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Signal Analysis Applied to the Parametric Identification of Non-Linear Vibratory Systems

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Abstract: Nowadays the use of mechanical non-linear devices to control the dynamic behavior of vibrating structures is increasing. Modern engineering applications use materials with non-linear properties such as springs, viscous, and Coulomb dampers, to reduce the structure vibration. The main objective of this paper is to study the vibration signals generated in a simulation of a vibratory system with non-linear stiffness using a non-linear signal analysis formulation. A two input and one output MISO model is proposed to represent the non-linear system dynamics. The global measured output is considered to be the sum of two outputs produced by one linear path associated in parallel with a non-linear path. This last path has a non-linear model that represents non-linear stiffness and another linear transfer function connected in series. Since the linear path is identified by traditional signal analysis, the non-linear function can be evaluated by using the global input/output relationships, and can be used to identify physical parameters of the non-linear stiffness. The input and output probability density functions and the coherence functions are used to quantify the system non-linear behavior. Simulations are conducted on a single degree of freedom system coupled to a cubic non-linear spring, to validate the non-linear model identified by the proposed methodology.

Keywords: Non-linear systems, Signal analysis, Vibration.

NOMENCLATURE

p = probability density functions (PDF).
x = global input of the non-linear MISO system.
y = global output of the non-linear MISO system.
y_h = linear part of global output.
v =output of the zero memory non-linear system.
y_v = non-linear part of global output.
u = output of the zero memory non-linear system statically uncorrelated with x .
y_u = non-linear part of global output statically uncorrelated with y_o .
y_o = linear part of global output statically uncorrelated with y_u .
S = power spectral densities.
H = transfer function.
L = transfer function.
A = transfer function.

Greek Symbols

γ = coherence function.

INTRODUCTION

Physical parameters of the nonlinear system cannot be identified by traditional linear analysis since this technique implies that the probability density function (PDF) of the inputs and outputs should have the same nature, condition not verified by nonlinear systems. (Bendat, 1998)

Volterra series theory can be used to represent nonlinear systems using sets of high order polynomial functions assembled in parallel. Bendat *et al.* (1992) proposed a new methodology that uses the well-known theory of multiple input/single output linear systems (MISO) modified to be applied to signal analysis of non-linear systems. Since the Volterra methodology is highly dependent on the probability density function of input signals, Bendat (1998) proposed a methodology that is independent of the PDF of the input signals and that produces results that are easy to interpret. This methodology is called nonlinear MISO signal analysis.

The nonlinear signal analysis is based on the hypothesis that the nonlinear characteristics of the system are additive to its linear characteristics and that the nonlinear effects acting on one input produce instantaneous output.

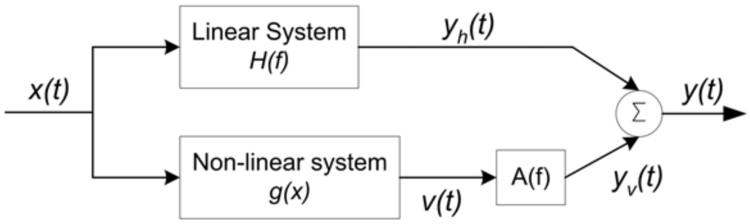

Figure 1 - Nonlinear MISO systems block diagram.

The nonlinear system represented in the Fig. 1 does not have limitations regarding the characteristics of the PDF of the signal input $x(t)$, as imposed by Volterra models.(Bendat, 1998)

To use this methodology a prior knowledge of the nonlinear function $g(x)$ is required. As in the linear MISO theory, the system represented in the Fig. 2, does not impose any restrictions on the correlation level of the input signals $x(t)$ and $v(t)$, so the system can be represented by Fig. 2, where $x(t)$ and $u(t)$ are uncorrelated.

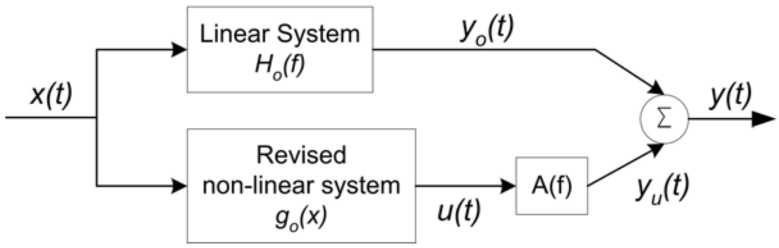

Figure 2 - Nonlinear MISO system with uncorrelated inputs.

