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Reduction size for Fluid-Structure interaction problems

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*Abstract: Dynamic analysis of industrial structures may be costly from a numerical point of view. For coupled fluid-structure finite element models, the importance of reducing size becomes obvious because the fluid degrees of freedom must be added to the structural ones. We propose a component mode synthesis (**modal synthesis**) method for large fluid-structure interaction problems. This method couples fluid sub-domains and dynamical sub-structuring. The fluid formulation is written in terms of the pressure, which implies a direct extension of synthesis method to fluid.*

One of the principal hypothesises in the use of component mode synthesis method is that the model is deterministic; it is to say that parameters used in the model have a defined and fixed value. Furthermore, the knowledge of variation response of a structure involving uncertain material, geometrical parameters, boundary conditions, tolerances of manufactures and loading conditions is essential in global process of conception. In order to do that, the modal synthesis method is extended to stochastic analysis of coupled fluid-structure finite element models. It is assumed that the probability distribution of random parameters is known and the eigen frequencies are calculated using Monte Carlo simulation and perturbation techniques.

The use of the modal synthesis leads to a considerable reduction of fluid-structure system dofs, with acceptable results. The study of dynamic behaviour of a structure coupled to a fluid, using the perturbation methods needs less calculation times than the widely used Monte Carlo simulation.

Keywords: fluid-structure interaction, modal synthesis, stochastic, Monte Carlo, Perturbation.

NOMENCLATURE

Dofs degrees of freedom
$\{u\}$ structure dofs
$\{P\}$ fluid dofs
c fluid celerity
$[M]$ mass matrix of the structure
$[K]$ stiffness matrix of the structure
$[E]$ mass matrix of the fluid
$[H]$ stiffness matrix of the fluid
$[L]$ interaction fluid-structure matrix
$\{F\}$ imposed efforts
$\{a\}$ imposed accelerations
$E(\ )$ random variable average
$Et(\ )$ standard deviation of a random variable
$Var(\ )$ random variable variance
n the normal

Greek Symbols

∇ gradient operator
Δ rotational operator
ρ density
σ stress tensor
Γ border

Subscripts

s relative to the structure
f relative to the fluid
i relative to the internal dofs
j relative to the junction
I relative to the interface

INTRODUCTION

The dynamic analysis of interactions fluid-structure is often costly and sometimes difficult due to the computer resources limitations. Furthermore, these mechanical systems are often made of several parts, which for organization reasons, are calculated and tested independently by different teams.

The sub-structuring methods constitute often the only resolution strategy. The use of these methods is then justified by the numerical benefit and by taking organization constraints of such big projects into account. One of the most used dynamic sub-structuring strategies is based on a component mode synthesis. In the reference (Craig, 1995) we find a synthesis of these methods. One of the pioneer works on the subjet are presented in that literature review (For instance, we can quote the ones proposed by (Craig and Bampton, 1968), (MacNeal, 1971) or (Rubin, 1975).

Furthermore, the understanding of the interaction mechanisms between a fluid and an elastic solid has a capital importance in several industrial applications. When a structure vibrates in the presence of a fluid, there is interaction between the eigen waves of two environments: the fluid generates a structural deformation and/or the movement of a solid provokes the movement of the fluid. These applications request a coupling.

One of the main hypothesises in the study of mechanical systems is that the model is deterministic. That means that the parameters used in the model are constant. However the experimental works show the limitations of such

assumption. This is because there are always differences between what we calculate and what we measure due mainly to the uncertainties in geometry, the material properties, the boundary conditions or the load, which has a considerable impact on the vibrating behaviour of mechanical systems. This is why it is important to use numerical methods in order to take these uncertainties into count.

DEFINITION AND MODELING OF FLUID-STRUCTURE INTERACTIONS

Definition of the problem

The study of fluid-structure interactions can be defined as the analysis of the coupled behaviour of two different media, the first one is an elastic structure and the second is a fluid:

- If the fluid is incompressible the problem is said: Hydro-Elastic ;
- If the fluid is compressible the problem is said: Elasto-Acoustic.

