

A Neural Network Model to Estimate the Aerodynamic Derivatives of Fixed Wing Aircraft

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Abstract: The flight dynamics of an aircraft can be represented through a set of nonlinear ordinary differential equations, as described in literature. The system formed by these equations, involves aerodynamic forces and moments of the aircraft and these can be found substituting the stability derivatives in the set of equations. But, the determination of these derivatives requires a difficult and high cost work, since normally they are obtained experimentally, either through flight tests either in wind tunnel. With the technological advance of the computers, some methods in the time and frequency domains have been considered for identification and estimation of these derivatives. In this context, the objective of this work is the implementation of neural networks for the estimation of the longitudinal stability derivatives of an aircraft. Several tests will be presented and discussed and in all them various inputs will be supplied. Such inputs are: for example, static inputs - aircraft mass, air density, moments of inertia, and dynamic inputs - variation of angle of attack, variation of vertical and horizontal velocity and pitching velocity. For this study the data of the A4-d aircraft, supplied in literature, will be used. The non-linear mathematical model was implemented in Simulink and was based on the equations presented by Etkin, 1996. Seven different flight conditions had been simulated, aiming to close a representative flight envelope of this aircraft. To get the responses of the aircraft, geometric, aerodynamic, stability derivatives and aerodynamic coefficients of the aircraft are used as input parameters. In a work previously done, the mapping of the aircraft in all flight envelope was accomplished, using recurrent neural networks, with sufficient success. Being thus, in this work, it is desired to solve the inverse problem, or in other words, from the answers obtained using the simulator, to esteem the stability derivatives of the aircraft.

Keywords: Simulation, flight envelope, aerodynamic derivatives, aircraft non-linear models, neural networks.

NOMENCLATURE

M = Mach number, dimensionless
 m = Aircraft's mass, Kg
 U_0 = Initial velocity of aircraft, m/s
 I = Moment of inertia about axis, Kg/m²
 X = Longitudinal Force, N
 Y = Vertical Force, N
 Z = Lateral Force, N
 L = Roll aerodynamic moment
 M = Pitch aerodynamic moment
 N = Sideslip aerodynamic moment

Greek Symbols

ρ = air density, dimensionless
 Δ = relative to variations
 δ = control surface deflexion, degree

Subscripts

r relative to rudder
e relative to elevator
a relative to aileron

u relative to linear velocity u
w relative to linear velocity w
y relative to lateral direction
z relative to vertical direction
q relative to pitch velocity
p relative to lateral velocity
r relative to roll velocity

INTRODUCTION

The stability and control derivatives, created to represent the aerodynamics forces and moments, are of extreme importance in the study of the aircraft flight dynamics. In Vasconcelos (2002), an approach to attain the control and stability derivatives can be found. The existing methods in this area are sufficiently efficient, but, for example, the attainment of derivatives from flight test data, presents high financial cost.

Sim (1997) presented the correlation between the flight-determined derivatives and wind-tunnel predictions for the first 21 flights of the X-24B research aircraft. The flight derivatives were obtained with a modified maximum likelihood estimation method used at the NASA Flight Research Center. Two aircraft configurations were used to obtain the data presented in this report. A subsonic configuration was used to achieve good aerodynamic performance with adequate stability at subsonic speeds, and a transonic configuration was used to achieve good stability at transonic and supersonic speeds. The flight derivatives were consistent and provided a good documentation of the aircraft's characteristics.

Mendonça and Góes (2003) had applied the output error method to identify parameters of a linear model of the longitudinal dynamics of a regional aircraft. They used data produced by “3-2-1-1” elevator input and identified the angle of attack of the aircraft, the acceleration of the aircraft and the pitch velocity. The assumption of a linear model showed to be reasonable, since the results of the model and the data of the flight test were coherent.

Raisinghani and Ghosh (2000) had shown artificial neural networks application to aeroelastic aircraft modeling and parameters estimation, without needing measures of elastic deflections or its derivatives. Specifically, a feedforward neural network was proposed associating the two developed methods called method Zero and method Delta to predict force and moment coefficients using only measured variables of movement and control. In this work, the results for different conditions of aircraft flexibility were sufficiently coherent, leaving clear the applicability of the neural networks for modelling and parameters estimation.

