INTERFACE SHAPE IN VERTICAL ANNULAR LIQUID-LIQUID FLOW

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Abstract. *The annular flow pattern formed by two immiscible liquids of very different viscosities (also called 'core annular flow') has found important applications in the transportation of heavy oils in horizontal pipes, through the addition of small quantities of a thinner fluid (usually water). Here the vertical flow is focused, in view of its possible application in heavy oil production. Including the interfacial tension and neglecting inertial terms in the annulus, equations are derived which govern the shape of the liquid-liquid interface. First, it is shown that the interface must be circular in the pipe cross section, as expected. Besides, the condition that pressure is continuous on each side of the interface leads to the conclusion that the interface profile generally presents axially symmetric waves, whose shape is governed by the Laplace-Young equation. Its solution reveals interface shapes which are entirely compatible with the "bamboo waves" observed by Bai, Kelkar & Joseph (1996) for upward flow. A simple model for wavelength prediction is proposed, which is in reasonably good agreement with presently available data.*

***Keywords**: Multiphase flow, Liquid-liquid flow, Interface shape, Vertical pipe, Heavy oil production*

1. INTRODUCTION

The production of heavy oil is a big problem worldwide due to the great technological difficulties associated with handling highly viscous crudes. In the Brazilian deep water scenario, these difficulties are still more dramatic in view of the low temperature at the sea bottom.

To facilitate the flow of thick oils through the well, the oil viscosity is usually decreased through the addition of light diluents, dispersants or heat. The former case demands the existence of light oil in the same production area, while the later is relatively expensive.

A new alternative for lifting heavy oils (above 10^2 cP) is based upon the great accumulated experience in the transport of highly viscous fluids by injection of small amounts of water, in such a way to create an appropriate lubrication of the oil and to establish an annular pattern of liquid-liquid flow called 'core flow' or 'core-annular flow'. In fact, this flow pattern is shown to require the smallest pumping power (Oliemans, 1986), since the highly viscous oil flows in the center and is surrounded by a water ring close to the tube wall. Laboratory experiments (Prada, 1999) with a very viscous oil (17,600 cP) and water, confirmed that the pressure drop in vertical core flow is comparable with the expected for single phase water flow at mixture volumetric flow rate. Besides, the thin water annulus requires injection of a small amount of water, thus making the core flow pattern an attractive alternative for the lift of heavy crudes. However, this idea has not yet been tested in real production apparatus.

Ooms *et al.* (1984) proposed the use of the lubrication theory for the pressure gradients determination in horizontal core annular flow. The theory assumes that the core moves at a single velocity and inertial terms can be neglected. Using a previously imposed periodic interface shape, they solve the resulting Reynolds differential equation of the lubrication.

Bai, Kelkar & Joseph (1996) took photographs and recorded movies of the interfacial waves, observing the axisymmetry of the flow. They also measured the oil volume fraction, interfacial wavelength and pressure drop. Based on data for their system, Bannwart (1998) proposed a method of indirect measure of the oil fraction through the measure of the speed of the interfacial waves.

The present work focus on the hydrodynamics of ascending vertical oil-water flow in the core-annular pattern, for possible application in the production of heavy crudes. Taking into account the interfacial tension (i.e. the liquid-liquid surface tension) and making use of common assumptions of small annulus-to-core viscosity ratio and thin water ring, we derive equations governing the shape of the liquid-liquid interface.

2. PROBLEM FORMULATION AND ANALYSIS

Considering the general flow picture shown in Fig. 1 the following assumptions are made: a) laminar incompressible isothermal flow of Newtonian fluids; b) the fluid in the center (phase 1) is much thicker than the annulus (phase 2); c) thin annulus: $R - r_i << R$. A cylindrical coordinate system (r,θ,z) with velocity components (u,v,w) respectively is taken.