Modelling

It is assumed that:

- The structure is elastic, linear, isotropic and without any initial stress or strain;
- The fluid is perfect around his rest position and a small perturbation in an adiabatic transformation is considered.

Equation for the structure:

$$\nabla\sigma - \rho_s \ddot{u} = 0 \tag{1}$$

If:

Γ_u : is the boundary where the displacements are imposed;

Γ_F : is the s boundary where the forces are imposed.

The limit conditions associated to the sub-structure s are:

$$u\big|_{\Gamma_u} = \overline{u} \qquad \sigma.n\big|_{\Gamma_F} = \overline{F} \tag{2}$$

Equation for the fluid:

$$\Delta P - \frac{1}{c^2}\ddot{P} = 0 \tag{3}$$

If:

Γ_P : is the f boundary where the pressures are imposed;

Γ_a : is the f boundary where the normal accelerations are imposed.

The limit conditions associated to the sub-domain fluid f are:

$$P\big|_{\Gamma_P} = \overline{P} \qquad \left.\frac{\partial P}{\partial n}\right|_{\Gamma_a} = -\rho_f \overline{a} \tag{4}$$

Coupling conditions in the fluid-structure interface:

If C is the fluid-structure interface:

The continuity of the normal acceleration gives:

$$\left.\frac{\partial P}{\partial n}\right|_{C} = -\rho_f \ddot{u} \tag{5}$$

And the continuity of the normal composing of the stress tensor gives:

$$\sigma.n\big|_{C} = P.n \tag{6}$$

After discretization and approximation by finite elements, the following matrix equations are found:

- For the structure:

$$[M]\{\ddot{u}\} + [K]\{u\} = [L]\{P\} \tag{7}$$

- For the fluid :

$$[E]\{\ddot{P}\} + [H]\{P\} = -\rho[L]^t\{\ddot{u}\} \tag{8}$$

Finally, the coupled system equations are:

$$\begin{bmatrix} M & 0 \\ \rho_f L^t & E \end{bmatrix} \begin{Bmatrix} \ddot{u} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} K & -L \\ 0 & H \end{bmatrix} \begin{Bmatrix} u \\ P \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{9}$$

To determine the eigen frequencies of the coupled system, the matrix must be symmetrical which is not the case. Therefore a symmetrization procedure such as **Irons method will be used.**

N.B: for a hydro-elastic problem (incompressible fluid) the fluid celerity tends to the infinite ($c \to \infty$), therefore Eq.(9) becomes:

$$\left([M] + \rho_f [L][H]^{-1}[L]^t \right) \{\ddot{u}\} + [K]\{u\} = \{0\} \tag{10}$$

MODAL SYNTHESIS FOR THE COUPLED SYSTEM

The structure is divided in N_s sub-structures and the fluid in N_f fluid sub-domains. The dofs vector of each sub-structure s (respectively for each fluid sub-domains f) is partitioned as follows:

- The internal dofs (index i) ;
- The junction dofs (index j) that correspond to the dofs situated on the interfaces between the sub-structure s (respectively between the fluid sub-domain f) and adjacent sub-structures (respectively the all fluid sub-domains).

The adopted modal synthesis method is the fixed interfaces method published by (Craig and Bampton, 1968).

Mass and stiffness matrices

For a sub-structure s, the equation of motion is:

$$[M^s]\{\ddot{u}^s\} + [K^s]\{u^s\} + \sum_{f=1}^{N_f} [L^{sf}]\{P^f\} = \sum_{\substack{s'=1 \\ s' \neq s}}^{N_S} \left\{ F_I^{ss'} \right\} \tag{11}$$

Where:

$$\{u^s\} = \left\{ u_i^s \middle| u_j^s \right\} \qquad [M^s] = \begin{bmatrix} M_{ii}^s & M_{ij}^s \\ M_{ji}^s & M_{jj}^s \end{bmatrix} \qquad [K^s] = \begin{bmatrix} K_{ii}^s & K_{ij}^s \\ K_{ji}^s & K_{jj}^s \end{bmatrix}$$