Neto et al. (2005) had presented an adaptive optimal linear estimation algorithm for the architecture known as Functional Link Network. To illustrate the proposed technique, a case study of longitudinal dynamic model using the model of an F-16 aircraft was shown, using simulated data, focusing on its aerodynamic derivatives estimation. For the simulation, non-linear modelling was used and as network input, it was supplied: aerodynamic speed, pitch velocity, angle attack variation, elevator deflection and sideslip angle. The obtained results had been sufficiently satisfactory and had shown that this method can be applied to control and identification problems. They had also concluded that this algorithm can easily be implanted in an on board computer for in flight identification of the derivatives.

In this context, the aim of this work is to show the accuracy of the artificial neural networks, with simple topologies for aerodynamic stability derivatives estimation. The methodology will be applied to an A-4D aircraft, using simulated data. Knowing the aerodynamic derivatives in some points of a flight envelope and using them to train the neural network, it is possible to estimate the derivatives in any another point of the envelope. This methodology implies in a reduction of sufficiently great cost in the derivatives estimation.

Stability Derivatives and Aircraft Forces and Moments

The aircraft stability derivatives relate the elementary variations of the force and moment vectors for small disturbances of the movement variables and the control inputs (Faria, 2002). The expressions of the stability derivatives are obtained through linearization techniques using the Small Disturbances Theory. They are functions of the aerodynamic incidence angles α and β , Euler angles and velocities, i.e., the stability derivatives are functions of the aerodynamic coefficients. In such a way, the dynamic equations of an aircraft can analytically be written as functions of the aerodynamic coefficients and this development can be found in Etkin and Reid, (1996).

Aerodynamic Forces and Moments Equations

Souza et al. (2005) shown the full development and implementation of an aircraft dynamic simulator. The non-linear mathematical model presented by Etkin and Reid (1996) was used, because, this model, although the use of some simplifications, presents coherent results. The developed software was applied in the simulation of the A4-D aircraft under some flight conditions. A recurrent neural network was trained using 3 of these flight conditions on 3 different points of the flight envelope and later it was applied to estimate the answers of the aircraft on other 4 points of the flight envelope. The work was carried through with sufficient success. As said previously, in this work the inverse problem will be solve, that is, from the answers, the aerodynamic derivatives will be obtained. For this, only the equations of forces and moments of an aircraft will be presented.

As it is known, the aerodynamic forces and moments acting on the aircraft are functions of the angle of attack and velocity components, and their variations are represented by the following equations:

$$\Delta X = X_u \cdot \Delta u + X_w \cdot \Delta w + X_{\dot{w}} \cdot \Delta \dot{w} + X_q \cdot \Delta q + X_{\delta_e} \cdot \Delta \delta_e \quad (1)$$

$$\Delta Y = Y_v \cdot \Delta v + Y_p \cdot \Delta p + Y_l \cdot \Delta l + Y_{\delta_a} \cdot \Delta \delta_a + Y_{\delta_r} \cdot \Delta \delta_r \quad (2)$$

$$\Delta Z = Z_u \cdot \Delta u + Z_w \cdot \Delta w + Z_{\dot{w}} \cdot \Delta \dot{w} + Z_q \cdot \Delta q + Z_{\delta_e} \cdot \Delta \delta_e \quad (3)$$

$$\Delta L = L_v \cdot \Delta v + L_p \cdot \Delta p + L_r \cdot \Delta r + L_{\delta_a} \cdot \Delta \delta_a + L_{\delta_r} \cdot \Delta \delta_r \quad (4)$$

$$\Delta M = M_u \cdot \Delta u + M_w \cdot \Delta w + M_{\dot{w}} \cdot \Delta \dot{w} + M_q \cdot \Delta q + M_{\delta_e} \cdot \Delta \delta_e \quad (5)$$

$$\Delta N = N_v \cdot \Delta v + N_p \cdot \Delta p + N_r \cdot \Delta r + N_{\delta_a} \cdot \Delta \delta_a + N_{\delta_r} \cdot \Delta \delta_r \quad (6)$$

