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Numerical and Experimental Investigations for Rotating Machine Balancing without Test Run

J. Mahfoud, X. De Lépine, N. Lévecque and J. Der Hagopian

LDMS – INSA de Lyon, Bâtiment Jean d'Alembert, 20, avenue Albert Einstein

69100 Villeurbanne (France)

Abstract: Unbalance is inherent to rotating machines and can induce important vibration levels that have to be reduced. Two basic methods are widely used: influence coefficients method and modal method. Based on these methods some variations or combinations had been implemented. Those methods require the measurements of vibration levels with and without trial weights, which can be expensive in term of experimental time consuming. Several methods have been proposed in the literature in order to reduce the trial measurements, through the use of a rotor model. Usually it is a linear finite element based model without gyroscopic effects. In this work we propose a new method for the balancing of rotating machines without trial weights. It is based on the modeling of the rotating machine with an equivalent reduced model, taking into account gyroscopic effects that have to be identified experimentally. The identification is performed using a limited number of exciting forces and radial displacement sensors located along the axis of the shaft. An optimization procedure based on the least-square method was used. The reduced model has the same number of degree of freedom as the number of sensors used. Once the state space matrices identified, the correction weights can be determined, in real time and for all speeds of rotation in the considered frequency range. The proposed method is assessed numerically and experimentally. The results obtained show the effectiveness of the method proposed.

Keywords: Balancing, identification, inverse model, rotor-dynamic, experiments

NOMENCLATURE

A = evolution matrix
A^* = speed dependant evolution matrix
B = command matrix
C = damping matrix, N.sec/m
F = force, N
K = stiffness, N/m
M = masse, Kg
x, y, z = system axis
X = state vector

Greek Symbols

δ = displacement, m
δ° = speed, m/sec
$\delta^{\circ\circ}$ = acceleration, m/sec2
ϕ° = rotational speed, rad/sec
$\phi^{\circ\circ}$ = angular acceleration, rad/sec2
Ω = rotational speed, rad/sec

Subscripts

b relative to bearings
g relative to gyroscopic

INTRODUCTION

The progression of rotating machines towards increasingly high specific powers induces high-level vibrations that have to be controlled especially for critical speeds crossing. Unbalances that cause synchronous vibrations, are one of the most important defects. Balancing is a way of limiting these vibrations by reducing directly the disturbing unbalance.

Balancing methods are divided into two basic categories: modal methods and influence coefficient methods. Based on these methods some variations or combinations had been implemented (Bishop and Glandell, 1959; Goodman, 1963; Darlow, Smalley and Parkinson, 1981; Rieger, 1986; Mahfoudh, Der Hagopian, and Cadoux, 1988). The modal method, based on a rotating structure model, determines experimentally the disturbing unbalance associated with a specific mode. The influence coefficient method uses an experimental model based on the linear relationship between displacements and forces (unbalances) and for small displacements. Those methods require the measurements of vibration levels and its angular phase with respect to a phase reference with and without trial weights, which can be expensive in term of experimentation and are time consuming. As the transition matrix between displacements and forces is speed dependent, if the rotor operating conditions evolve, the balancing procedure must be carried out again, and the interruption of normal operation is required.

Recent works and research propose new approaches that reduce the time consuming due to trial weights. Gnielka (1983) proposed a model balancing method without test runs. The modal characteristics of the model and the modal unbalance forces are identified for each considered mode by using an iterative procedures. Edwards, Lees and Friswell (1997) determine the unbalance and support parameters simultaneously by using a run-down unbalance response that is compared with a rotor model. The optimization procedure was based on the least-square method. Xu, Qu and Sun (2000) proposed an influence coefficients based method without trial weights. Measured system response is compared with a finite element model response. The unbalance is determined by using genetic algorithm. El-shafei, Kabbany and

Younan (2004), proposed a modal based method without trial weights. Assis and Steffen (2003) developed strategies in order to use optimization techniques for determining parameters of gyroscopic systems and they commented about the difficulties that arise in using classical optimization algorithms due to the presence of local minima. The identification of the unbalance of flexible rotors can be characterized as an inverse problem. Saldarriaga and Steffen (2003) proposed a numerical study for a method that does not use trial weights, applied to rotors mounted on non-symmetrical bearings. Saldarriaga et al (2006) applied and validate the method experimentally. Most of the proposed methods use a mathematical model to calculate the unbalance forces, natural frequencies and vibration mode shapes whish is based on the finite element method. The model must be precise enough to describe closely the dynamic behavior of the studied system with, in some cases, an important number of parameters to be identified, that is not practical especially in the case of non-symmetric or highly damped structures.