To the systems of the Fig. 1 and Fig. 2 the spectral densities, transfer and coherence functions are calculated by:

$$\begin{array}{lll} L_{xv} = \dfrac{S_{xv}}{S_{xx}} & S_{uy} = S_{vy} - L_{xv} * S_{xy} & H_o = S_{xy}/S_{xx} \\ S_{y_o y_o} = \left|H_o\right|^2 * S_{xx} & S_{uu} = S_{vv} * \left(1 - \gamma_{xv}\right) & A = S_{uy}/S_{uu} \\ H = H_o - \left(S_{xy}/S_{xx}\right) * A & S_{y_h y_h} = \left|H\right|^2 * S_{xx} & S_{y_u y_u} = \left|A\right|^2 * S_{uu} \\ S_{y_v y_v} = \left|A\right|^2 * S_{vv} & \gamma_{xv} = \left|S_{xy}\right|^2 / S_{xx} * S_{vv} & \end{array} \tag{1}$$

The system presented by Fig. 1 and Fig. 2 have only one nonlinear function, however it should be noticed that this methodology can represent systems with more than one nonlinear behavior. In this case the nonlinear MISO system representing each nonlinear function is precede by a linear frequency response function (FRF), which represents the frequency dependency. The nonlinear MISO formulation permits an easy correlation of estimated parameters with physical properties of the assumed model of nonlinear systems. (Lepore *et al.*, 2004)

VIBRATORY SYSTEMS WITH NONLINEAR PROPERTIES

The proposed methodology could be applied in two different cases of analysis:

a) Analysis and identification of a non-linear system.

b) Identification of the non-linear forces applied to a linear system.

The case (a) consists in characterize the dynamical behavior of a vibratory system by means of the proposed methodology. This is the case of the several vibratory systems encountered in most common engineering applications. The case (b) consists of a signal treatment to be used when is necessary to measure non-linear forces using linear instrumentation devices.

These two cases cover a major part of non-linear problems in mechanical sciences and have significant scientific interest as demonstrated by the following research works, which are examples where the proposed methodology has direct application: Goge *et al.* (2005) identified 15 different types of non-linearities that were present as excitation of linear subsystems or even due non-linear behavior of the materials and structures. These forces were directly used in the proposed non-linear MISO analysis. Lepore *et al.* (2005) applied the non-linear MISO analysis to estimate the friction force in material wear tests. Leonard *et al*. (2001) used a non-linear formulation to represent the internal force produced in a cracked beam. Cveticanin (1998) used a non-linear formulation to represent the dynamical behavior of a variable mass rotor/fluid system.

This non-linear signal analysis methodology is applied to the mechanical system shown in Fig. 3. The linear parameters m, c and k are, respectively, the mass, viscous damper and stiffness physical properties. A generic nonlinear device $g(x)$ is connected in parallel to the suspension, and an external force excites the system. The time domain mathematical model is presented in Eq. (2) and the correspondent frequency domain model in the Eq. (3) is obtained by taking the Fourier Transform of the Eq. (2).

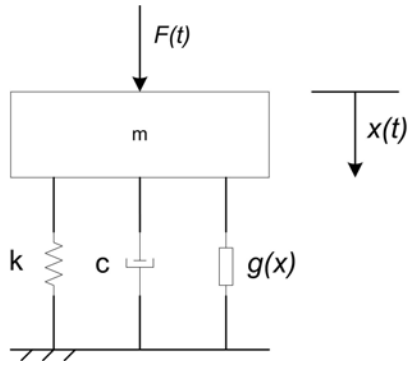

Figure 3 - One degree of freedom mechanical system with a nonlinear device.

$$m\ddot{x}+c\dot{x}+kx+ag(x)=F(t) \tag{2}$$

$$F(f)=[H]X(f)+A(f)\mathscr{F}(g(x)) \tag{3}$$

The term $A(f)$ represent the frequency dependencies of the nonlinear device and $\mathscr{F}(g(x))$ is the Fourier transform of the nonlinear input $v(t)$ as shown by Fig 1. Eq. (2) and Eq. (3) show that there is no restriction on the nature of nonlinear function $g(x)$ and that the nonlinear and linear terms are additive, according MISO theory. (Bendat, 1998)

The same procedure can be applied to systems with several degrees of freedom and several nonlinear effects without loss of generality. This formulation can be applied to identify the nonlinear force acting on a linear system, using the measured system response.