where X_u , X_w , $X_{\dot{w}}$, X_q , X_{δ_e} , Y_v , Y_p , Y_r , Y_{δ_a} , Y_{δ_r} , Z_u , Z_w , $Z_{\dot{w}}$, Z_q , Z_{δ_e} , L_v , L_p , L_r , L_{δ_a} , L_{δ_r} , M_u , M_w , $M_{\dot{w}}$, M_q , M_{δ_e} , N_v , N_p , N_r , N_{δ_a} and N_{δ_r} represent the aerodynamic derivatives of stability, Δu , Δv and Δw represent the

instantaneous values of the linear velocities perturbations, Δp , Δq and Δr represent the instantaneous values of the angular velocities perturbations and $\Delta\delta_e$, $\Delta\delta_a$ and $\Delta\delta_r$ represent variations of the control surfaces.

For the simulation of the flight dynamics, the altitude, the density in function of the altitude and the total velocity of the aircraft are defined. After the aerodynamic derivatives are supplied, the values of the forces and aerodynamic moments in equations (1) to (6) are calculated, using the initial values for the velocities (u , v , w , p , q and r). In this work, the longitudinal behavior of the A4-D aircraft was simulated with data given in MacRuer (1973). These data was used to train the artificial neural networks and to identify a flight envelope.

The resolution of inverse problem of that presented in Souza et al. (2005) is considered now. From the velocities responses and data of the aircraft, the stability derivatives will be estimated using static neural networks. Figure 1 shows a representative block diagram of the problem to be solved. In this paper, results of some tests will be shown, showing the capacity of the neural networks to estimate parameters with computational low cost.

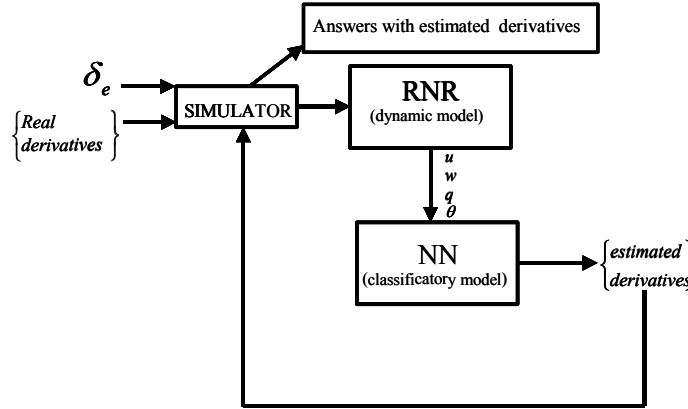


Figure 1 – Representative Block Diagram of the problem.

Artificial Neural Networks

Artificial neural networks are information processing systems with the capability of learning through examples (Haykin, 1994). Based on concepts derived from neuro-biology, neural networks are composed by a set of interconnected processing units, called *neurons*. The neurons process the signals presented to the neural network by accumulating each stimulus and by transforming the total value using a function; that is, the *activation function*. The stimuli to and from a neuron are modified by the real value called *synaptic weight*, which characterises the respective connection between neurons.

Figure 2 shows a typical representation for a generic neuron j , where x_1, x_2, \dots, x_p are the stimulus signals, $w_{j1}, w_{j2}, \dots, w_{jp}$, are the synaptic weights, θ_j is a bias value, v_j is the activation potential, o_j is the neuron output signal, and $\varphi(\cdot)$ is the activation function (generally adopted as a non-linear sigmoid function).

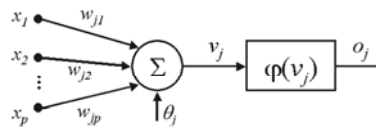


Figure 2. Typical neuron representation.

Then, from Figure 2, one can observe that the neuron output is given by:

$$o_j = \varphi \left(\theta_j + \sum_{i=1}^p w_{ji} x_i \right) \quad (7)$$

Network *architecture* is the name given to the arrangements of neurons into layers and how they are connected. Typical neural networks have the following architecture: (1) *input layer* – where the input stimulus is presented to the network; (2) *hidden layers* – internal layers of a network, and (3) *output layer* – the last layer of the network, where the outputs are given. Such typical network architecture is commonly referred to as a *multi-layer neural network*.