It is noted that:

$[L^{sf}]$: The matrix interaction between the sub-structure s and the fluid sub-domain f ;

$\left\{ F_I^{ss'} \right\}$: The inter-facial forces vector transmitted from the sub-structure s' to the sub-structure s;

For a fluid sub-domain f

$$[E^f]\{\ddot{P}^f\} + [H^f]\{P^f\} + \sum_{s=1}^{Ns} [L^{sf}]\{\ddot{u}^s\} = \sum_{\substack{f'=1 \\ f' \neq f}}^{N_f} \left\{ a_I^{ff'} \right\} \tag{12}$$

Where:

$$\{P^f\} = \left\{ P_i^f \middle| P_j^f \right\} \qquad [E^f] = \begin{bmatrix} E_{ii}^f & E_{ij}^f \\ E_{ji}^f & E_{jj}^f \end{bmatrix} \qquad [H^f] = \begin{bmatrix} H_{ii}^f & H_{ij}^f \\ H_{ji}^f & H_{jj}^f \end{bmatrix}$$

It is noted that:

$[L^{sf}]$: The matrix interaction between the fluid sub-domain f and the sub-structure s;

$\left\{ a_I^{ff'} \right\}$: The inter-facial accelerations vector imposed by the fluid sub-domain f' on the fluid sub-domain f;

For the complete system

After assembling the N_s sub-structures and the N_f fluid sub-domains in a global vector containing all structure and fluid dofs, the following equation is obtained:

$$[M_g]\{\ddot{u}_g\}+[K_g]\{u_g\}=\{F_{Ig}\} \tag{13}$$

Where:

$$\{u_g\}=\{u \mid P\} \qquad [M_g]=\begin{bmatrix} \sum_{s=1}^{N_s} M^s & 0 \\ \rho_f\left(\sum_{f=1}^{N_f}\sum_{s=1}^{N_s} L^{sf}\right)^t & \sum_{f=1}^{N_f} E^f \end{bmatrix}$$

$$[K_g]=\begin{bmatrix} \sum_{s=1}^{N_s} K^s & -\sum_{f=1}^{N_f}\sum_{s=1}^{N_s} L^{sf} \\ 0 & \sum_{f=1}^{N_f} H^f \end{bmatrix} \qquad \{F_{Ig}\}=\left\{\sum_{\substack{s'=1\\ s'\neq s}}^{N_S}\{F_I{}^{ss'}\} \;\middle|\; \sum_{\substack{f'=1\\ f'\neq f}}^{N_f}\{\nu_I{}^{ff'}\}\right\}$$

Calculation of the local eigen modes

For the sub-structures

According the Graig & Bampton method, the selected local modes correspond to the fixed interfaces modes. They are solution of the following eigen values problem:

$$[K_{ii}^s-\omega^2 M_{ii}^s]\{u_i^s\}=\{0\} \qquad s = 1 \ldots N_s \tag{14}$$

It is noted that the fluid- structure interfaces are implicitly free (no action of the fluid on the sub-structures).

These orthogonal modes are enriched by static modes of liaison. The local modal basis of a sub-structure s is then given by:

$$[\varphi^s]=\begin{bmatrix} \Psi^s & -K_{ii}^{s\,-1}K_{ij}^s \\ 0 & I_{jj}^s \end{bmatrix} \tag{15}$$

$[\Psi^s]$: is the matrix of truncated fixed interfaces modes (only the modes corresponding to the first eigen frequencies are kept).

The physical dofs of each sub-structure are decomposed in terms of local ones:

$$\{u^s\}=[\Psi^s]\{\alpha^s\} \qquad s = 1 \ldots N_s \tag{16}$$

$\{\alpha^s\}$ is the vector of the generalized coordinates associated with the sub-structure s, containing:

- the coefficients associated with the fixed interfaces modes ;
- the physical dofs of the sub-structure junctions.

For the fluid sub-domains

The pressure formulation of the fluid domain, leads to an algebraic system which authorizes a direct extension of the Graig & Bampton method.

