Proceedings of the XII International Symposium on Dynamic Problems of Mechanics (DINAME 2007),
Varoto, P. S. and Trindade, M. A. (Editors), ABCM, Ilhabela, SP, Brazil, February 26 - March 2, 2007

Identification of Viscoelastic Vibration Absorbers in a Frequency Band

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Abstract: The behavior of viscoelastic materials under the action of dynamic loads exhibits characteristics that cannot be modeled in terms of the classical Hooke's Law. Usually a complex modulus (that depends on the load frequency) is adopted for expressing the relationship between stress and strain. From this complex modulus, it is possible to determine two other characterization parameters, namely the storage modulus and the loss factor. By knowing the frequency response, it is possible to determine the stiffness of a longitudinal viscoelastic vibration absorber together with its loss factor. By using the shape factor of the viscoelastic element, the storage modulus of the material can also be obtained. The present contribution presents an experimental procedure for the determination of the parameters used to characterize viscoelastic absorbers in a given frequency range. The resulting characterization of the vibration absorber can be used in the design of vibration reduction devices, including those for rotating machinery applications as it is shown in the remaining of this article.

Keywords: Viscoelastic Absorbers, Viscoelastic Damping, Identification of Dynamical Systems.

NOMENCLATURE

M = Mass, kg
K = Stiffness, N/m
C = Residual damping, Kg/s
G = Shear modulus, N/m^2
i = $\sqrt{-1}$
F = External force, N
S = Dynamic stiffness, N/m
x = Displacement, m
R = Dynamic accelerance, 1/kg
U_1= Unbalance force matrix for $\omega = 2 \cdot n \cdot \pi$, with n=0,1,2…
U_2= Unbalance force matrix for $\omega = 2 \cdot \left(n + \frac{1}{2} \right) \cdot \pi$, with n=0,1,2…

Greek Symbols

σ = Stress, N/m^2
ε = Strain, dimensionless
ω = Circular frequency, rad/seg
η = loss factor, dimensionless
γ = shape factor

Subscripts

v relative to viscoelastic material
T relative to rotor matrix

Superscripts

* relative to complex value
‘ relative to real part of a complex value.
” relative to imaginary part of a complex value.
T relative to transpose matrix

INTRODUCTION

The reduction of the vibration level of dynamical structures is an important issue in the industrial context in a wide range of engineering applications. The most usual practice is to add energy dissipation devices to the structure (known as vibration absorbers), which can reduce the undesirable vibrations. Vibration absorbers are commonly used in civil, automotive and aerospace structures. For the case of viscoelastic absorbers, such as the one represented schematically by Fig. 1, there are several analytical and experimental studies e.g: Mahmoodi (1969), Shen and Soong (1995), Park (2001), Xu et al (2004). The use of this type of devices for the passive vibration control of dynamical systems is limited by conditions such as high temperatures and large displacements. However, due to the low cost, versatility and simplicity, viscoelastic absorbers are considered as an appropriate choice in several applications.

For using viscoelastic absorbers in mechanical systems, it is necessary to know its dynamical behavior, considering the different parameters affecting the dynamics of the system (frequency, temperature, etc.). An appropriate knowledge of the dynamical characteristic of the absorber allows a good choice of location, size and number of necessary absorbers in order to assure the most effective energy absorption by means of the optimal design of the structure-vibration absorber system (Xu et al, 2004).

In this article the experimental procedure adopted with the purpose of identifying the dynamical characteristics of a viscoelastic absorber is explained. In the following sections of this article the general characteristics exhibited by viscoelastic materials are presented, then, the mathematical model and the corresponding dynamical equations used to identify the complex stiffness of the absorber are shown. After, the results obtained from experimental tests are compared with a FEM model of the absorber aiming at checking the accuracy of the experimental procedure. At the remaining, the viscoelastic parameters obtained for two different configurations are used in order to build a model of flexible rotor with viscoelastic absorbers placed on the supports.