Once trained, one can assume that the network stored the knowledge supplied to it. However, the knowledge in a neural network is not stored in a particular localization. It depends on its topology and the magnitude of the weights in the input layer.

The generalization of an artificial neural network is the capacity to reproduce desired signals for different input signals that have not been used during the network training, or either, that it is able to catch the dynamics of the system being emulated (Saravanan and Dyear, 1994).

Neural Network Training

To achieve a desirable set of synaptic weights of a pre-defined network architecture, a training process is needed. A training process is generally based on an optimisation scheme to adjust the network parameters (mainly, the weights) in relation to a set of input-to-output to be matched by the neural network model (supervised learning scheme). The backpropagation algorithm based on a gradient descent technique (Haykin, 1994) has been widely applied for general neural network training. A more efficient training scheme can be achieved by using the Levenberg-Marquardt Algorithm (LMA).

Levenberg-Marquardt Algorithm (LMA)

This algorithm is a variation of the Newton's method for minimizing functions that are sums of squares of other non-linear functions (Hagan et al., 1996). The LMA provides better performance when compared with typical backpropagation algorithms.

From Newton's method the network update rule is:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mathbf{H}_n^{-1} \mathbf{g}_n, \quad (8)$$

where, \mathbf{w} is the network weight matrix, n is a step of iteration, \mathbf{H} is the Hessian matrix and \mathbf{g} is the gradient matrix.

For the performance index as a sum of square functions, the Hessian matrix can be approximated in terms of the Jacobian matrix, \mathbf{J} , which contains first derivatives of the network errors with respect to the weights and biases. Thus,

$$\mathbf{H} \cong \mathbf{J}^T \mathbf{J}. \quad (9)$$

When the approximation in Eq. (9) is substituted into Eq. (8), the Gauss-Newton method is obtained, that is:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - [\mathbf{J}_n^T \mathbf{J}_n]^{-1} \mathbf{g}_n. \quad (10)$$

A problem that may arise in the Gauss-Newton method is that the matrix $[\mathbf{J}^T \mathbf{J}]$ may not have an inverse. This can be overcome by assuming a modification to the matrix $[\mathbf{J}^T \mathbf{J}]$ that leads to the LMA:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - [\mathbf{J}_n^T \mathbf{J}_n + \mu_n \mathbf{I}]^{-1} \mathbf{g}_n, \quad (11)$$

where, \mathbf{I} is the identity matrix and μ is a scalar.

The scalar μ presents an important role to the LMA. When μ_n is zero, the weight update is basically the Gauss-Newton method. When μ_n is sufficiently large, Eq. (11) becomes gradient descent with small step size. By choosing the proper value of μ the LMA provides an efficient compromise between the great performance of the Newton's method and the guaranteed convergence of the gradient descent approach.

Aerodynamic Derivatives Estimation of the A4-D Aircraft

First, it was thought to implement only one neural network to estimate the longitudinal aerodynamic derivatives of the aircraft, but due to the great number of inputs the network had to have and to the consequent high computational cost, it was decided to implement separate networks to estimate the derivatives. One neural network was implemented to estimate the u derivatives, in the case, $X_u, Z_u \in M_u$, another to estimate the w derivatives, $X_w, Z_w \in M_w$ and another to estimate the derivatives $M_{\dot{w}} \in M_q$.

For the attainment of all these derivatives, static neural networks were used. They were of the type feedforward, with two hidden layers of neurons and to train them, the backpropagation algorithm with the Levenberg-Marquardt optimization technique was used. In all the networks, two types of inputs had been used: the dynamic ones, which in the case are the velocity responses of the aircraft and the static ones, which correspond to the aircraft characteristics and air density, specifically $m, \rho, U_0, M, I_x, I_y, I_z$ and I_{zx} .

Estimation Results of the Longitudinal Aerodynamics Derivatives $X_u, Z_u \in M_u$

First, the estimation results of the derivatives X_w, Z_u and M_u will be presented. As said previously, beyond the static inputs, dynamic inputs had been used also, in this in case, the time responses of the variations of linear speed u and

angular speed q . These signals in time had been discretized and taken 1 point at each 1000 of the total of 30000 points. That is, with these signals two vectors of size 30 had been formed to be supplied as input to the neural network. It could be verified that the signal continued to be characterized, making this discretization. Figure 3 shows a representative block diagram of the neural network used in this case. These inputs had been chosen after an analytical study on which ones have more influence on the results.