The method proposed in this paper enable the identification of the unbalance of flexible rotors with small amplitude of radial displacements. The unbalance is calculated from the measured responses of the system and by using the state system model that had been identified previously. The state space representation enables the use of a reduced model that describe closely the behavior of the studied system for a given frequency range. De Lépine, Der Hagopian and Mahfoud (2006) applied this method for the identification and the control of a truss. In this paper, we will present first, the method, then the results obtained in both numerical simulation and experiments and finally, we will conclude.

METHOD DESCRIPTION

The dynamic behavior of a rotor can be expressed by the following matrix differential equation (Lalanne and Ferraris, 1998):

$$[M]\ddot{\delta}+\left[C_b+\dot{\phi}\cdot C_g\right]\dot{\delta}+\left[K+\ddot{\phi}\cdot K_g\right]\delta=F \tag{1}$$

In steady state domain, or for small speed variations, the equation 1 can be written as:

$$[M]\ddot{\delta}+\left[C_b+\dot{\phi}\cdot C_g\right]\dot{\delta}+[K]\delta=F \tag{2}$$

The state system presentation is more convenient for control purposes especially in the case of multi-inputs. Here, balancing is considered as a control action. The equation of motion becomes:

$$\begin{bmatrix}\dot{\delta}\\ \ddot{\delta}\end{bmatrix}=\begin{bmatrix}0 & I\\ -M^{-1}\cdot K & -M^{-1}\cdot C_b-M^{-1}\cdot C_g\cdot\dot{\phi}\end{bmatrix}\begin{bmatrix}\delta\\ \dot{\delta}\end{bmatrix}+\begin{bmatrix}0\\ M^{-1}\end{bmatrix}F \tag{3}$$

For practical raisons, the dynamic matrix can be split into a speed dependant matrix (A^*) that expresses the gyroscopic effects, and a speed non-dependant matrix (A):

$$\dot{X}=[A]\cdot X+\left[A^*\right]\cdot X+[B]\cdot F \tag{4}$$

Where:

$$X=\begin{bmatrix}\delta\\ \dot{\delta}\end{bmatrix}\quad;\quad A=\begin{bmatrix}0 & I\\ -M^{-1}\cdot K & -M^{-1}\cdot C_b\end{bmatrix}\quad;\quad A^*=\begin{bmatrix}0 & 0\\ 0 & -M^{-1}\cdot C_g\cdot\dot{\phi}\end{bmatrix}\quad;\quad B=\begin{bmatrix}0\\ M^{-1}\end{bmatrix}$$

The identification of the unbalance of flexible rotors can be characterized as an inverse problem. From equation 4, the perturbation forces, here only unbalance forces are considered, can be calculated as follow:

$$[B]\cdot F=\dot{X}-\left[[A]\cdot X+\left[A^*\right]\cdot X\right] \tag{5-a}$$

And as the command matrix B is not square, it becomes:

$$F=\left[B^t\cdot B\right]^{-1}\cdot[B]^t\cdot\left[\dot{X}-\left[[A]\cdot X+\left[A^*\right]\cdot X\right]\right] \tag{5-b}$$

The state system matrices have to be identified first, and then the unbalance forces could be calculated. The state vector represents the system responses (displacement, speed and acceleration) that can be measured. The system identification passes trough three steps. First, the free motion of the system at rest is considered. Equation 4 becomes:

$$\dot{X} = [A] \cdot X \tag{6}$$

When the impulse response of the system is measured, the dynamic matrix A can be determined by using an optimization procedure based on the least-square method. The impulse response could be considered as the free motion response if the measurements that correspond to the impulse duration are neglected. The second step consists of the determination of the input matrix B. By measuring the system response due to a sinusoidal excitation at rest, and as the matrix A was already determined, the matrix B can be calculated as follow:

$$\dot{X} = [A] \cdot X + [B] \cdot F \tag{7}$$

The third step is to determine the speed dependant dynamic matrix A^*. This step can be achieved by using the system impulse response for a given speed of rotation, and as the matrix A was already determined, the matrix A^* can be calculated as follow:

$$\dot{X} = [A] \cdot X + \left[A^*\right] \cdot X \tag{8}$$

Once the state system matrices are identified, the excitation forces could be calculated by using equation 5. Thus the obtained system is an equivalent model reduced to the number of degree of freedom considered (number of measurement points).