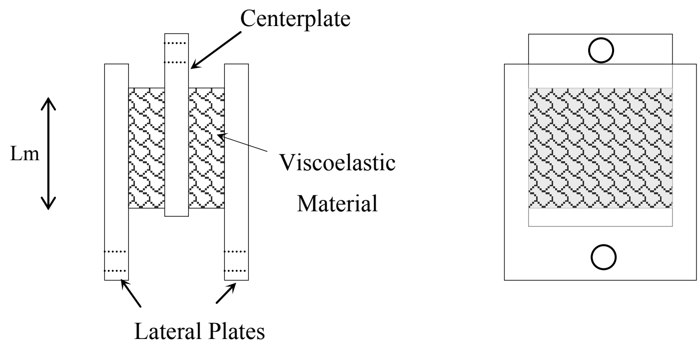


Figure 1 - Viscoelastic Absorber

VISCOELASTIC BEHAVIOUR

It is considered that a material exhibits viscoelastic behavior when its dynamic response presents characteristics of elastic solid and viscous fluid at the same time. Viscoelastic materials recover their original shape after being deformed, the same manner as an elastic solid. However, this recovery is rather slow, leading to consequences in the future loading cycles. The relation between stress and strain in viscoelastic materials is affected by a number of variables. Some of the most important are the temperature and the loading frequency. In this work only the influence of the loading frequency is considered for performing identification.

The frequency-domain response can be related to the time-domain response by using both the creep and relaxation functions. The way as these functions are related depends on the model adopted in order to reproduce the viscoelastic behavior. The determination of the unknown parameters of these models is based on two different approaches. The first one encompasses the so-called quasi-static methods, which are based in the concept of creep (or relaxation) function. In this approach, a constant force (or strain) is applied to the material and the time-dependant responses of the material for several loading conditions are used to build the model. The other approach uses dynamic methods. Now, a sinusoidal loading condition is applied to the viscoelastic material and the resulting response is used to build the corresponding frequency response function, which is used to determine the parameters of the model. In both cases, time-depending functions can be used to construct frequency-depending functions and vice-versa.

The frequency-temperature superposition principle is frequently used to facilitate the identification procedure. This principle is based on the equivalence between frequency and temperature effects, considering that the change in the material behavior is affected in the same manner by increasing temperature or reducing the loading frequency (Etchessahar et al, 2005). This principle is valid for most viscoelastic materials used in passive control applications. From this principle it is possible to characterize the dynamical response of a material along a broad frequency band by using experimental data acquired in a narrow frequency band and controlling the temperature of the test specimen (Yu and Haddad, 1996).

The internal damping mechanism inherent to the viscoelastic material offers the capability to implement this material in industrial applications as passive absorbers of vibration and noise (Nashif et al, 1985). In the case of rotating systems, this type of materials also has proved to be successful for vibration control applications (Childs, 1993).

MATHEMATICAL MODELS AND DYNAMIC EQUATIONS

Stress in viscoelastic materials is not only proportional to strain; instead it also depends on the strain variation. In this case the relation between stress and strain depends on the frequency of the applied loads. In the case the viscoelastic material is subjected to varying frequency sinusoidal loading the strain lags behind the stress, which is denoted by a phase angle between the two signals, being this lag dependant on the frequency too. Considering this behavior, the relation between stress and strain can be written by means of a complex modulus, according Eq. (1).

$$\sigma(\omega) = G^*(\omega) \cdot \varepsilon(\omega) \tag{1}$$

The real and imaginary parts of the complex modulus can be separated in order to better understanding the viscoelastic properties. For this reason, it is common to find many authors writing the complex modulus as in Eq. (2).

$$G^*(\omega) = G'(\omega) \cdot [1 + i \cdot \eta(\omega)] \tag{2}$$

By using the same approach, it is possible to write the stiffness of a viscoelastic absorber as being a complex stiffness that depends on the frequency and is related with the complex modulus through a shape factor, which depends on the dimension and geometric characteristics of the specimen, as given by Eq. (3).

$$K_v^*(\omega) = \gamma \cdot G'(\omega) \cdot [1 + i \cdot \eta(\omega)] \tag{3}$$

Viscoelastic absorber attached to a Single Degree of Freedom System

Considering the hypotheses presented in the previous section, the equation that represents the dynamics of a single DOF system attached to a viscoelastic absorber (Fig. 2.) can be represented by Eq. (4).

$$M \cdot \ddot{x} + C \cdot \dot{x} + (K + K_v) \cdot x = F \tag{4}$$

Considering that the parameters of the single-DOF system are known and a sinusoidal excitation is applied to the system, the real and imaginary parts of the viscoelastic stiffness can be calculated by using Eqs. (5) and (6).