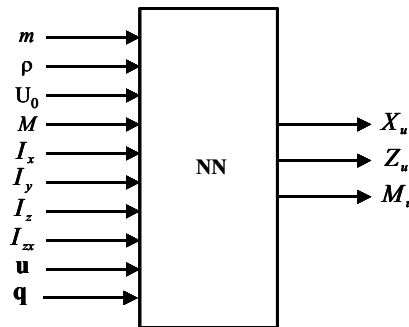


Figure 3 - Block Diagram showing the neural network used to estimate the longitudinal u derivatives.

Chosen the inputs, the next step was then to choose the topology of the neural networks to be used. First a neural network with 3 layers of neurons was implemented: the first one to receive the net inputs, an hidden layer and an output one with respectively 68, 20 and 3 neurons, using the activation functions sigmoidal and linear tangents. To use these functions, as all the data were very different, it was necessary to normalize them, dividing each one of the data sets by its maximum absolute value, therefore the function of sigmoidal tangent activation, always varies between -1 and 1 . Chosen the topology, training tests had been carried out, successfully. The training time was around 15 minutes. Aiming to an optimization of the training time, it was added another hidden layer and the result was sufficiently satisfactory, resulting on a training time of around 2 minutes. In this first test, the numbers of neurons used in the input, the two intermediate and the output layers respectively, was 68, 12, 9 and 3. It could be verified then, that with two hidden layers, the training time was well lesser. The results of two training and generalization tests will be presented. The used data for training had been that of points 3, 6, 1 and 7 of the flight envelope studied in this work, whose scheme is presented in Figure 4.

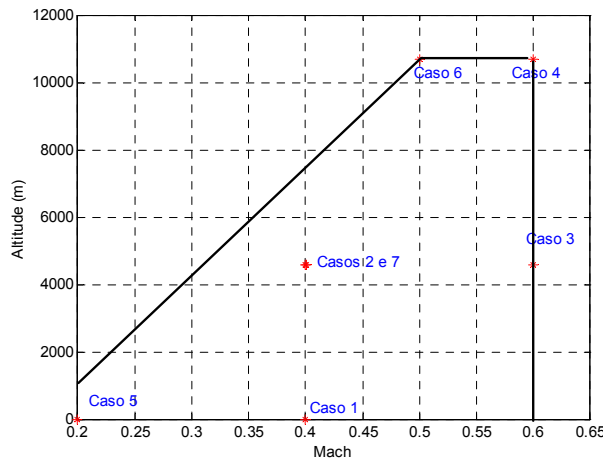


Figure 4 - Scheme showing the flight envelope.

After the training, the “real” results correspondent to cases 4, 5 and 2 had been calculated in order to verify the network capacity of generalization. The obtained results are presented in the Tab. 1, Tab. 2 and Tab. 3.

Table 1. – Comparison of X_u estimated and real.

X_u	Case 4	Case 5	Case 2
Real	-108.9554	-847.3290	-125.9797
Estimated	-101.4711	-33.7752	-123.9310

Table 2. - Comparison of Z_u estimated and real.

Z_u	Case 4	Case 5	Case 2
Real	-0.9704(10 ³)	-2.6671(10 ³)	-1.3619(10 ³)
Estimated	-0.9634(10 ³)	4.3821(10 ³)	-1.2949(10 ³)

Table 3. - Comparison of M_u estimated and real.

M_u	Case 4	Case 5	Case 2
Real	46.0713	377.8643	57.5891
Estimated	47.2230	294.4156	56.8199

For a better visualization of the results, the real ones were plotted against the estimated ones and shown in Figure 5.