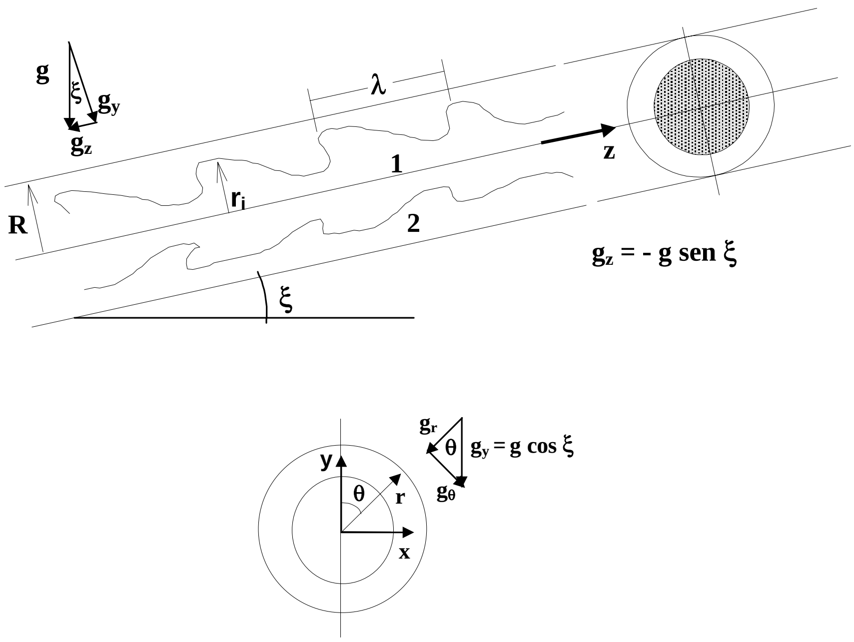

Figure 1 – Problem geometry

From assumption b), fluid 1 can be considered as moving axially at a single velocity W and pressure can be considered hydrostatic over this phase. Then, for a referential moving with fluid 1, we have $u_1 = v_1 = w_1 = 0$ and the annulus flow is steady over a wavelength λ. From assumption c), it can be concluded from magnitude analysis that the radial velocity component of the annulus flow is much smaller than the other components, i.e. $u_2 << v_2$ and $u_2 << w_2$. Under these conditions the Navier-Stokes equations for each phase become:

- *fluid 1:*

$$-\frac{\partial p_1}{\partial r} + \rho_1 g_r = 0 \tag{1}$$

$$-\frac{1}{r}\frac{\partial p_1}{\partial \theta} + \rho_1 g_\theta = 0 \tag{2}$$

$$-\frac{\partial p_1}{\partial z} + \rho_1 g_z = 0 \tag{3}$$

- *fluid 2:*

$$\frac{\partial u_2}{\partial r} + \frac{1}{r}\frac{\partial v_2}{\partial \theta} + \frac{\partial w_2}{\partial z} = 0 \tag{4}$$

$$-\frac{\partial p_2}{\partial r} + \rho_2 g_r = 0 \tag{5}$$

$$\rho_2\left(u_2\frac{\partial v_2}{\partial r} + \frac{v_2}{r}\frac{\partial v_2}{\partial \theta} + w_2\frac{\partial v_2}{\partial z}\right) = -\frac{1}{r}\frac{\partial p_2}{\partial \theta} + \rho_2 g_\theta + \mu_2\frac{\partial^2 v_2}{\partial r^2} \tag{6}$$

$$\rho_2\left(u_2\frac{\partial w_2}{\partial r} + \frac{v_2}{r}\frac{\partial w_2}{\partial \theta} + w_2\frac{\partial w_2}{\partial z}\right) = -\frac{\partial p_2}{\partial z} + \rho_2 g_z + \mu_2\frac{\partial^2 w_2}{\partial r^2} \tag{7}$$

where $g_r = - g_y \cos\theta$, $g_\theta = g_y \sin\theta$, $g_y = g \cos\xi$, $g_z = - g \sin\xi$. Note that w_2 represents the axial velocity component of phase 2 relative to phase 1. From Eq. (5), the relation between pressures p_2 and p_{2i} (pressure of fluid 2 at the interface) is

$$p_2 = p_{2i} + \rho_2 g_y (r_i - r)\cos\theta \cong p_{2i} \tag{8}$$

The jump condition relating pressures at the interface (Laplace-Young law) is

$$p_{1i} - p_{2i} = 2\frac{\sigma}{R_i} \tag{9}$$

where σ is the interfacial tension and R_i is the radius of curvature of the interface (not to be confused with r_i , which represents the radial position of the interface). Using Eqs. (8-9) and (1-3) to eliminate pressure gradients in Eqs. (6-7) and since $r \cong r_i$ in the annulus, leads to

$$\frac{\partial u_2}{\partial r}+\frac{1}{r_i}\frac{\partial v_2}{\partial \theta}+\frac{\partial w_2}{\partial z}=0 \tag{10}$$