The analyses of linear systems submitted to non-linear forces are of great interest to many fields of research, since in some real experiments the excitation force applied to the linear system is non-linear.

A generic linear system excited by a nonlinear force is represented in Fig. 4 and its mathematical model is given by Eq. (4).

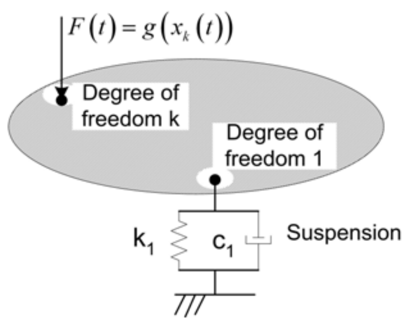

Figure 4 - Linear system subjected to a nonlinear force.

$$[m]_{nxn}\{\ddot{x}\}_{nx1}+[c]_{nxn}\{\dot{x}\}_{nx1}+[k]_{nxn}\{x\}_{nx1}=\{F\}_{nx1} \tag{4}$$

The vector $\{F\}$ includes the nonlinear forces. Using the same notation of Eq. (2) the nonlinear force can be written:

$$\{F\}_{nx1}=-[A]_{nxn}\{g(x)\}_{nx1} \tag{5}$$

Matrix $[A]$ is diagonal with non-zero elements only at the positions corresponding to the degrees of freedom where the nonlinear forces act. The same condition is verified for the vector $\{g(x)\}_{nx1}$ where no null nonlinear functions exist only at each corresponding degree of freedom. Assuming that the system is connected to the inertial reference by a spring and damper, located at the degree of freedom “1”, representing for example, a load cell that connects this degree of freedom to the ground and produces F_{load_cell} , Eq. (4) and Eq. (5) can be rewritten as follows.

$$\begin{aligned}&[m]_{nxn}\{\ddot{x}\}_{nx1}+[c']_{nxn}\{\dot{x}\}_{nx1}+[k']_{nxn}\{x\}_{nx1}=-[A]_{nxn}\{g(x)\}_{nx1}+\{F_{load\_cell}\}\\&\{F_{load\_cell}\}=-[c'']_{nxn}\{\dot{x}\}_{nx1}-[k'']_{nxn}\{x\}_{nx1}\end{aligned} \tag{6}$$

Where:

$$\begin{aligned}&[c']_{nxn}=[c]_{nxn}-[c'']_{nxn}\\&[k']_{nxn}=[k]_{nxn}-[k'']_{nxn}\\&[c'']_{nxn}=\begin{bmatrix}c_1&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&0\end{bmatrix}_{nxn}\\&[k'']_{nxn}=\begin{bmatrix}k_1&\cdots&0\\\vdots&\ddots&\vdots\\0&\cdots&0\end{bmatrix}_{nxn}\end{aligned} \tag{7}$$

$$[m]_{nxn}\{\ddot{x}\}_{nx1}+[c']_{nxn}\{\dot{x}\}_{nx1}+[k']_{nxn}\{x\}_{nx1}+[A]_{nxn}\{g(x)\}_{nx1}=\{F_{load\_cell}\} \tag{8}$$

The frequency domain representation of the linear system, subjected to nonlinear forces, is finally obtained by taking the Fourier transform on both sides of Eq.(8).

$$\{F_{load\_cell}(f)\}_{nx1}=[H'(f)]_{nxn}\{X(f)\}_{nx1}+[A(f)]_{nxn}\{\mathscr{F}(g(x_k))\}_{nx1} \tag{9}$$

Using Eq.(9) and the concept presented in Fig. 1, the correct representation of a MISO system with nonlinear excitation force is constructed.

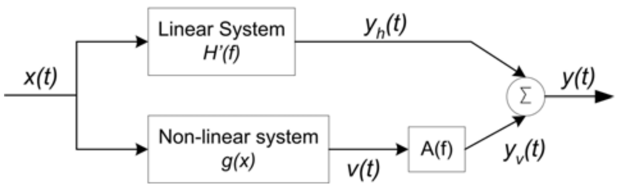

Figure 5 - MISO system used to indirectly determine the nonlinear force.