RESULTS

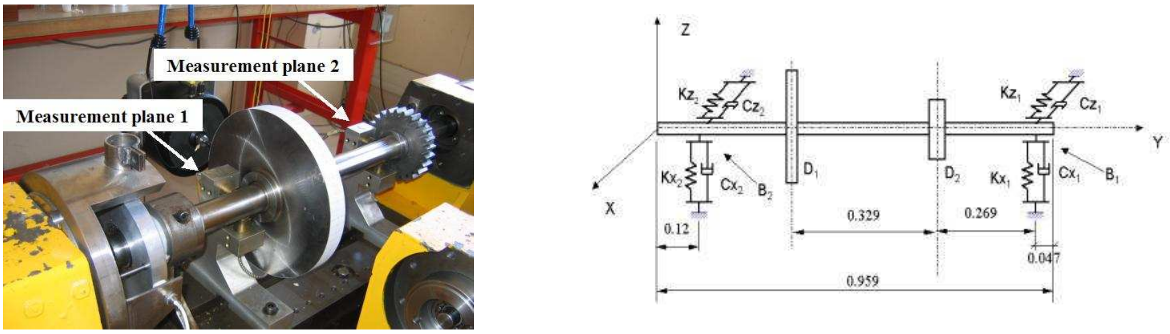

Figure 1 – Rotor test rig

The developed method is applied to a rotor test rig (Fig. 1). The test rig is composed of a horizontal steel flexible shaft of 0.04 m of diameter and two rigid steel discs of 0.03 m of thickness and 0.25 m for D1 and 0.13 m for D2 of diameter . The shaft is supported by bearings, a ball bearing at the right hand end and a roller bearing at the left hand. The bearing characteristics are presented in Table 1. An electrical motor that can accelerate the rotor up to 6000 rpm drives the system. The displacements were measured by using four proximity sensors (Vibrometer TQ 103) along the x and z directions arranged in two planes and quoted 2, 1, 3 and 4 respectively.

Table 1 – Bearing characteristics

Direction	Stiffness N/m	Damping N.sec/m
x_1	$5.0 *10^6$	800
z_1	$5.0 *10^6$	800
x_2	$1.6 *10^6$	800
z_2	$2.4 *10^6$	800

Numerical Simulation

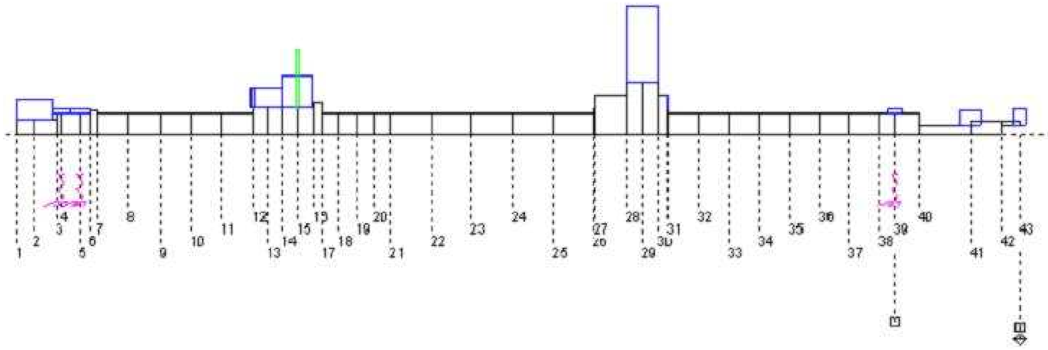

Figure 2 – Rotor finite element model

A finite element model of the rotor test rig was used to carry out numerical simulations (Fig. 2). The rotor model is composed by the following elements: rigid discs with only kinetic energy contribution; flexible shafts exhibiting both kinetic and strain energy; bearings with elastic and viscous dissipation characteristics. Regarding the shaft, it is represented by a Timoshenko beam element with two nodes and 4 degrees of freedom, namely, two displacements and two rotations per node. The software ROTORINSA® was applied to obtain the modal matrices of the rotor necessary for the modal state system presentation. The ten first modes were considered. The simulation of the system behavior was done using MATLAB® and SIMULINK®. Two measurement planes were used placed on nodes #18 and #32 in the both directions Z and X. In this case, the identified system will be equivalent to a system with four degrees of freedom. The modal characteristics, for the six first modes, of the rotor-bearings model are summarized in Tab. 2. The sampling frequency used for data acquisition was 10 KHz.

