eq. (20) together with its initial conditions. Once this problem is solved, we invoke the inversion formulae, Eq. (16), to obtain the desired original potential . The truncated version of Eq. (20) and its transformed initial conditions are given by:

$$\begin{cases} [A]\left[\dfrac{d\bar{\theta}(X)}{dX}\right] = -[B].\left[\bar{\theta}(X)\right] \\ \bar{\theta}_i(0) = F_i(0) \end{cases} \tag{23}$$

where,

$$[A] = \{a_{i,j}\}; \qquad a_{i,j} = \sum_{j=1}^{N}\left[I_{ij} + \gamma\sum_{k=1}^{N} L_{ijk}.\bar{\theta}_k(X)\right] \tag{24}$$

$$[B]=\{b_{i,j}\}; \quad b_{i,j}=\sum_{j=1}^{N}\Delta_{ij}.\beta_i^2.\left[\frac{1}{4}+2.\gamma.\sum_{k=1}^{N} M_k.\bar{\theta}_k(X)\right] \tag{25}$$

$$I_{ij}=\frac{1}{(N_i N_j)^{1/2}}.\int_0^1 \frac{(R-R^3)}{2}.\psi_i(R).\psi_j(R).dR \tag{26}$$

$$L_{ijk}=\frac{1}{(N_i N_j)^{1/2}}.\int_0^1 R.F_k(R).\psi_i(R).\psi_j(R).dR \tag{27}$$

Determination of Axial Velocity U(X,Y)

After substitution for the inversion formulae, Eq. (16), Eq. (8) becomes the following expression for the determination of axial velocity *U(X,R)* in terms of the transformed potentials, which are in their turn, obtained from the numerical solution of Eq.(23), as described above:

$$U(X,R)=\frac{1}{2}.\left(\frac{1-R^2}{2}+\gamma\sum_{k=1}^{N} F_k(R).\bar{\theta}_k(X)\right)\Bigg/\left(\frac{1}{8}+\gamma\sum_{k=1}^{N} M_k.\bar{\theta}_k(X)\right)=$$
$$\frac{1}{2}.\left\{\left[\frac{1-R^2}{2}+\gamma\sum_{k=1}^{N}\frac{\bar{\theta}_k(X)}{\beta_k.N_k^{1/2}}\left[J_1(\beta_k)-R.J_1(\beta_k R)\right]\right]\Bigg/\left[\frac{1}{8}+\gamma\sum_{k=1}^{N}\frac{\bar{\theta}_k(X)}{\beta_k.N_k^{1/2}}\left[\frac{J_1(\beta_k)}{2}-\frac{J_2(\beta_k)}{\beta_k}\right]\right]\right\} \tag{28}$$

Determination of Average Temperature

From the definition of average temperature (Shah & London, 1978), the following expression is obtained in terms of the transformed variables:

$$\theta_M(X)=\sum_{j=1}^{N}\left\{\left[I_j^*.\bar{\theta}_j(X)+\gamma\sum_{k=1}^{N} L_{jk}^*.\bar{\theta}_j(X).\bar{\theta}_k(X)\right]\Bigg/\left[\frac{1}{8}+\gamma\sum_{k=1}^{N} M_k.\bar{\theta}_k(X)\right]\right\} \tag{29}$$

where,

$$I_j^*=\frac{1}{N_j^{1/2}}.\int_0^1 \frac{(R-R^3)}{2}.\psi_j(R).dR \tag{30}$$

$$L_{jk}^*=\frac{1}{N_j^{1/2}}.\int_0^1 R.F_k(R).\psi_j(R).dR \tag{31}$$

Determination of Local Nusselt Number

The local Nusselt number for internal tube flow is defined as (Shah & London, 1978):

$$Nu(X)=\frac{2\left.\frac{\partial\theta(X,R)}{\partial R}\right|_{R=1}}{\theta(X,1)-\theta_M(X)} \tag{32}$$

where,

$$\left.\frac{\partial\theta(X,R)}{\partial R}\right|_{R=1}=\sum_{j=1}^{N}\frac{\psi_j^{'}(1).\bar{\theta}_j(X)}{N_j^{1/2}}=-\sum_{j=1}^{N}\frac{\beta_j.J_1(\beta_j).\bar{\theta}_j(X)}{N_j^{1/2}} \tag{33}$$

$$\theta(X,1)=0 \tag{34}$$

3. RESULTS AND DISCUSSION

In order to study the effects of viscosity variation on liquid flow in tubes with fully developed velocity profile at inlet and developing temperature profile, subjected to a prescribed temperature at the tube wall, numerical simulations were carried based on the model originally proposed by Yang (1962) and following the GITT approach.

The numerical code was written in FORTRAN, and the computations were performed in a PC PENTIUM 200 MMX. Also, the eigenvalues and related eigenquantities were efficiently calculated by the "Sign-Count Method" (Mikhailov and Ozisik, 1993) providing as many expansion terms, N, in Eq. (23) as desired.