$$K_v^{'}(\omega) = M \cdot \omega^2 - K - S'(\omega) \tag{5}$$

$$K_v^{''}(\omega) = -(S''(\omega) + C \cdot \omega) \tag{6}$$

where:

$$S^*(\omega) = S'(\omega) + i \cdot S''(\omega)$$

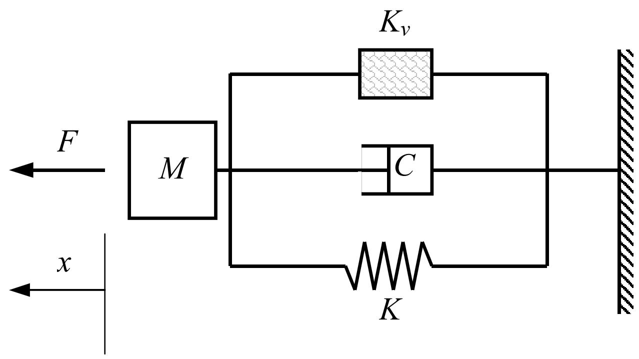


Figure 2 - Single DOF mechanical system attached to a viscoelastic absorber

Considering S^* as being the dynamic stiffness, it is the inverse of the dynamic compliance. The dynamic compliance is defined as the ratio between the complex amplitudes of the frequency dependant displacement and force.

From Eqs. (3), (5) and (6), it is possible to write the loss factor of the absorber, according to Eq. (7).

$$\eta(\omega) = \frac{K + S'(\omega) - M \cdot \omega^2}{S''(\omega) + C \cdot \omega} \tag{7}$$

The values of the complex dynamic compliance can be determined by using either frequency or time domain methods, by exciting the structure with a shaker and measuring the forces and resulting vibrations.

EXPERIMENTAL AND NUMERICAL RESULTS

Experimental Set-up

In order to measure the dynamic response of the viscoelastic absorbers a single-DOF system coupled with a shaker was used. In this way, an accelerometer and a force transducer were used for measuring the vibration of the system and the force applied by the shaker. The test rig is shown in Fig. 3. A signal analyzer SD380 was used to perform the measurements. This equipment is able to calculate the dynamic accelerance. Using this information (and knowing the parameters of the single-DOF mechanical system) it is possible to calculate the dynamic stiffness, and finally the absorber parameters. The parameters of the single-DOF system are exposed in Tab. 1. Considering that the damping characteristics of a dynamical system are more evident in the neighborhood of the natural frequencies, an appropriate stiffness to mass ratio was selected in such a way that the natural frequency is close to the center of the analyzed frequency band.