$$\rho_2\left(u_2\frac{\partial v_2}{\partial r}+\frac{v_2}{r_i}\frac{\partial v_2}{\partial \theta}+w_2\frac{\partial v_2}{\partial z}\right)=(\rho_2-\rho_1)g_y\mathrm{sen}\theta+\frac{2\sigma}{r_i}\frac{\partial(1/R_i)}{\partial \theta}+\mu_2\frac{\partial^2 v_2}{\partial r^2} \tag{11}$$

$$\rho_2\left(u_2\frac{\partial w_2}{\partial r}+\frac{v_2}{r_i}\frac{\partial w_2}{\partial \theta}+w_2\frac{\partial w_2}{\partial z}\right)=(\rho_2-\rho_1)g_Z+2\sigma\frac{\partial(1/R_i)}{\partial z}+\mu_2\frac{\partial^2 w_2}{\partial r^2} \tag{12}$$

For vertical upward flow, taking $v_2 = 0$, Eqs. (10-12) become

$$\frac{\partial u_2}{\partial r}+\frac{\partial w_2}{\partial z}=0 \tag{13}$$

$$\frac{\partial(2/R_i)}{\partial \theta}=0 \tag{14}$$

$$\rho_2\left(u_2\frac{\partial w_2}{\partial r}+w_2\frac{\partial w_2}{\partial z}\right)=(\rho_1-\rho_2)g+2\sigma\frac{d(1/R_i)}{dz}+\mu_2\frac{\partial^2 w_2}{\partial r^2} \tag{15}$$

Equations (13-15) form a set in the variables u_2, w_2 and R_i. Equation (14) is satisfied if $R_i = R_i(z)$, i.e. the interface must be circular in the pipe cross section. In the following we suggest an approach to determine $R_i(z)$ from Eq. (15).

3. LUBRICATION THEORY ASSUMPTION

The basic assumptions of lubrication theory are

$$R-r_i \ll R \quad , \quad R-r_i \ll \lambda \, , \quad \frac{\rho_2 W\lambda}{\mu_2}\frac{(R-r_i)^2}{\lambda^2} \ll 1 \tag{16}$$

where λ is the interfacial wavelength. Under these conditions inertial terms can be neglected in Eq. (15) and the problem simplifies considerably. These assumptions are discussed by Oliemans (1986). In this situation, Eq. (15) becomes

$$(\rho_2-\rho_1)g-2\sigma\frac{d(1/R_i)}{dz}=\mu_2\frac{\partial^2 w_2}{\partial r^2}=\mathrm{const} \tag{17}$$

Thus:

$$\frac{d^2 w_2}{dr^2} = C_o \tag{18}$$

and

$$\frac{d}{dz}\left[\frac{2}{R_i(z)}\right] = C_1 \tag{19}$$

Solution of Eq. (18) is straightforward and leads to the traditional parabolic velocity profile. Focusing on the analysis of Eq. (19), the average curvature of the interface can be expressed as (see Fig. 2)

$$\frac{2}{R_i} = \frac{1}{R_{i2}} + \frac{1}{R_{i1}} = \frac{1}{r_i\sqrt{1+r_i'^2}} - \frac{r_i''}{\left(1+r_i'^2\right)^{3/2}} = \frac{1}{r_i}\frac{d}{dr_i}\left(\frac{r_i}{\sqrt{1+r_i'^2}}\right) \tag{20}$$

where $r_i' = \frac{dr_i}{dz}$; $r_i'' = \frac{d^2 r_i}{dz^2}$.

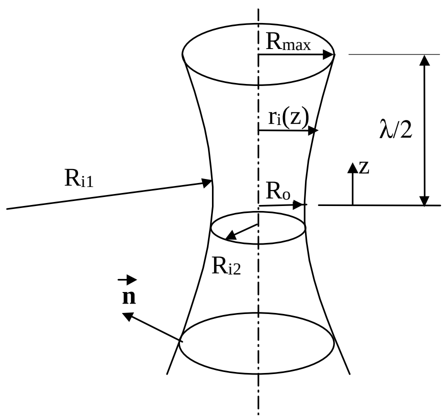

Figure 2 – Radius of curvature of the interface

Since pressure must be continuous on each side of the interface, its curvature must be a continuous function as well. Furthermore, the curvature must be periodic, i.e.

$$R_i(z) = R_i(z+\lambda) \quad \text{for any z} \tag{21}$$

It can be concluded that $C_1 = 0$ in Eq. (20). As a consequence, interfacial tension causes no net force on the flow and the interface is characterized by a constant curvature (C_2) i.e.