Table 2 – Initial model characteristics

Mode	Frequency Hz	Modal damping factor
1	52.08	0.054
2	58.07	0.034
3	157.90	0.089
4	170.71	0.082
5	322.45	0.009
6	323.91	0.010

System Identification

The impulse response at rest was generated by applying a half-sinus of 0.004 sec with 1/0.004 N amplitude on node #36. As we are interested by the four first modes, a low pass filter bounded to 290 Hz was applied to the measured responses. This step enables the identification of the dynamic matrix A for the selected frequency range. the modal characteristics obtained are presented in table 3.

Table 3 – Identified model characteristics

Mode	Frequency Hz	Modal damping factor
1	52.26	0.054
2	58.19	0.034
3	158.94	0.089
4	171.88	0.090

The system was then excited by using sinusoidal forces (5-180 Hz in 10sec) applied successively on nodes #15 and #29 (corresponding to the disc positions) in both z and x directions. This step enables the identification of the command matrix B. The use of two measurement planes enables the use of two balancing planes placed at nodes #15 and #29 that corresponds the position of the two discs.

The impulse response for a speed of rotation of 2500 rpm enables the identification of the speed-dependent dynamic matrix A*. The impulse response was generated by using a half-sinus of 0.004 sec with 1/0.004 N amplitude applied to node #36.

The identified system response is compared with the initial system in both transient and steady state domains. An unbalance of 120 g.mm is placed in node #29, and the system was excited by an impulse on node #15 for a rotational speed of 2000 rpm. Only a slight difference in the transient domain can be noticed. This difference is due to the fact that the model identified is an equivalent model reduced to the four first modes.

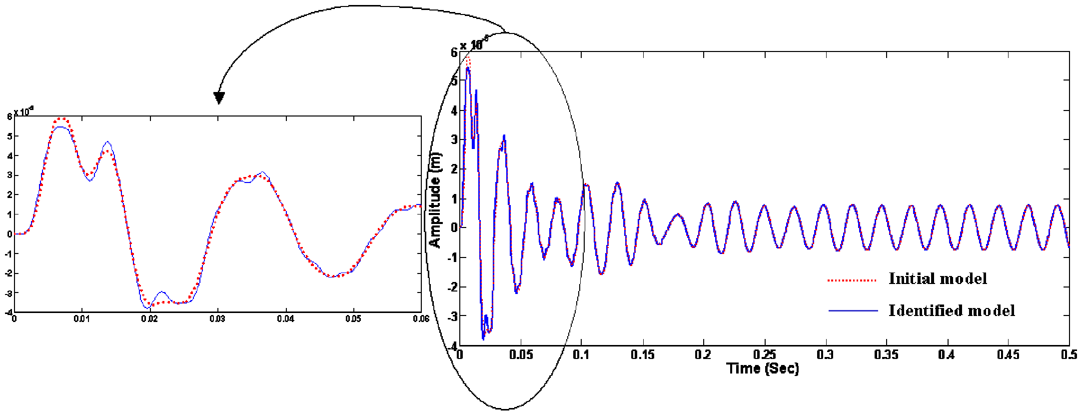

Figure 3 – Impulse system response, 2000 rpm, x direction, node #18

Unbalance Identification

An initial unbalance of 120 g.mm is placed at node #29 at zero degree. Equation 5 enables the determination of the unbalance, using the two possible planes (Tab.4). Calculations were done for a speed of rotation (2300 rpm). The unbalance calculated at node #15 is almost negligible.

Table 4 – Identified unbalance

Node #	Modulus g.mm	Phase °
15	1	121.3
29	120.1	180.4

The aim is to identify the transition matrix between the forces applied and the displacements measured for the whole range of operation speeds. The system matrices (A & B) were identified at rest, and only the impulse response for one speed was needed to identify the gyroscopic effect matrix. The unbalance was then calculated for several speeds of rotation (1000 – 15000 rpm). The variations of the phase and the modulus are presented in Fig.4. The results obtained show that the maximum of variation occurs in the vicinity of critical speeds.