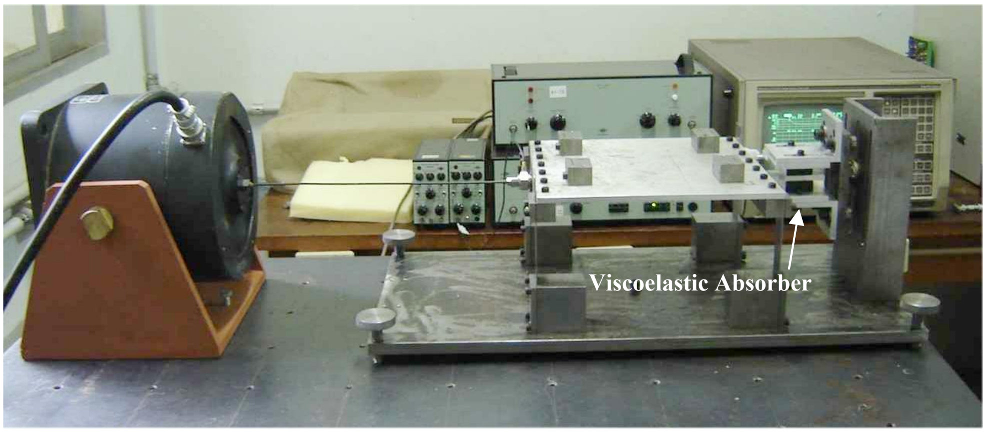


Figure 3 - Test rig

Table 1 - 1-DOF Parameters

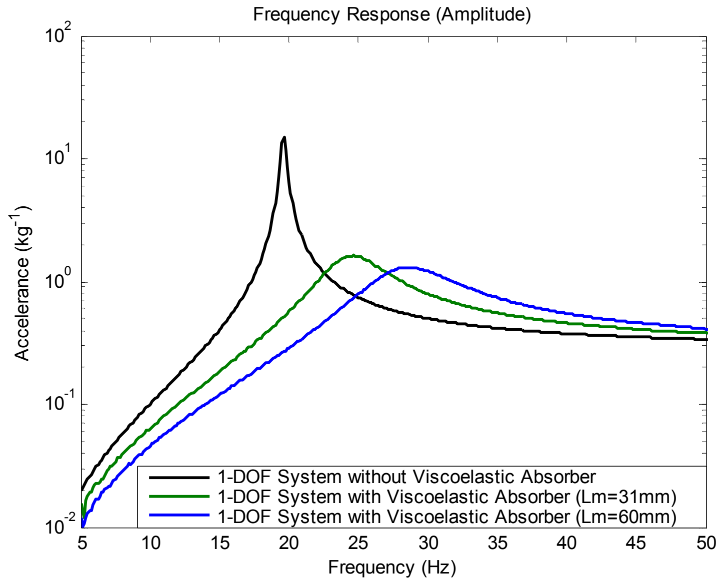
Parameter	Value
Mass	3.49 Kg
Stiffness	52919 N/m
Residual Damping	8.13 kg/s

Experimental Results

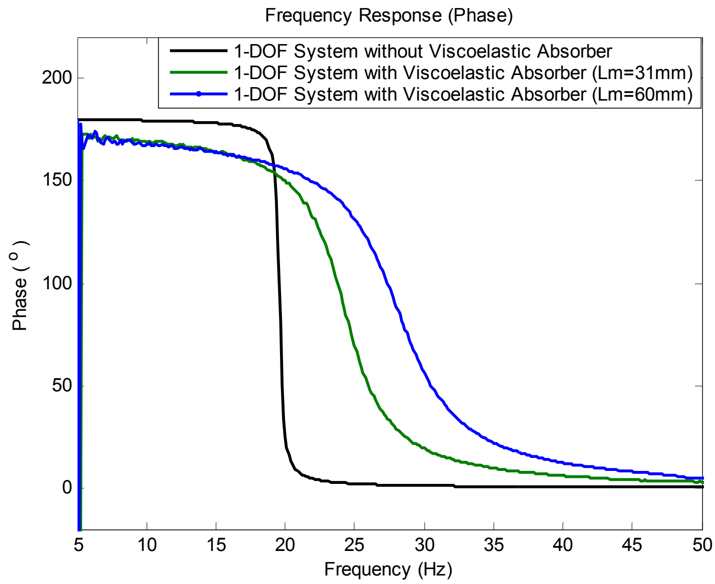
Two configurations of absorbers were evaluated, by changing the length Lm (see Fig. 1). Fig. 4 shows the accelerance frequency response measured in the studied band of frequency. As noticed, the introduction of the dampers produces an evident reduction of the vibration response, together with an increment in the stiffness of the system, thus increasing the resonance frequency.

The dynamic stiffness can be approximated using the accelerance frequency response by taking into account Eq. (13).

$$S^*(\omega) = \omega^2 \cdot \frac{R'(\omega) + i \cdot R''(\omega)}{[R'(\omega)]^2 + [R''(\omega)]^2} \tag{13}$$



a)



b)

Figure 4 – Accelerance Frequency Response of the system: a) Amplitude; b) Phase

In Figures 5 and 6 the frequency dependant functions of the absorbers (real part of the stiffness and loss factor) are shown. The viscoelastic functions are compared with the responses obtained by using a FEM model developed by Saldarriaga et al (2006). The results of the numerical model have shown a good agreement with the experimental data. The difference between analytical and experimental results is probably due to the preloads and fixture dynamics, which are always a source of uncertainties. These effects can be minimized, however it is not possible to avoid them completely.

Another source of uncertainties is related to changes in the temperature of the room and these are not considered in the model. In the present contribution it was observed that temperature variation was negligible since the material is in its rubbery region. Besides, uncertainties in the transducers are more evident in the low frequencies, as can be seen in the figures in the next page.