The local modes with perfectly complaisant interfaces (null pressure) are defined by :

$$[H_{ii}^f-\omega^2 E_{ii}^f]\{P_i^f\}=\{0\} \qquad f = 1 \ldots N_f \tag{17}$$

Where the fluid-structure interfaces are implicitly assumed perfectly rigid (no action of the structure on the fluid sub-domain).

By analogy with the sub-structures, the orthogonal modes are enriched by static modes of liaison. The local modal basis of a fluid sub-domain f is given by:

$$\left[\phi^f\right]=\begin{bmatrix}\Psi^f & -H_{ii}^{f\,-1}H_{ij}^f\\ 0 & I_{jj}^f\end{bmatrix} \tag{18}$$

$\left[\varphi^f\right]$: is the matrix of complaisant interfaces modes disposed in columns.

The physical dofs of each fluid sub-domain are written in term of local modal bases:

$$\left\{P^f\right\}=\left[\phi^f\right]\left\{\beta^f\right\} \qquad f = 1 \ldots N_f \tag{19}$$

$\left\{\beta^f\right\}$: is the vector of the generalized coordinates associated with the fluid sub-domain f, containing:

- the coefficients associated with the complaisant interfaces;
- the physical dofs (the nodal pressures) of the sub-domain junctions.

The modal synthesis

The local decompositions Eq.(16) and Eq.(19) are assemble as followed:

$$\left\{u_g\right\}=\left[\varphi_g\right]\left\{q\right\} \tag{20}$$

Where:

$$\left[\varphi_g\right]=\begin{bmatrix}\varphi^1 & 0 & 0 & 0\\ & \ddots & & \\ 0 & \varphi^{N_s} & 0 & 0\\ 0 & 0 & \phi^1 & 0\\ & & & \ddots\\ 0 & 0 & 0 & \phi^{N_f}\end{bmatrix},\ \left\{q\right\}=\begin{Bmatrix}\alpha^1\\ \vdots\\ \alpha^{N_s}\\ \beta^1\\ \vdots\\ \beta^{N_f}\end{Bmatrix}$$

After projection, Eq.(13) becomes:

$$\left[M_q\right]\left\{\ddot{q}\right\}+\left[K_q\right]\left\{q\right\}=\left[\varphi_g\right]^t\left\{F_{Ig}\right\} \tag{21}$$

Where:

$$\left[M_q\right]=\left[\varphi_g\right]^t\left[M_g\right]\left[\varphi_g\right] \quad \text{and} \quad \left[K_q\right]=\left[\varphi_g\right]^t\left[K_g\right]\left[\varphi_g\right]$$

N.B.:

- It is necessary to take into account the continuity conditions in the structure/structure and fluid/fluid interfaces ;
- The dofs {q} are not linearly independent;
- The relationships between these dofs are results from the equality of the displacements in the structure/structure interfaces and equality of the pressure in the fluid/fluid interfaces;
- The dofs {q} can be expressed by a global connectivity matrix [S] :

$$\{q\}=[S]\{y\} \tag{22}$$

- $\{y\}$ Contains only the linearly independent dofs;
- $[S]$ characterizes both the connectivity between the sub-structure and the connectivity between the fluid sub-domains ;
- for the Graig & Bampton method $[S]$ is Boolean and easy to express.

According to the normal components continuity conditions of the stiffness tensors and the accelerations, The following compatibility equations is obtained:

$$\begin{cases}\left\{F_I^{ss'}\right\}+\left\{F_I^{s's}\right\}=0\\ \left\{\nu_J^{ff'}\right\}+\left\{\nu_J^{f'f}\right\}=0\end{cases}$$ It can be shown that these equations lead to: $\left[S\right]^t\left[\varphi_g\right]^t\left\{F_{Ig}\right\}=\left[S\right]^t\left\{F_{Ig}\right\}=0$ (23)

The final system to be resolved:

$$[M_y]\{\ddot{y}\}+[K_y]\{y\}=\{0\} \tag{24}$$

Where:

$$[M_y]=[S]^t[M_q][S] \quad \text{and} \quad [K_y]=[S]^t[K_q][S]$$

After the modal synthesis the number of coupled system unknowns is only the sum of the truncated local eigen modes and the Junctions dofs.