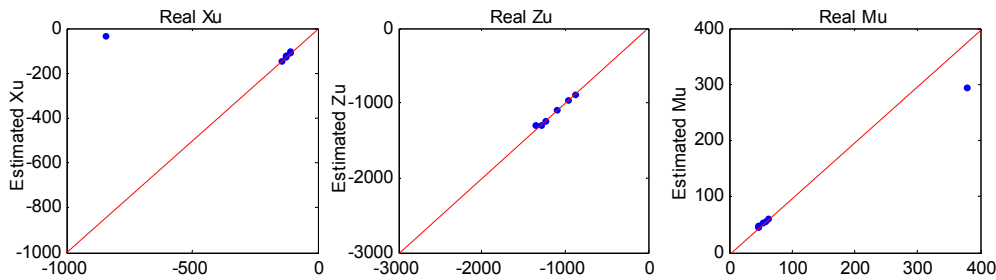


Figure 5 - Comparison between the real and estimated values of X_u , Z_u and M_u .

Estimation Result of the Longitudinal Aerodynamics Derivatives M_w e M_q

Again, the answers of the aircraft in points 3, 6, 1 and 7 of the flight envelope will be used for training and verification of the time responses of the network for cases 4, 5 and 2 respectively. As input for the neural network to get the derivatives M_w and M_q , the discrete w and q time histories and the static data representing the characteristics of the aircraft in each point of the envelope will be used for network training. Figure 6 shows the representative diagram of this neural network.

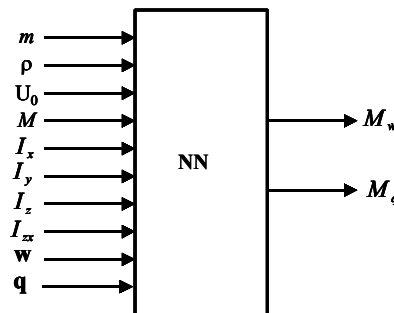


Figura 6 - Block Diagram showing the neural network used to estimate the derivatives M_w and M_q .

Some tests had been carried out and some of the results will be presented. The neural network used in this first test presented 11, 8 and 2 neurons in the intermediate and output layers and the results were sufficiently satisfactory. First the results used for the training of this network had been tested in the neural network and could be verified that the training was good. After that, the estimations for cases 4, 5 and 2 were done and these generalization results were satisfactory, as can be observed in Tab. 4 and Tab 5.

Table 4. Comparison of M_w estimated and real.

M_w	Case 4	Case 5	Case 2
Real	-31.5557	-84.1725	-55.5469
Estimated	-27.8213	-93.6557	-55.0157

Table 5. Comparison of M_q estimated and real.

M_q	Case 4	Case 5	Case 2
Real	-1.6991 (10^4)	-1.9063 (10^4)	-2.3521 (10^4)
Estimated	-2.0508 (10^4)	-1.7013 (10^4)	-2.3733 (10^4)

Figure 7 shows a comparison between the real and estimated derivatives.

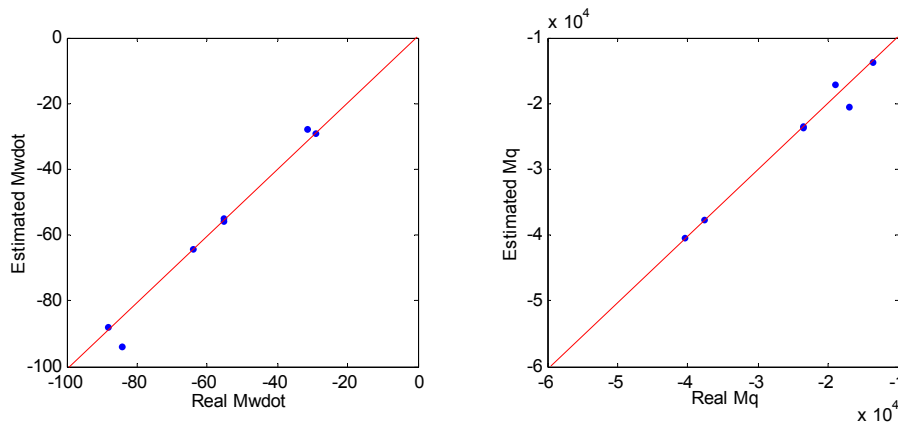


Figure 7 - Comparison between the M_w and M_q “real” derivatives and those estimated by the neural network.