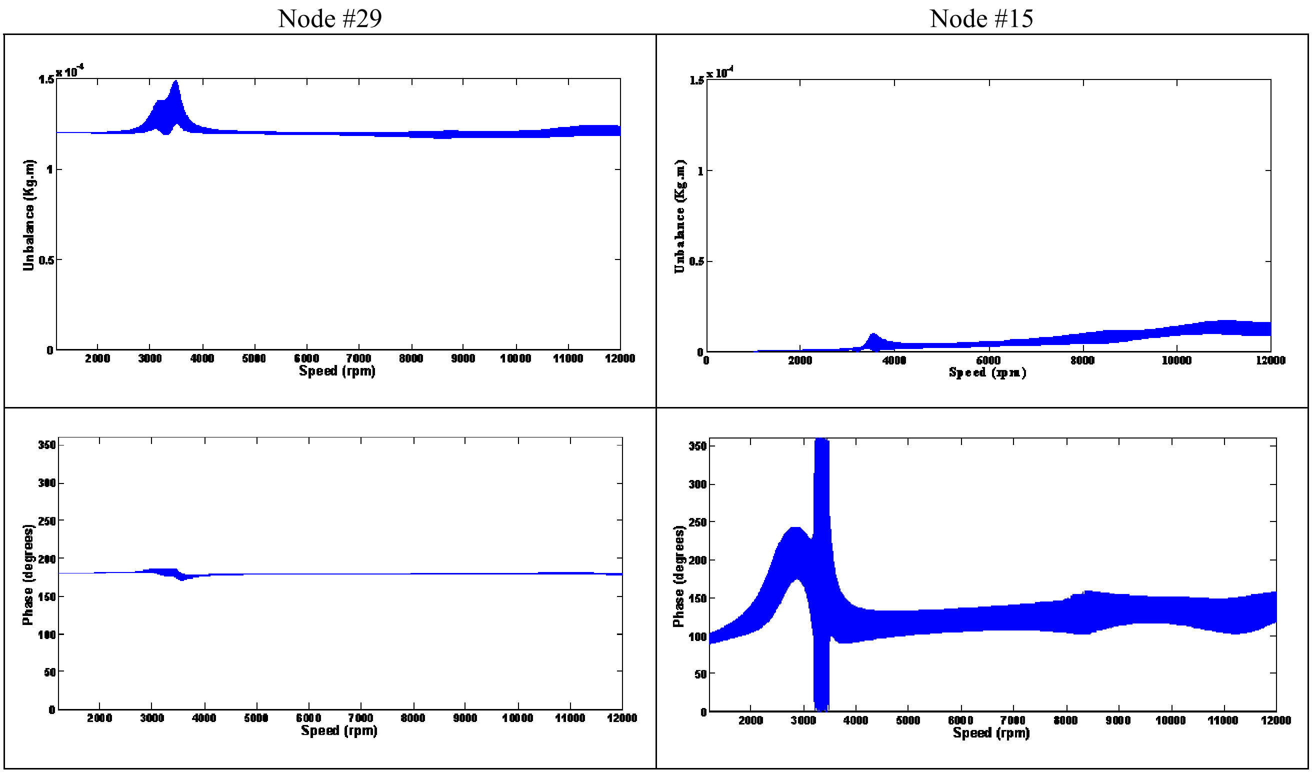

Figure 4 – Unbalance variation / speed

Regarding disc #1 (node #29), the phase variations are negligible; the variations of the unbalance modulus are also negligible excepted around the critical speed. This is due to the fact that the small variations of the transfer matrix of the identified system are relatively important in the vicinity of critical speed.

The method developed is efficient for values of damping factor up 0.1, which represent a high damping factor for mechanical systems (Tab. 5).

Table 5 – Initial and identified model characteristics

	Initial model		Identified model	
Mode	Frequency Hz	Modal damping factor	Frequency Hz	Modal damping factor
1	52.07	0.099	52.26	0.099
2	58.07	0.034	58.19	0.034
3	157.90	0.089	160.29	0.103
4	170.71	0.082	170.30	0.114

Experiments

The data acquisition device was the SCADA III interface of LMS® that enables a real time and simultaneous data acquisition. Several units of LMS® modal analysis software were used for data processing. The sampling frequency was 4096 Hz. An electromagnetic shaker Brüel & Kjaer (B&K), and an impact hammer (B&K 8202) with a piezoelectric force sensor (B&K 8200) were used to generate the system responses under several excitations.