The solution of the O.D.E.'s system (23) was obtained through the subroutine DIVPAG from IMSL library, which performs an automatic local error control with user prescribed tolerance, and is suitable for the solution of stiff O.D.E.'s systems. For all cases studied a 10^{-6} precision was imposed. A convergence of at least 3 digits was obtained when truncating the series in Eq. (23) at N =30 terms. For the sake of simulation, values of γ equals to 9.0 , 6.0 and 3.0 were selected for the case of cooling the liquid while the values –0.9 , -0.6 and –0.3 were chosen for the case of heating.

Figures 1 and 2 show, respectively , the evolution of velocity profile $U(X,R)$ along axial position X , for the cases of fluid heating and cooling. It is noticed that, by heating the liquid, the accompanying viscosity variation flattens the initially parabolic velocity profile, especially at the entrance region.

For the case of cooling, viscosity variation extends the velocity profile. In the asymptotic region, the velocity profile recovers its parabolic shape.

Figures 3 and 4 present axial velocity profile evolution at the center line in the X direction for the cases of liquid heating and cooling, respectively. From these figures one observes that the higher the absolute value for the viscosity parameter, the higher the deviation of axial velocity with respect to the fully developed condition (γ=0). Also, entry length increases proportionally to the increase of the absolute value of parameter γ.

Figures 5 and 6 depict local Nusselt number behavior for the cases of heating and cooling, respectively. For heating, Fig. 5 shows that, for a fixed position X, as γ decrease in absolute value the local Nusselt number decreases. The heating of the liquid causes the decrease in the value of viscosity near the tube wall, thus accelerating the flow in this region when compared to the isothermal situation (γ = 0). The higher the absolute value of γ, the higher the influence of viscosity on Nusselt number and flow velocity. Its also noticeable from this figure the asymptotic behavior of Nusselt number, which tends to the classical value of 3.657 reported in the literature (Shah & London, 1978).

Thermal entry length is also affected by viscosity variation, but in a weaker fashion than that for the case of axial velocity profile. The variation of viscosity, as measured by γ, progressively extends the thermal entry length.

For the situation when the liquid is being cooled, Fig. 6 illustrates the behavior of local Nusselt number as a result of variations in viscosity parameter γ. Such a behavior is similar to the one encountered in the case of heating discussed above, where the decrease in Nusselt number follows the decrease in the absolute value of parameter γ. It is also noticeable the attainment of the asymptotic value 3.657.