Matrix $[H]$ is replaced by matrix $[H']$ to represent the FRF of the system in Fig.4 with the terms k_1 and c_1 removed from stiffness and damping matrix and transferred to the right side of Eq.(8). These are representing the force measured by the load cell. It should be noted that the force measured by the load cell includes the linear component

$S_{y_h y_h}$ and the nonlinear component $S_{y_v y_v}$. Moreover, to obtain the true nonlinear force, by means of nonlinear MISO signal analysis, a prior knowledge of nonlinear function $g(x)$ is required.

STATISTICAL METHODOLOGY TO IDENTIFY NON-LINEAR FUNCTIONS

Bendat (1998) affirms that the following statements, related to the system represented in Fig. 6, are true:

a) When $p(x)$ is Gaussian and $g(x)$ is linear, then the resulting $p_2(y)$ will be Gaussian.

b) When $p(x)$ is Gaussian and $g(x)$ is non-linear, then the resulting $p_2(y)$ will be non-Gaussian.

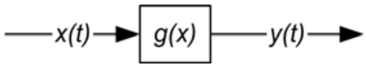

Figure 6 – Zero memory non-linear system

Therefore, the non-linear behavior of the system can be verified comparing the probability density functions of the input with that of the output.

Additionally, if the non-linear function $g(x)$ is a single-valued and one-to-one function, it can be determined by setting the probability distribution function $P_2(y)$ equal to the $P(x)$ and the relation between the probability distribution functions of input and output is written in Eq. (10). (Bendat, 1998)

$$P_2(y_0) = \int_{-\infty}^{y_0} p_2(y)\,dy = \int_{-\infty}^{x_0} p(x)\,dx = P(x_0) \tag{10}$$

Equation (10) is calculated with each value of x_0 determining the $P(x_0)$ from the measured $p(x)$. The value y_0 is determined from measured $p_2(y)$, since $P_2(y_0) = P(x_0)$ this procedure will produce the variable pair (x_0, y_0) that is the representation of $y_0 = g(x_0)$. A schematic representation of the suggested procedure to identify the non-linear function $y = g(x)$ is shown in Fig.7, where the horizontal red line represents the equal probability of occurrence of the input x_i and of the output $y_i = g(x_i)$. The procedure is done for all probability values from zero (at the bottom of the graphic) to one. (at top of the graphic), resulting a set of ordered input and output values that are the numerical estimation of non-linear function $y = g(x)$.

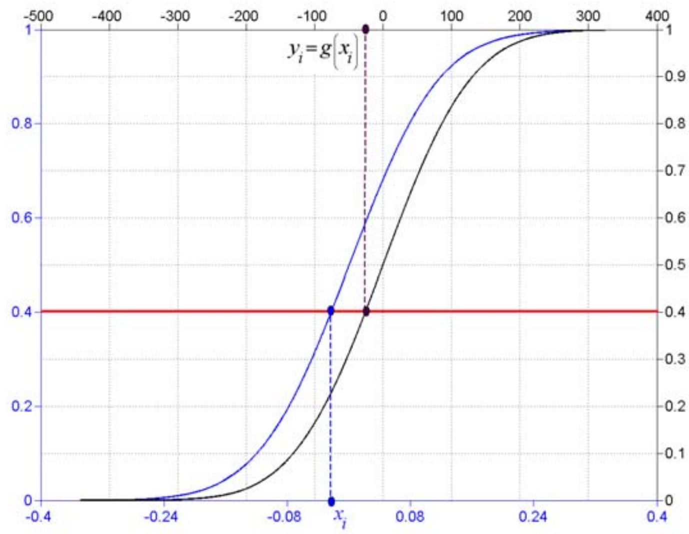

Figure 7 – Representative diagram of Eq. (10)

The two statements presented by Bendat (1998) are useful to identify the presence of a non-linearity in the vibratory system. However, sometimes the methodology presented in Fig. 7 and in the Eq. (10) could be not directly applied to real systems. Generally in these systems is not possible measure directly the effects of non-linearity, since the theorem shown in the Eq. (10) can be applied only to the measured global input, represented by $x(t)$, and to the measured global output, represented by $y(t)$ as shown in the Fig. 1. In this case the system of Fig. 1 is not a zero-memory non-linear system, which is an important condition imposed by Bendat (1998).

In this paper, it is shown that the theorem presented in Eq. (10) can be applied using the global input and global output of the non-linear system as present in Fig. 1. However the estimated function is useful only to separate the non-linear parts from the linear parts, represent by y_h and y_v, and this procedure does not allow to reconstruct the non-linear function, as proposed by Bendat (1998). The application of Eq.(10) using the measured global input and output remains useful to identify the linear path of the system, to separate the linear and non-linear parts of the global output. These characteristics are presented in the following section.