The system impulse responses stemming from a shock oriented 45° with respect to the sensors, applied at 40 mm to the left of the disc #1 are presented in figure 5.

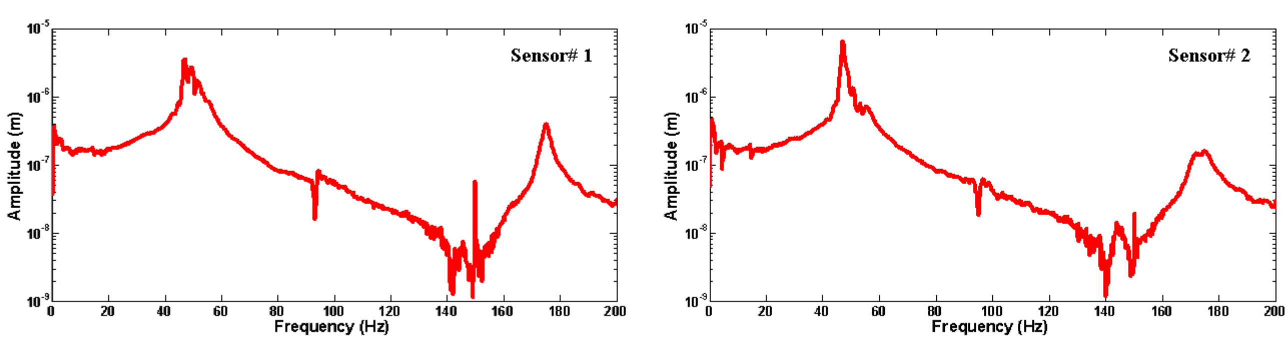

Figure 5 – The system impulse responses - frequency domain.

The frequencies that had been identified and that correspond to the structure were: 47, 48.1, 172.3 and 175.4 Hz. The frequencies correspond to the two first flexible modes in both directions x and z. The system seems to be slightly asymmetric. On the other hand, the position of the disc #1 correspond to a node of the flexible second mode of the rotor, that is why this mode was not excited when the shock was applied to disc #1. As we intend to use two balancing planes two measurement planes were necessary and the system identified is an equivalent system with 4 degrees of freedom.

The different eigenvalues of the identified dynamic matrix (frequencies identified) issued from several impacts at several positions are compared. Only slight variations were noticed.

This step enables the identification of the evolution matrix by using Eq.6. The state vector was built by using the displacements issued from measurements, and the numerical derivative of the displacement produces the velocity. The average results from four acquisitions (frequencies and damping factors) obtained are presented in table 6.

Table 6 – Identified system characteristics

Mode	Frequency Hz	Modal damping factor
1	47.5	0.0056
2	49.3	0.0239
3	170.7	0.0102
4	176.2	0.0063

The measurements were repetitive; the maximum variation coefficient (standard variation/mean) of the frequencies was less than 2%.

A sinusoidal excitation (20-250 Hz in 8 sec) applied in both z and x directions consecutively at rest enables the identification of the command matrix by using the equation 7. The excitations were applied to the two discs as they supposed to be the balancing planes.

The non-coincidence between excitations and measurements leads to a slight modification of the command matrix. The new command matrix corresponds to the initial one with a coefficient of reciprocity displacements-forces.

The measurements of the system responses are compared to the identified model, excited by using sinusoidal forces (20-250 Hz in 8 sec) applied in both z and x directions respectively at rest (Fig. 6).