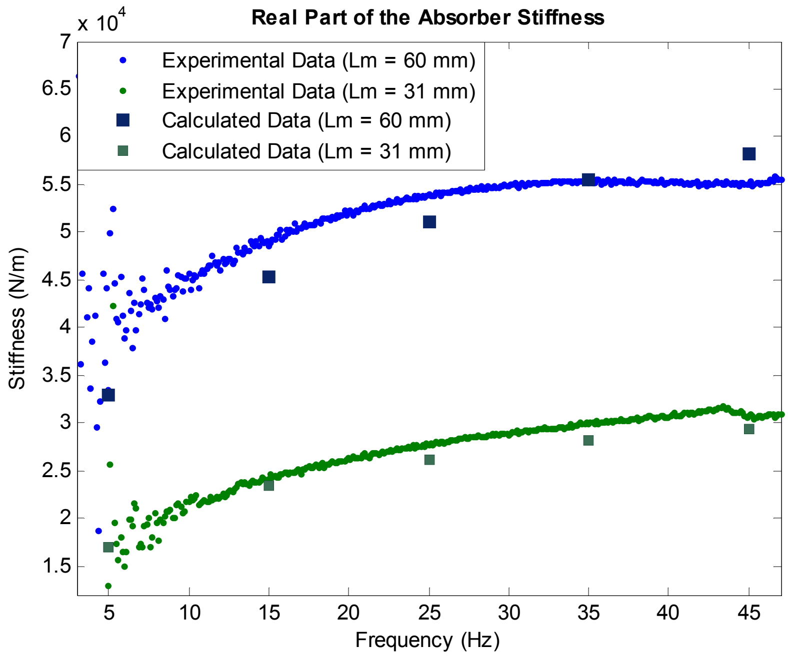


Figure 5 - Real Part of the Absorber Stiffness

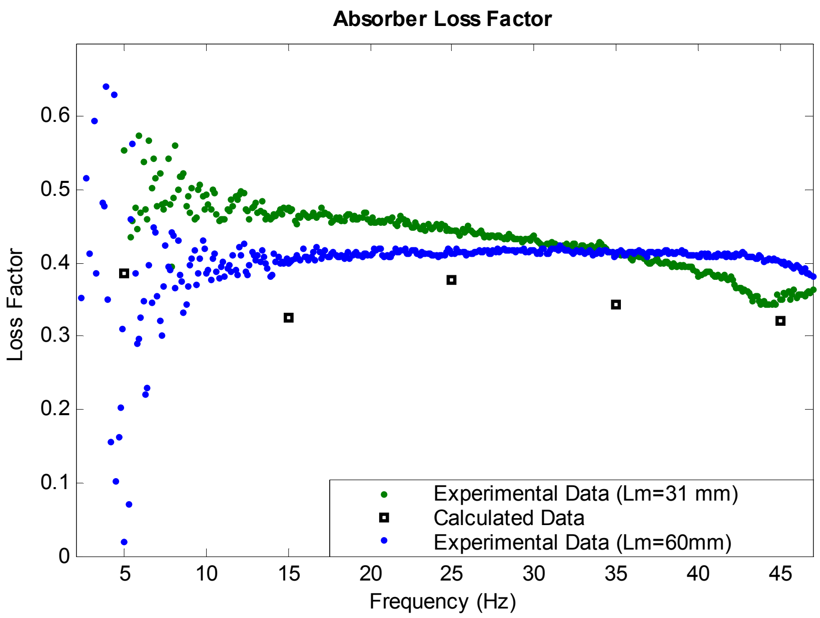


Figure 6 – Viscoelastic Absorber Loss Factor

AN APPLICATION: ROTORDYNAMIC SYSTEMS WITH VISCOELASTIC ABSORBER IN THE SUPPORTS

In order to show the application of viscoelastic absorbers in complex systems, the results obtained from the identification procedure are now used to perform the modeling of a gyroscopic system supported by viscolastic absorbers. In this way, it is possible to calculate the response of dynamic systems in which viscoelastic absorbers are used as a strategy of vibration passive control. The model is based on a test rig that is dedicated to the study of dynamic behavior of drilling machines in the petroleum industry. The test rig was modelled considering five beam and three rigid disc elements. The supports were modelled as being non-symmetric with constant coefficients of stiffness and damping. The coupling used to provide torque transmission is modelled as a symmetric stiffness with concentrated mass. The rotor model is based on the finite element model and the Lagrange equations. The details of the model elements can be seen in Fig. 7.