NUMERICAL SIMULATIONS AND RESULTS

A model of a single degree linear vibratory system coupled to a non-linear spring was studied. Two separated cases of cubic spring were simulated: one has hardening spring and the other has a softening spring. The imposed excitation force is a zero mean white noise with RMS amplitude equal to 10 N. The Runge-Kutta method was used to integrate the system non-linear equation of motion, using a 0.5 milliseconds time step, so that there are at least 100 sampled points in each natural period of the linear system. Figure 8 shows the physical model of the vibratory system, the values of its linear parameters, and the behavior of the non-linear spring, which were used in all computational simulations.

A first simulation of the linear vibratory system without the non-linear function was done to obtain its the linear response to be used as reference. The methodology shown in Fig. 1, Fig. 2 and in the Eq. (1) was applied with the functions $g(x)$ described in Fig. 8 and also with the function $g(x)$ estimated by the application of the Eq. (10) using the global input $x(t)$ and the global output $y(t)$. The numerical results are compared using the estimated linear transfer functions (TF), the cumulative coherence functions, the power spectral density of the linear part of the global output and the power spectral density of the non-linear part of the global output.

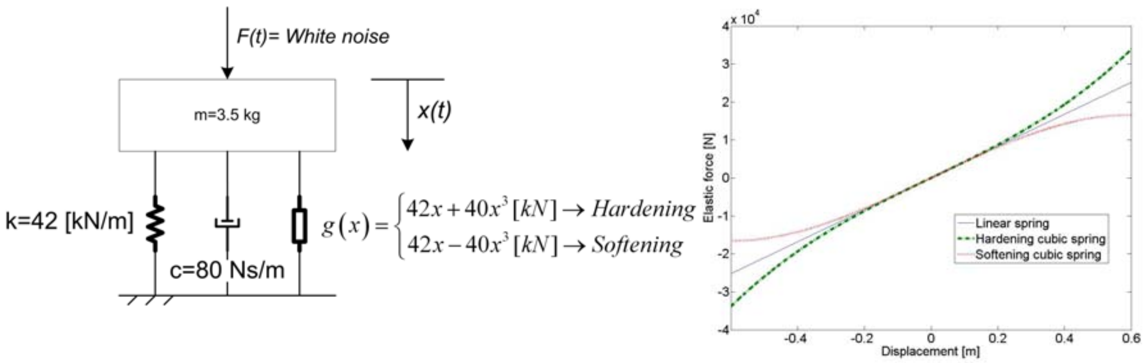

Figure 8 – Non-linear vibratory system.

Figure 9 and Fig. 10 respectively show the numerical results of the non-linear system with a hardening cubic spring and a softening cubic spring. In these figures were considered as exact non-linear functions the functions $g(x)$ represented in Fig. 8. The power spectral densities were calculated using the PWELCH methodology, as show in Press *et al.* (1992), with samples of 1024 data points. The hanning window was used to reduce leakage error and the power spectral density functions were calculated by averaging 40 samples.