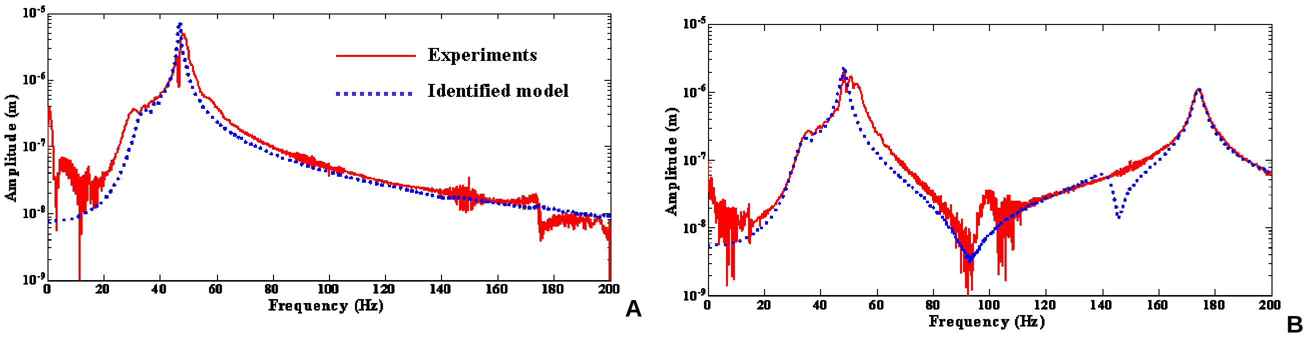

Figure 6 – system responses in frequency domain: A- Excitation disc #1 in horizontal direction, sensor #2; B-Excitation disc #2 in vertical direction, sensor #4

The identified model describes globally the dynamic behavior of the studied system. It can be noted that the measurement noises influence slightly the results obtained.

The system responses issued from measurements and from the identified model due to a harmonic excitation of 60 Hz at rest were compared in figure 7.

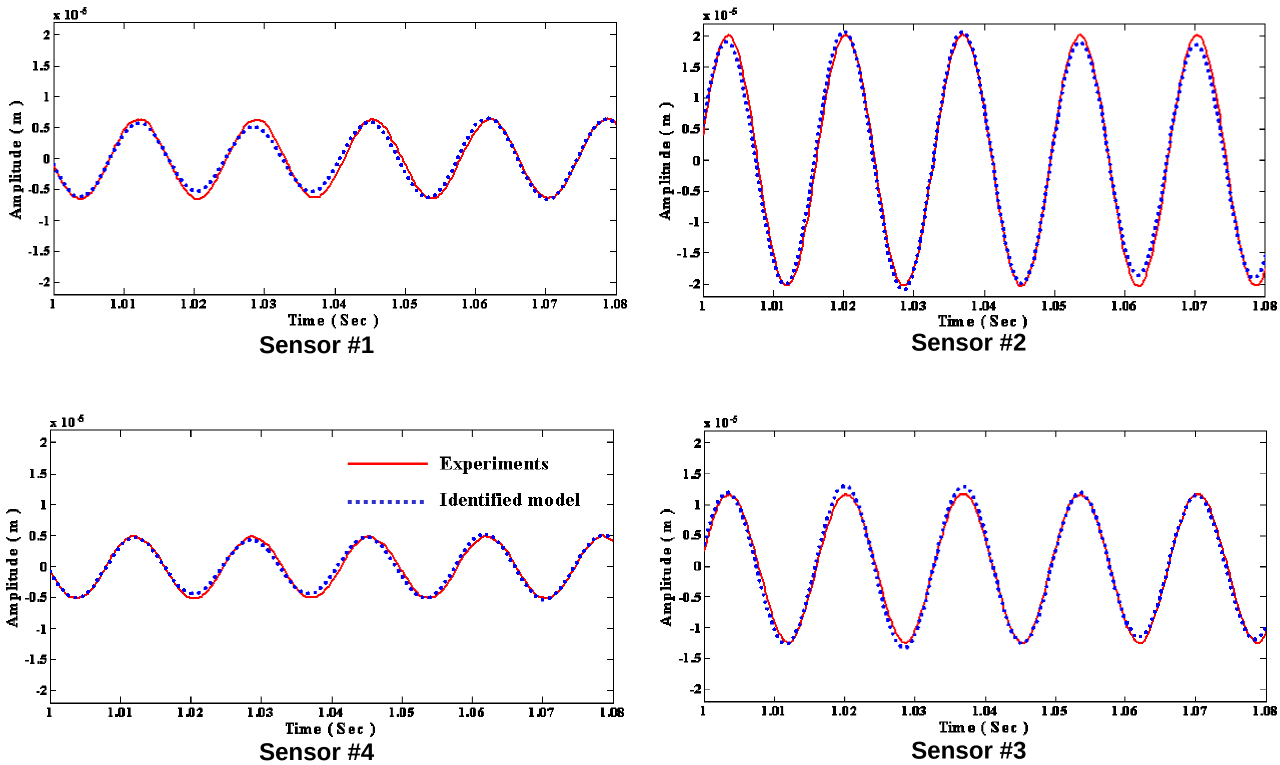

Figure 7 – Harmonic responses due to a 60 Hz excitation

The force was applied using the electro-dynamic shaker on the disc #1 in the x direction. The differences between measured and estimated displacements are almost negligible which indicates that the command matrix was successfully identified.