$$\frac{1}{r_i}\frac{d}{dr_i}\left(\frac{r_i}{\sqrt{1+r_i'^2}}\right) = C_2 \tag{22}$$

which is the Laplace-Young equation. Since Eq. (22) remains unchanged under the substitution of $z \to -z$, it can be concluded that interfacial waves are symmetric (i.e. $r_i(z)$ is an even function) with respect to a certain cross section plane. Taking the origin $z = 0$ at such plane (see Fig. 2) three boundary conditions are required, which are taken as:

$$r_i(0) = R_o \quad ; \quad r_i'(0) = 0 \quad ; \quad \frac{r_i''(0)}{\left[1 + r_i'^2(0)\right]^{3/2}} = \frac{k}{R_o} \tag{23}$$

where the parameters k, R_o and the wavelength λ are to be determined from known information. At z = 0, Eq. (22) then gives $\frac{1-k}{R_o} = C_2$. Integration of Eq. (22) leads to

$$r_i' = \frac{dr_i}{dz} = \sqrt{\frac{\left(\frac{r_i}{R_o}\right)}{\frac{(1-k)}{2}\left[\left(\frac{r_i}{R_o}\right)^2 - 1\right] + 1} - 1} \tag{24}$$

Solution of Eq. (24) is possible when k is in the range $-1 \le k \le 1$. For $-1 \le k < 0$, the periodic solutions form a chain of beads like a pearl necklace and satisfy $(1+k)/(1-k) \le r_i/R_o \le 1$. For $0 < k \le 1$, the solutions are similar to the "bamboo waves" observed by Bai *et al.* (1996) and satisfy $1 \le r_i/R_o \le (1+k)/(1-k)$. It is important to note that the later pattern was observed for upward oil-water flow, whereas the former seems to correspond to downflow. For $k = 0$ the interface is cylindrical and no waves are formed. Integration of Eq. (24) gives

$$\tilde{z} = \int_1^{\tilde{r}_i} \frac{d\tilde{r}_i}{\sqrt{\frac{\tilde{r}_i}{\frac{(1-k)}{2}\left(\tilde{r}_i^2 - 1\right) + 1} - 1}} \tag{25}$$

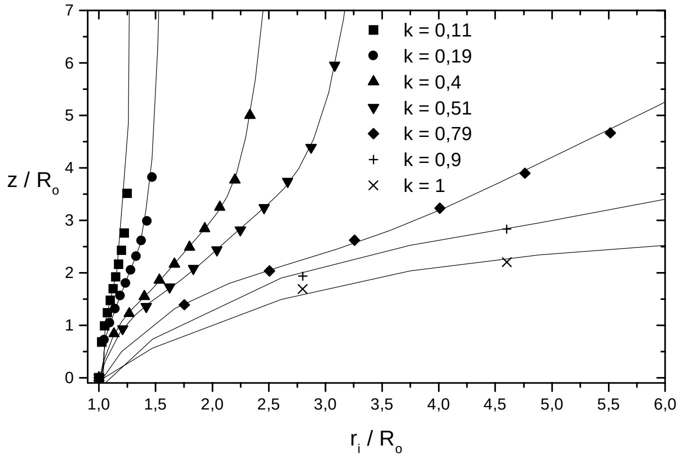

Figure 3 – Interface shape for several values of k

where $\tilde{r}_i = \frac{r_i}{R_o}$ and $\tilde{z} = \frac{z}{R_o}$. Some illustrative interface profiles according to Eq. (25) are shown in Fig. 3 for representative values of k.

Joseph & Renardy (1993) observed that the filaments connecting the crests of the waves thicken and the average wavelength decreases when the superficial velocity of the oil increases for a constant superficial velocity of water. Analyzing the results presented in Fig. 3, it can be noticed that the wavelength decreases and the amplitude increases when k increases. The development of the interfacial wave with the increase of k corresponds to the expected result for increasing volumetric fraction of the oil core.

4. ADJUSTING THE MODEL WITH AVAILABLE DATA

The three parameters to be adjusted are k, R_o and R_{max}. The parameter k was adjusted through comparison with available experimental data of Bai *et al.* (1996), who employed a motor oil (density = 0.905 g/cm^3, viscosity = 6 poise) and water in a tube of 0.9525 cm ID. Interfacial tension for this system was 0.0225 N/m. The value that best fitted all interface shapes was $k \cong 0.229$, as can be seen in the example of Fig. 4.