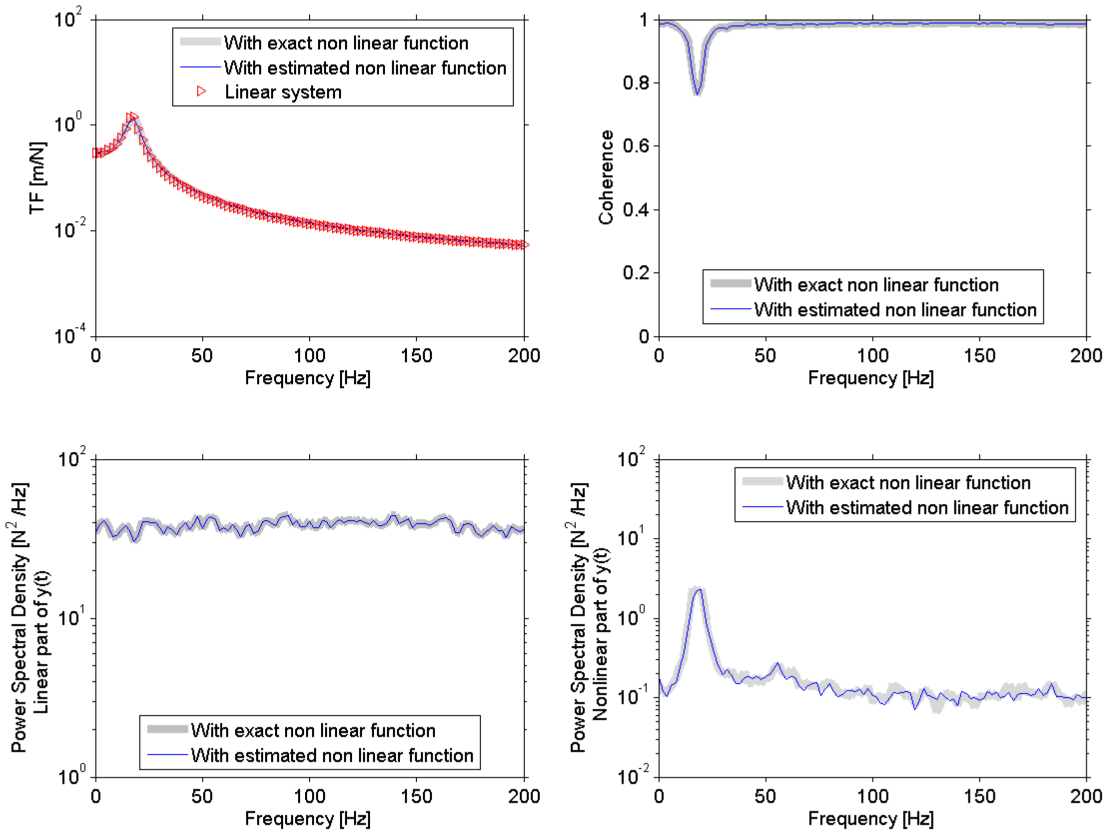

Figure 9 – Simulations results of a linear system with a hardening cubic spring.

It can be seen in the left upper corners of Fig. 9 and Fig. 10 that the estimated linear systems transfer functions are very close to the reference TF calculated for the linear system, regardless the type of used non-linear spring. This result indicates that the proposed methodology is capable to precisely identify the linear system path.

The same conclusions are valid for the $y(t)$ power density functions of the linear and the non-linear paths estimated by the proposed methodology, shown at the bottom of Fig. 9 and Fig. 10. The non-linear analysis performed with the non-linear function estimated by the MISO methodology produce results very close to those calculated with the exact non-linear function. Therefore it is possible to assume that the non-linear path, defined by the product of $g(x)$ by $A(f)$, is correct. But, it should be noted that the estimated non-linear function using Eq. (10) with the global input and output of the non-linear vibratory system is not the same $g(x)$ function used in the simulations. This is due to the difference between the probability functions of $y(t)$ and the probability function of the output of the zero memory non-linear system, since the hypothesis of a zero memory non-linear, as supposed by Bendat (1998), is not verified in this case. This result indicates that the estimated non-linear function is responsible only to provide the nature of the non-linearity to the input $x(t)$ and that $A(f)$ adjusts the gain and the time delay, in such way that the estimated output is coincident with that output produced by the exact model.

An exact mathematical proof of this theory is out of the scope of this paper and will be studied in the future. However the hypothesis that only the non-linear function changes the nature of the probability density functions (PDF) of the output, when compared with the input PDF, provides an initial support to the non-linear MISO analysis that uses the global measured input and output, without the assumption of a zero-memory model at the non-linear path.

Therefore it is expected that the estimated non-linear function, using the Eq. (10) with the global input and output, is capable to represent the non-linearity nature providing a good identification of the non-linear path in the MISO analysis.