Estimation Results of the Longitudinal Aerodynamics Derivatives X_w , Z_w e M_w

To esteem the derivatives X_w , Z_w e M_w , in a first test, a neural network was implemented, using as dynamic input w and q time histories and as static input all the data used in the previous tests. Figure 8 presents a representative block diagram of the neural network used.

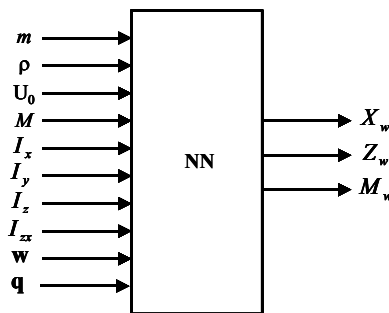


Figure 8 - Block Diagram showing the neural network used to estimate the derivatives X_w , Z_w and M_w

Again, a feedforward and static neural network is used, that is, without delays in the time. The results obtained in the training of a neural network with 68, 12, 8 and 3 neurons respectively in the layers will be presented. Again the answers of the training had been very good and the results of generalization for cases 4, 5 and 2 respectively, are presented in Tab..6, Tab. 7 and Tab. 8.

Table 6. Results of X_w estimated and real.

X_w	Case 4	Case 5	Case 2
Real	-300.4027	-320.0565	-250.7358
Estimated	-305.2752	-162.3623	-305.7163

Table 7. Results of Z_w estimated and real.

Z_w	Case 4	Case 5	Case 2
Real	-3.1818 (10^3)	-3.1493 (10^3)	-4.4191 (10^3)
Estimated	-2.9792 (10^3)	-2.6338 (10^3)	-4.3454 (10^3)

Table 8. Results of M_w estimated and real.

M_w	Case 4	Case 5	Case 2
Real	-1.0654 (10^3)	-1.3290 (10^3)	-1.5354 (10^3)
Estimated	-1.4561 (10^3)	-2.2865 (10^3)	-1.6545 (10^3)

Figure 9 shows o gráfico representativo dos resultados obtidos por esta rede.

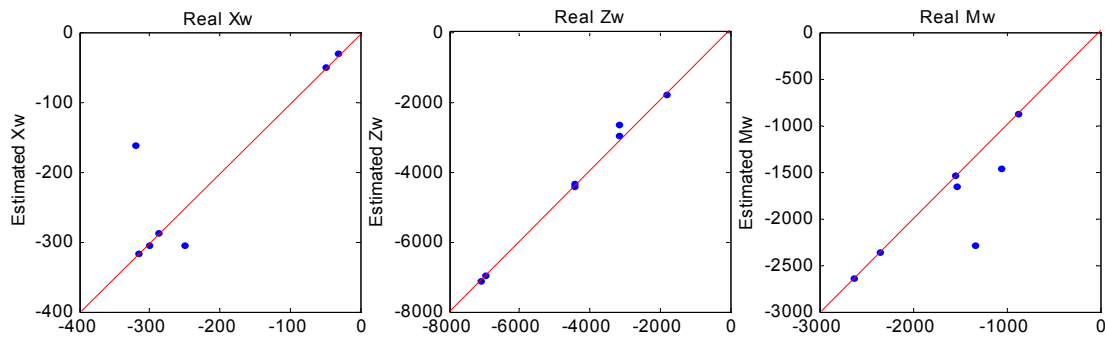


Figure 9 - Comparison between “real” X_w , Z_w and M_w and those estimated by neural network model.

Aircraft simulation using “real” and estimated derivatives for Case 4

To validate the estimation process, the “real” and estimated derivatives were separately loaded into the aircraft simulator to obtain time histories responses corresponding to “real” and estimated derivatives. Then, these time histories were accordingly plotted to carry out a comparison between them. In this work the results only Case 4 of the flight envelope will be presented in order to not to extend the paper, although other cases had been tested presenting satisfactory results.

Figure 10.a shows the input elevator used, which was obtained adding noise to the step input and Figure 10.b shows the u time histories responses using the real derivatives and the estimated ones loaded separately into the simulator.