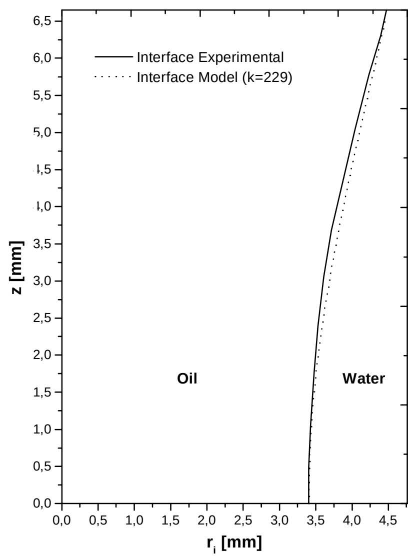

Figure 4 – Comparison between the observed oil-water interface for J_1 = 0.100 m/s, J_2 = 0.0468 m/s (Bai *et al.*, 1996) with the predicted by Eq. (25) with k = 0.229.

In their work, Bai *et al.* (1996) provided wavelength data at different flow conditions. From Eq. (25), the wavelength can be related to R_o and R_{max} by

$$\lambda = 2R_o \int_1^{\tilde{R}_{max}} \frac{d\tilde{r}_i}{\sqrt{\dfrac{\tilde{r}_i}{\dfrac{(1-k)}{2}\left(\tilde{r}_i^2 - 1\right) + 1} - 1}} \qquad (26)$$

where $\tilde{R}_{max} = \frac{R_{max}}{R_o}$.

Finally, information about the volumetric fraction α of the oil can be obtained from the correlation developed by Bannwart (1998), which was adjusted from data of the same system investigated by Bai *et al.* (1996):

$$J_1(1-\alpha) - J_2\alpha - 0.0223 V_{ref}\,\alpha^2 (1-\alpha)^2 = 0 \tag{27}$$

where J_1 and J_2 are the superficial velocities of the oil and water, respectively, $V_{ref} = \frac{(\rho_2 - \rho_1) g D^2}{16\mu_2}$, D is the inner diameter of the tube and μ_2 is the dynamic viscosity of the annulus fluid. Using Eqs. (24-25), the following expression for α can be derived

$$\alpha = \frac{\int_1^{R_{max}} \pi r_i^2 \left(\frac{dz}{dr_i}\right) dr_i}{\frac{\pi D^2 \lambda}{8}} = \frac{4R_o^2}{D^2} \frac{\displaystyle\int_1^{\tilde{R}_{max}} \frac{\tilde{r}_i^2 d\tilde{r}_i}{\sqrt{\frac{\tilde{r}_i}{\frac{(1-k)}{2}(\tilde{r}_i^2 - 1) + 1} - 1}}}{\displaystyle\int_1^{\tilde{R}_{max}} \frac{d\tilde{r}_i}{\sqrt{\frac{\tilde{r}_i}{\frac{(1-k)}{2}(\tilde{r}_i^2 - 1) + 1} - 1}}} \tag{28}$$

Equations (26) and (28) make it possible to adjust the parameters R_o and R_{max} from data for λ and α, for a specified k. The results are shown in Table 1 for the data points supplied by Bai *et al.* (1996). Although few data are presently available, the excellent quality of the wavelength fit is encouraging.

Table 1 – Adjusted values of R_o and R_{max} (k = 0.229)

Data from Bai *et al.* (1996)					**Fit with present theory**		
Run	**J_1 [m/s]**	**J_2 [m/s]**	**λ [m]**	**α**	**λ [m]**	**R_o [m]**	**R_{max} [m]**
1	3.56E-01	1.85E-01	0.0121	0.645	0.0121	0.0035	0.00446
2	2.55E-01	1.85E-01	0.0131	0.565	0.0131	0.0031	0.00420
3	1.55E-01	1.85E-01	0.0141	0.435	0.0140	0.00265	0.00388
4	1.04E-01	1.85E-01	0.0122	0.341	0.0121	0.00239	0.00345
5	1.04E-01	9.07E-02	0.0137	0.499	0.0138	0.00284	0.00405
6	1.55E-01	9.07E-02	0.0179	0.604	0.0179	0.00295	0.00450
7	2.05E-01	9.07E-02	0.0134	0.675	0.0133	0.00348	0.00455
8	2.55E-01	9.07E-02	0.0117	0.725	0.0116	0.00370	0.00455
9	3.06E-01	9.07E-02	0.0090	0.761	0.0090	0.00402	0.00455
10*	1.00E-01	4.68E-02	0.0124	0.633	0.0124	0.00337	0.00437
11*	1.93E-01	4.68E-02	0.0082	0.789	0.0082	0.00410	0.00453
12*	2.85E-01	4.68E-02	0.0054	0.852	0.0054	0.00436	0.00455

(* average wavelength obtained from photographs)

5. A SIMPLE MODEL FOR WAVELENGTH PREDICTION

As seen in the precedent section, a constant value of the parameter k can fit different flow conditions. Assuming this is generally true for fully developed wavy core flow, determination of the interface becomes a closed problem if the wavelength λ can be predicted, besides the volumetric fraction of the core.