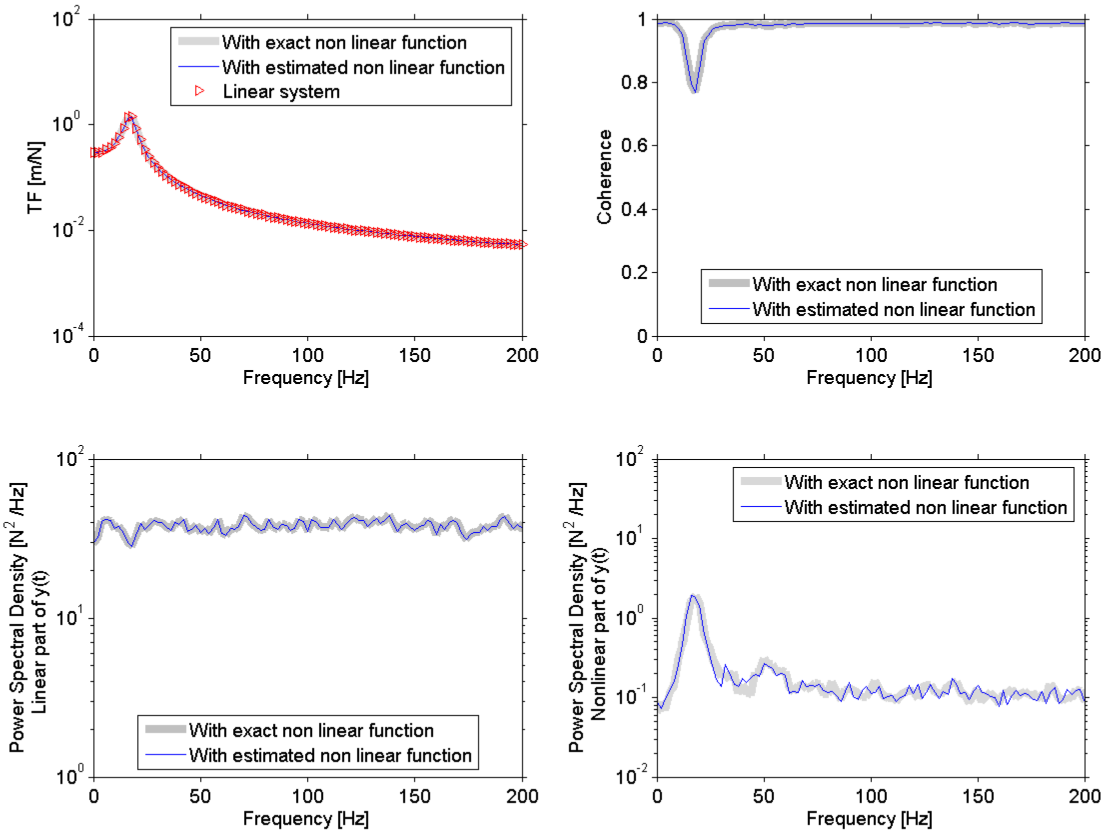

Figure 10 – Simulations results of a linear system with a softening cubic spring.

As shown in the graphs at the right upper corners of Fig. 9 and Fig. 10, the cumulative coherence functions calculated with the exact and with the estimated non-linear functions are coincident, and have values near unity along the analysis frequency band. These good correlations as well as those between the transfer functions (TF) are indicative of the good estimative of the parameters in the non-linear MISO models.

The cumulative coherence functions have lower values around the natural frequency of the linear system. These reductions may be caused by the 2 Hz frequency resolution used in the simulations, and by the adopted estimator of the ordinary coherence functions. According to Bendat & Piersol (1986) the estimator used to calculate the coherence functions has bias error and under-estimates the real one. This bias error is proportional to the square of the relation between the physical time delays, imposed by the system transfer function, and the acquisition total time. Therefore to reduce this error the solution is to increase the total acquisition time, which is proportional to the sample length used to estimate the auto spectral and the cross spectral densities.

Figure 11 and Fig. 12 compare the transfer function and coherence functions obtained by the proposed non-linear MISO analysis and by traditional linear analysis, applied to the same mechanical system with the hardening and softening cubic springs.

The main differences between transfer functions calculated by the both methodologies are in the region near the natural frequency. This can be explained by the large displacement of moving mass of the system in the Fig. 8, in these frequency bands. Due to the stiffness cubic behavior, large displacement amplitudes increase the non-linear part of the system response in the measured global system response, promoting a significant difference between the TF estimated by both methodologies. The same analysis could be based on the graphic of the Fig. 8 where is verified that the difference between the linear spring and the non-linear springs are greater at larger displacements $x(t)$.