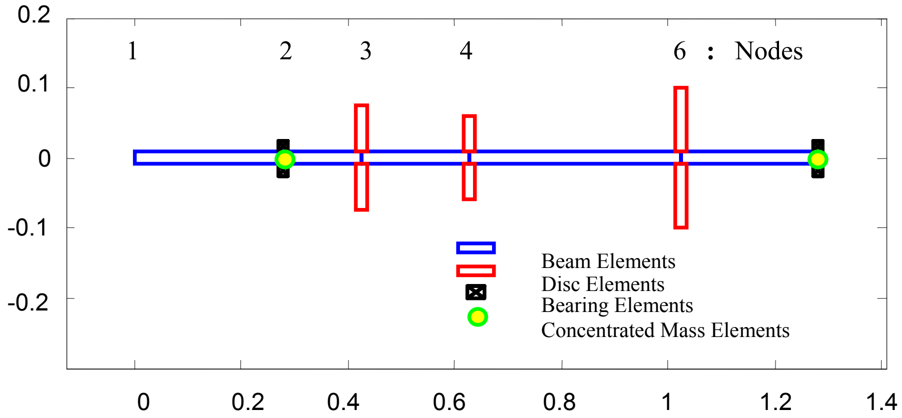


Figure 7- FEM rotor Model (dimension in meters)

The complete rotor model (including the viscoelastic dampers) is represented by the following matrix differential Eq. (14):

$$M_T \cdot \ddot{\delta} + C_T(\omega) \cdot \dot{\delta} + \left[K_T + K^*_{Tv}(\omega)\right] \cdot \delta = F_T \tag{14}$$

where $K^*_{Tv}(\omega)$ is the matrix due to the viscoelastic effect of the absorber located at the support positions. This matrix is built according to equation (15):

$$K^*_{Tv}(\omega) = K'_{Tv}(\omega) + i \cdot K''_{Tv}(\omega) = V^T \cdot \left[K'_v(\omega) + i \cdot K''_v(\omega)\right] \cdot V \tag{15}$$

where V is as a vector with dimension equal to the number of generalized coordinates of the rotor system. The values relatives to displacements at the support position are ones, and the others are nulls.

By considering only unbalance forces, it is possible to rewrite Eq. (14) as follows:

$$M_T \cdot \ddot{\delta} + C_T(\omega) \cdot \dot{\delta} + \left[K_T + K^*_{Tv}(\omega)\right] \cdot \delta = U_1 \cdot \cos(\omega \cdot t) + U_2 \cdot \sin(\omega \cdot t) \tag{16}$$

The solution for Eq. (16) is given by:

$$\delta = \Delta_1 \cdot \cos(\omega \cdot t) + \Delta_2 \cdot \sin(\omega \cdot t) \tag{17}$$

After writing the sine and cosine terms based on Euler's formula (as sums of complex exponentials), and collecting the real and imaginary terms, it is possible to write Eq. (18)

$$\begin{bmatrix} K_T + K'_{Tv}(\omega) - M_T \cdot \omega^2 & -\omega \cdot C_T(\omega) - K''_v(\omega) \\ \omega \cdot C_T(\omega) + K''_v(\omega) & K_T + K'_v(\omega) - M_T \cdot \omega^2 \end{bmatrix} \cdot \begin{Bmatrix} \Delta_1 \\ \Delta_2 \end{Bmatrix} = \begin{Bmatrix} U_1 \\ U_2 \end{Bmatrix} \tag{18}$$

Then, knowing the unbalances and their corresponding angular positions, the displacement vector of the system for each frequency of rotation can be calculated. Figure 8 shows the unbalance response of the model including viscoelastic absorbers for both configurations (Lm=31mm and Lm =60mm) located at the bearing position in the transversal directions. It is possible to notice that in both cases the viscoelastic device allows to reduce the vibration due to unbalance effects. When the stiffest absorbers are used, an increment in the stiffness of the entire system is obviously observed, as compared to the other cases in which more flexible absorbers are used. Furthermore, it not clear a better vibration attenuation when the length of the viscoelastic element is increased, for frequencies over 10 Hz. Under the frequency of 10 Hz, rigid body modes govern the dynamic behaviour of the system; in this case a high stiffness value at the support leads to smaller vibration amplitudes.