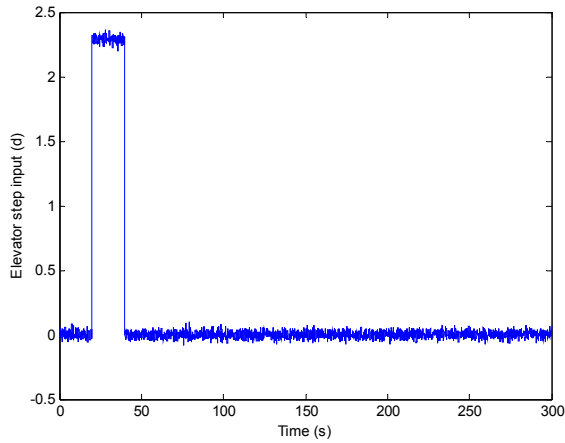


Figure 10.a – Elevator step input used for simulation.

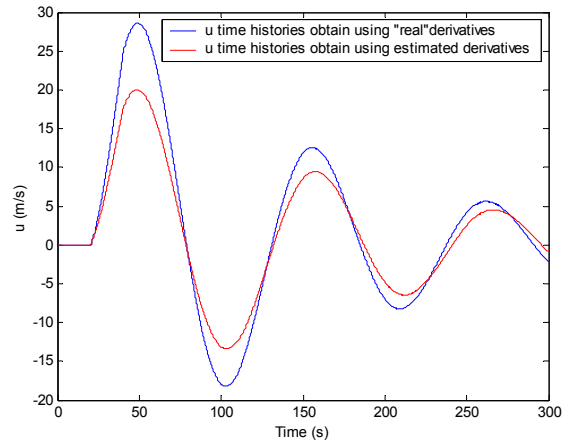


Figure 10.b - u time histories obtain using “real” and estimated derivatives for Case 4.

Figure 11.a and Fig. 11.b show the w and q (pitch) velocities also due to input presented in Fig. 10.a.

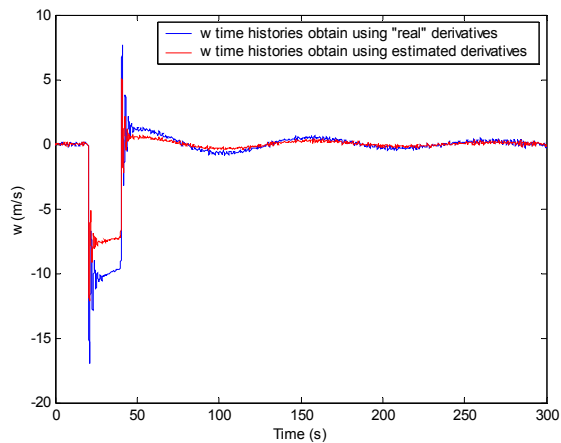


Figure 11.a - w time histories obtain using “real” and estimated derivatives for Case 4.

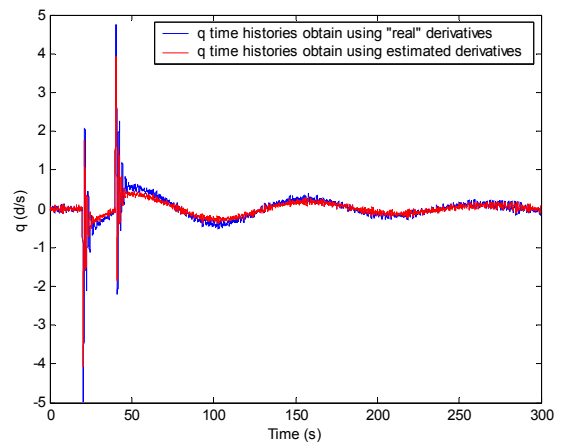


Figure 11.b - q time histories obtain using “real” and estimated derivatives for Case 4.

CONCLUSIONS

Neural networks had been presented to esteem stability derivatives of the A4-D aircraft. Networks with relatively simple topologies had been used, that is, multilayer perceptron with two hidden layers and statics. The results had been satisfactory. Other tests will be carried out, training the nets for other points of the envelope and will be presented future.

These first results had encouraged the authors to continue the research and, as soon as possible, to use real flight tests data to obtain aerodynamic derivatives of a real aircraft.

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