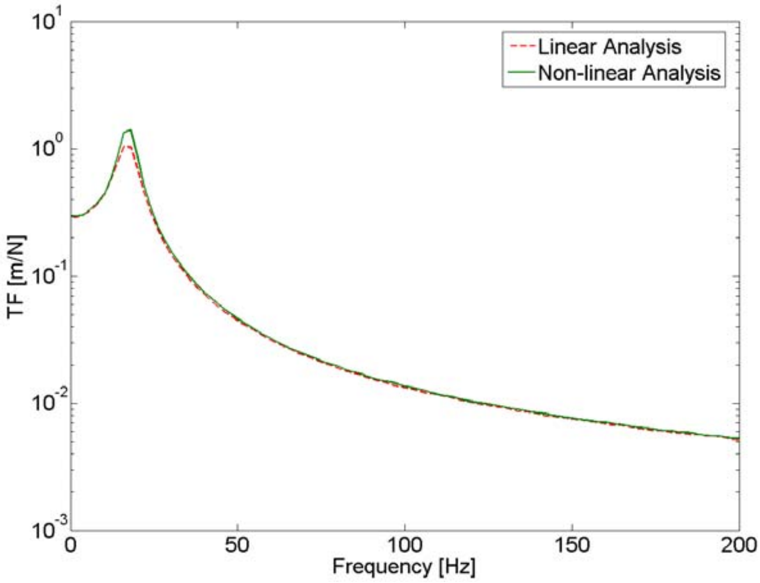

Figure 11 – Comparison between estimated transfer functions.

The coherence functions shown in Fig. 12 have the same behavior verified in Fig. 11 in the regions near the natural frequency. Outside this frequency band the coherence functions obtained with the non-linear analysis show that the non-linear frequency analysis provides a better estimative of linear system transfer function. Higher differences can be expected in experimental tests of a real vibratory system, since in these simulations the noise is only due to numerical errors and in real cases the noise captured by the instrumentation is always present.

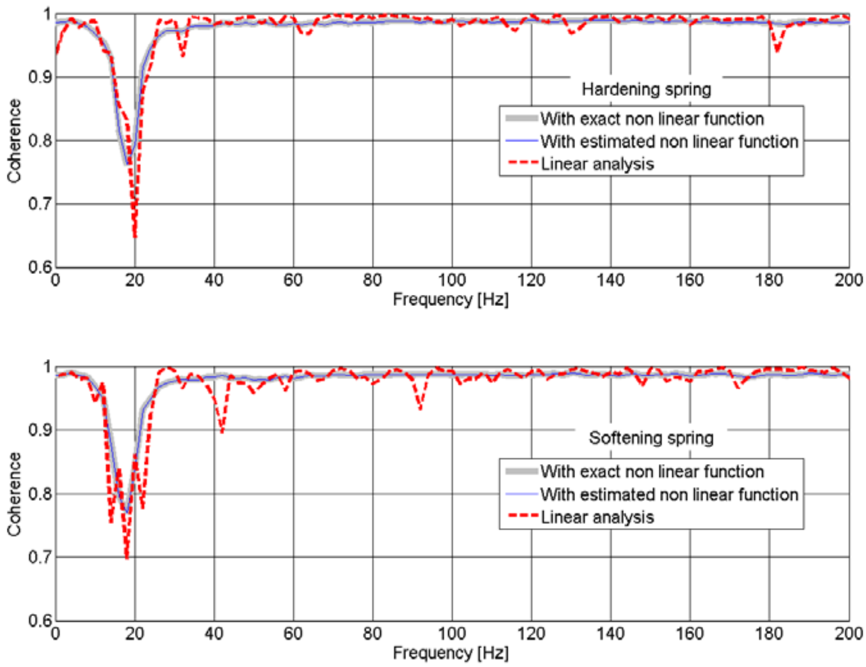

Figure 12 – Comparison between coherences functions.

CONCLUSIONS AND FUTURE WORKS

The analysis and interpretation of nonlinear behavior that occurs in a mechanical system is feasible by the usage of nonlinear MISO technique. The proposed methodology is useful to identify nonlinear effects that act on linear system without restrictions on the statistical nature of the measured signals. If the nonlinear behavior is well modeled, the proposed methodology is capable to identify and separate the exact nonlinear part from the global response of the system.

An alternative methodology to identify the non-linear path in the MISO models was proposed. This methodology is capable to estimate a non-linear function. This methodology when used in the non-linear MISO analysis produced the same results obtained with the exact form of the non-linear function. However a detailed mathematical study should be done to validate this assumption.

In case of mechanical systems with known linear properties the MISO representation is simple and easy to be physically interpreted than by other nonlinear analysis techniques. Since the methodology does not impose any restrictions on the nature of the nonlinear function, it finds usefulness in the identification of physical parameters of highly nonlinear mechanical systems.

A group of experiments in a well know non-linear test rig should be carried out. These experiments are in order to verify the performance of the both proposed methodology to analyze real systems.

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