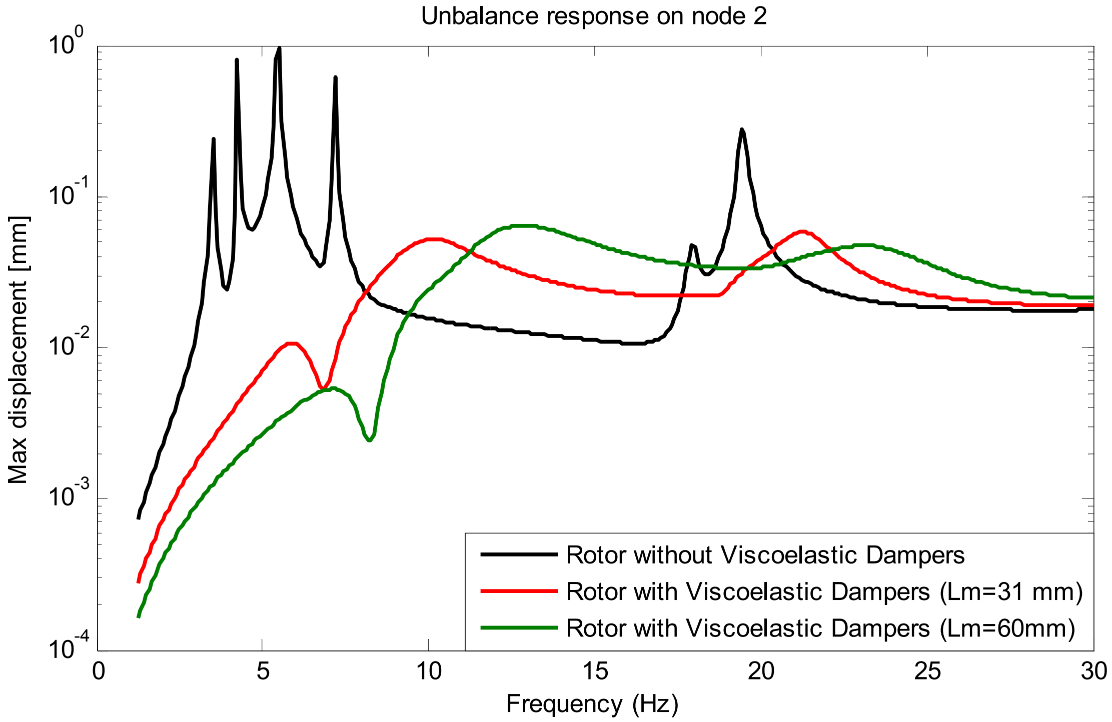


Figure 8 – Unbalance Response

CONCLUSIONS

An experimental procedure for the determination of the parameters used to characterize viscoelastic absorbers in a given frequency range was presented. The results obtained were compared with those from a numerical model, resulting good agreement. The procedure was performed for a narrow frequency band; however, a similar procedure can be applied for larger frequency ranges. For this purpose, the whole range is subdivided in several sub-bands and the tests are performed for each sub-band separately, using a different stiffness to mass ratio for the single-DOF system. In every case, the natural frequency of the system is maintained as close as possible to the center of the sub-band, in such a way that the average values of the viscoelastic parameters can be obtained regarding the whole range of interest.

The results of the identification procedure are introduced in the model of a flexible rotor to illustrate the potential use of viscoelastic absorbers to reduce vibration in rotating machinery. The authors are encouraged by the presented identification procedure of viscoelastic parameters and consider that the introduction of viscoelastic materials in the support of complex mechanical systems (such as rotating machinery) can be very helpful for designing effective passive vibration control absorbers.

ACKNOWLEDGEMENTS

The first author would like to thank VICONDIA (alfa 2 project – EC) project for supporting this research work and to FAPEMIG Foundation for his doctorate scholarship. The second author is thankful to CNPq for his research grant.

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