In section 4, it has been shown that interfacial tension does not exert any net force on the core. However, it is not meant that force does not act. Like in ascending bubbles, interfacial tension keeps the core from breaking up into slugs, preserving it as a continuous phase. Besides, by analogy with the critical bubble diameter concept, we propose that interfacial tension determines the average stable wavelength of the waves. In fact, while the net buoyant force stretches the core against shear, interfacial tension acts oppositely to stabilize the core as a continuous phase. The ratio between Archimedes and capillary forces is (Fig. 5)

$$Eo_\lambda = \frac{F_a}{F_c} = \frac{\frac{\pi}{4}\Delta\rho g \alpha \lambda D^2}{2\pi R_o \sigma} \tag{29}$$

where Eo_λ is the Eötvos number based on the wavelength. Making the approximation $2R_o \cong D\sqrt{\alpha}$, we propose

$$Eo_v = \frac{\Delta\rho g \lambda D \sqrt{\alpha}}{4\sigma} \cong 1 \tag{30}$$

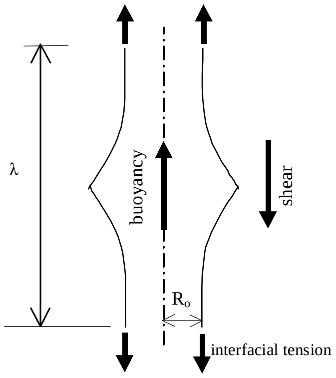

Figure 5 – Schematic picture of forces on a core wavelength

Table 2 - Results for the Eötvos number Eo_v

Run	**J_1 [m/s]**	**J_2 [m/s]**	**λ [m]**	**α**	**Eo_v**
1	3.56E-01	1.85E-01	0.0121	0.645	0.908
2	2.55E-01	1.85E-01	0.0131	0.565	0.920
3	1.55E-01	1.85E-01	0.0141	0.435	0.869
4	1.04E-01	1.85E-01	0.0122	0.341	0.666
5	1.04E-01	9.07E-02	0.0137	0.499	0.904
6	1.55E-01	9.07E-02	0.0179	0.604	1.300
7	2.05E-01	9.07E-02	0.0134	0.675	1.029
8	2.55E-01	9.07E-02	0.0117	0.725	0.931
9	3.06E-01	9.07E-02	0.0090	0.761	0.734
10	1.00E-01	4.68E-02	0.0124	0.633	0.922
11	1.93E-01	4.68E-02	0.0082	0.789	0.681
12	2.85E-01	4.68E-02	0.0054	0.852	0.564

Equation (30) agrees with the experimentally observed trend that λ decreases with increasing α. Using again Eq. (27) to determine α from the flow rates and the wavelength data given in Table 1, Table 2 is obtained. Though a certain spread is observed, the results are reasonably supportive of the idea of a unity Eötvos number, whose average was 0.87 and standard deviation 18 %. More data are clearly necessary to confirm Eq. (30).

6. CONCLUDING REMARKS

A new formulation is proposed for the equations governing the upward oil-water flow in core-annular pattern, when the liquid-liquid interfacial tension is taken into account. The new feature is the determination of the interface shape as part of the solution of the hydrodynamics. Neglecting inertial terms and turbulence in the annulus flow, the equation governing the interface shape is the Laplace-Young equation. It is shown that interfacial tension does not exert any net force in axi-symmetric flow, although its action in shaping the interface is decisive. This contrasts with the case of horizontal flow, where curvature gradients may cause a net capillary force on the core (Bannwart, 2000). The interfacial shapes obtained are constant curvature, periodic solutions entirely compatible with the "bamboo waves" observed by Bai *et al.* (1996) for upward flow, provided that a positive parameter ($k \cong 0.229$) is selected. A simple theory for wavelength prediction as a function of the volumetric fraction is proposed, whose agreement with the limited data available is nevertheless encouraging. Experiments are currently being done at UNICAMP to confirm the theory proposed.

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