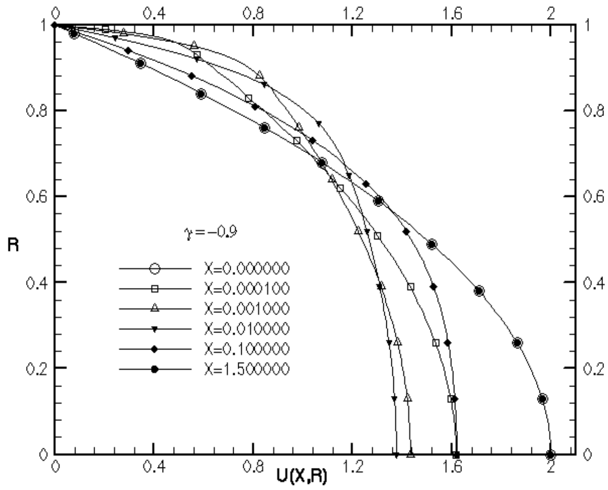

Figure 1: Axial velocity profile evolution – fluid heating

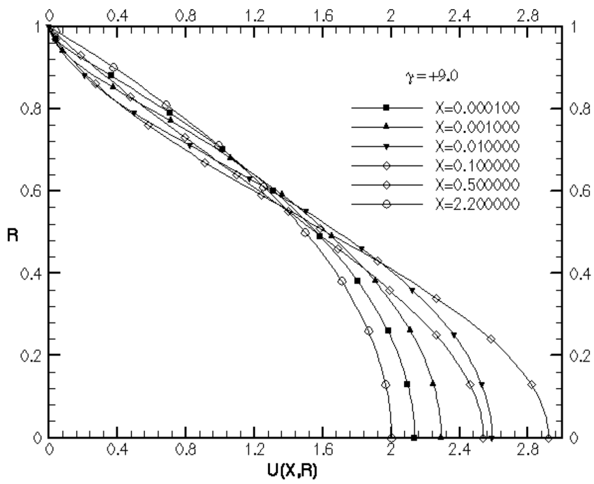

Figure 2: Axial velocity profile evolution – fluid cooling

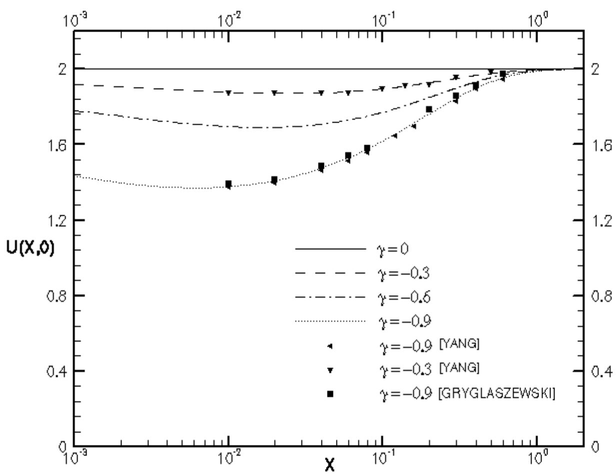

Figure 3: Axial velocity at the centerline - fluid heating

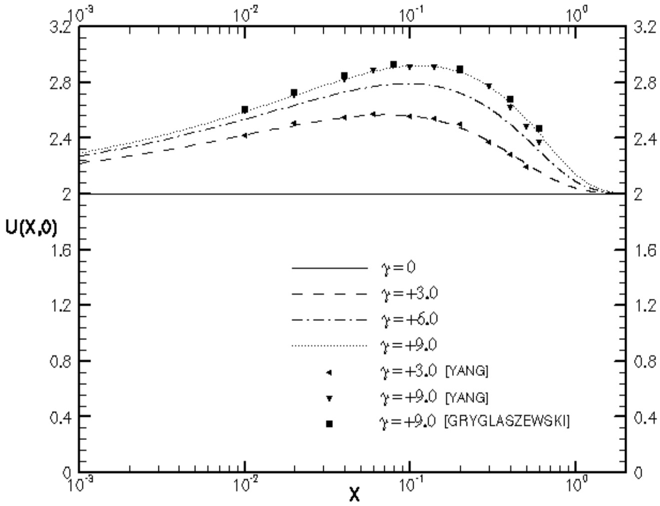

Figure 4: Axial velocity at the centerline - fluid cooling

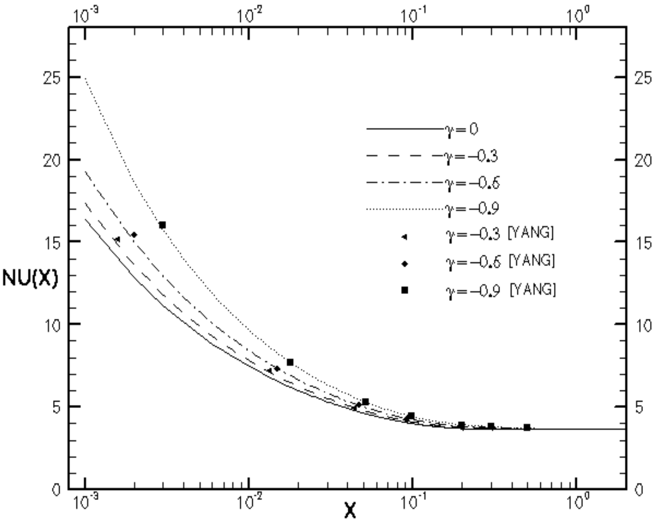

Figure 5: Local Nusselt number variation – fluid heating

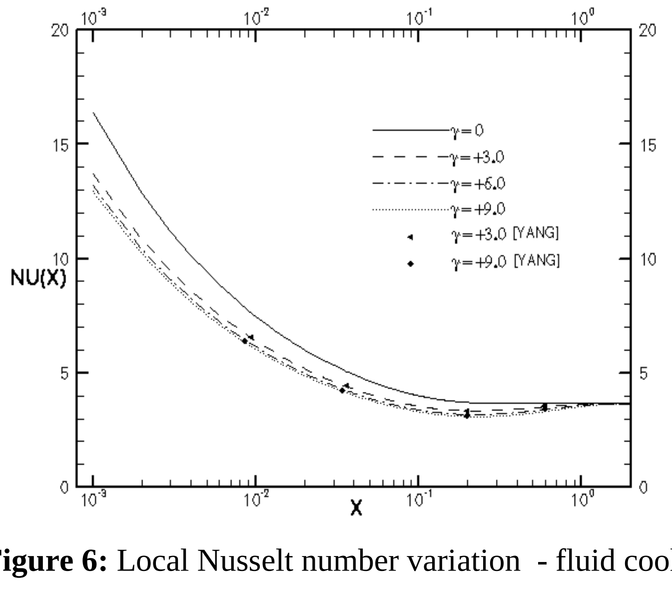

Figure 6: Local Nusselt number variation - fluid cooling

The analysis of the figures shows that, for the case of heating, Nusselt number decreases monotonically, while for the case of cooling it reaches a minimum value and then increases toward the asymptotic value. This characteristic is due to the influence of viscosity variation specially in the neighborhood of the tube walls, which alters the velocity in this region and consequently propagates this effect up to the centerline. This modification in velocity profile differs from heating to cooling situations.

The results displayed in figures 3 to 6 show good agreement when compared with the previously reported contributions of Yang (1962) and Gryglaszewski et al. (1980).

As a conclusion, the GITT approach has been successfully extended to handle internal forced convection problems of non-linear nature. The investigation has produced viscosity dependent hydrodynamic and thermal entry solutions for both cases of cooling and heating the liquid flowing in the tube. Results are in good agreement with the literature.

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