The identification of the speed dependant dynamic matrix A^* is achieved by using the measurement stemming from the impulse response of the system for a speed of rotation of 2055 rpm. The speed was selected such to have significant displacement and a stable orbit. A shock using the impulse hammer was applied to bearing #2. The speed dependent dynamic matrix can be identified from Eq.8.

As a rotating machine is never perfectly balanced, the measured responses contain the displacements due to the shock and the displacements due to the unbalance simultaneously. The impulse response for the speed of rotation selected could therefore be obtained by subtracting the effect of the unbalance.

The system response was measured first at 2055 rpm, which represents the unbalance effects, and then, the shock was applied and the measured response represents the unbalance and the impulse effects. This operation imposes to have the two signals to be synchronized and to have the same number of samples per period, which was relatively difficult due to the slight variation of the speed and due to the fact that the sampling frequency was time dependent.

The effect of the speed dependent dynamic matrix could be neglected especially for low speeds of rotation. Even in this case, the unbalance forces could not be calculated. The proximity sensors used to measure the displacement for the nominal speeds, take into account a run out which is due to heterogeneity of the shaft material. This problem could be overcome by subtracting the initial run out that can be measured at low speed, in this case the two signals have to be synchronized and must have the same number of samples per period.

The measured impulse response of the system for a speed of rotation of 2000 rpm is compared to that generate with the identified model neglecting the gyroscopic effects. The shock on the identified model was applied to disc #2, for the test rig the shock was applied 400 mm to the left of disc #1. As the test rig wasn't perfectly balanced, an unbalance of 880 g.mm/120° was added on disc #1, and 490 g.mm/-40° on disc#2 for the identified model. Those masses correspond to the unbalance estimated by using the influence coefficient method. Figure 8 represents the measurements issued from sensor #2. It can be noticed that the generated response is similar to that measured with less harmonics, which is normal as the system was identified for a limited frequency range (0-290 Hz).

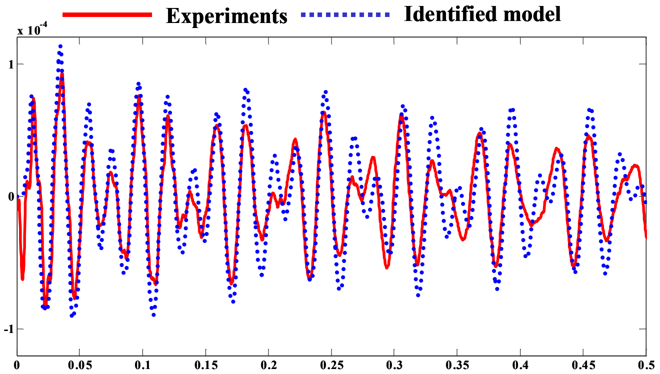

Figure 8 – Impulse system responses, 2000 rpm, sensor #2

CONCLUSIONS

A new method for the balancing of rotating machines without trial weights is proposed. It is based on the modeling of the rotating machine with an equivalent reduced model, taking into account gyroscopic effects that have to be identified experimentally. The identification is performed using a limited number of exciting forces and displacement sensors located along the axis of the shaft. The reduced model has the same number of degree of freedom as the number of sensors used. Numerical simulations show that the reduced identified model reproduces closely the dynamic behavior of the studied system in both transient and steady state domains. The calculated unbalance corresponds to the initial one. The unbalance could be calculated for all speeds in the considered frequency range. It was noticed that calculation must not be done in the vicinity of critical speeds. Simulations were carried out for relatively high damping factors (up to 0.10) and the method still efficient. Additional researches are carrying out in order to optimize the unbalance distribution based on pseudo-random methods.

The experimental identification of the system was achieved at rest successfully, the obtained reduced model describe relatively closely the behavior of the studied system. In this contribution, the gyroscopic effects and the unbalance forces were not determined due to the use of time sampling method. Researches are now carrying out in order to apply the angular sampling.

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