Eigen modes of the coupled system

The eigen modes of the coupled system are the solution of the following eigen values problem:

$$[K_y-\omega^2 M_y]\{y\}=\{0\} \tag{25}$$

STOCHASTIC FLUID-STRUCTURE INTERACTION

Monte Carlo Simulation

The estimation of the frequency response function (F.R.F.) moments (average and variance) (respectively the eigen frequencies or the temporal response) could be obtained by the Monte Carlo simulation (Shinosuka, 1971). This is a very costly method from a numerical point of view. The F.R.F. F is seen as a random variable image of the basic variables. The simulation is carried out by constructing a sample (F_1, F_2,…, F_n) of the random variable F and to treat this sample by the usual statistics techniques. The n simulations are done in an independent way according to the distribution law of the random vector.

The average E(F) of F is given by :

$$E(F)=\frac{1}{n}\sum_{i=1}^{n}F_i \tag{26}$$

The variance var(F) of F is given by:

$$var(F)=\frac{1}{n-1}\sum_{i=1}^{n}[F_i-E(F)]^2 \tag{27}$$

The simulation methods necessitate all the more as simulations than the variation coefficient of the function is great. The precision of the results is independent of the variables number.

Perturbation methods

The perturbation methods are very widely used in the stochastic finite elements domain. They are based on a development in Taylor series of the F.R.F. (respectively the eigen frequencies or the temporal response) in relation to the basis random physical variables, mechanical properties, geometrical characteristics or applied forces. The perturbation methods calculate the average and the standard deviation of the F.R.F. of a mechanical structure that has uncertain variables. This method is used in many areas in order to solve linear and non-linear problems, for either static or dynamic modes.

We note:

$$[M_T]=\begin{bmatrix} M & 0 \\ \rho_f L^t & E \end{bmatrix} \quad , \quad [K_T]=\begin{bmatrix} K & -L \\ 0 & H \end{bmatrix}$$

And:

λ_i : is the ith eigen frequency;

F_λ : is the F.R.F.;

F_t : is the temporal response.

For a coupled system with uncertain variables it is assumed that the mass matrix $[M_g]$ and the stiffness matrix $[K_g]$ are functions of the random variables $\{\alpha_p\}_{(p=1,.....,P)}$.

The vector of the average parameters is defined by $\{\overline{\alpha}\}$, and the quantity $d\alpha$ is defined by $\{d\alpha\}=\{\alpha\}-\{\overline{\alpha}\}$.

The following notation is used to simplify the writing:

$$[A]^0 = [A]_{\{\bar{\alpha}\}} \qquad [A]^n = \left.\frac{\partial [A]}{\partial \alpha_n}\right|_{\{\bar{\alpha}\}} \qquad [A]^{np} = \left.\frac{\partial^2 [A]}{\partial \alpha_n \partial \alpha_p}\right|_{\{\bar{\alpha}\}}$$

$[A]^0$, $[A]^n$ *et* $[A]^{np}$ are deterministic.

Perturbation method with second-order Taylor series

This method was published by (Keiber and Hien, 1992), and it can be used for a mechanical system that have depending or independent random parameters. It is based on a development into a second-order Taylor series:

$$[K] = [K]^0 + [K]^n \{d\alpha_n\} + \frac{1}{2}[K]^{np}\{d\alpha_n\}\{d\alpha_p\} \qquad [M] = [M]^0 + [M]^n \{d\alpha_n\} + \frac{1}{2}[M]^{np}\{d\alpha_n\}\{d\alpha_p\}$$

$$\lambda_i = (\lambda_i)^0 + (\lambda_i)^n \{d\alpha_n\} + \frac{1}{2}(\lambda_i)^{np}\{d\alpha_n\}\{d\alpha_p\} \qquad F_\lambda = (F_\lambda)^0 + (F_\lambda)^n \{d\alpha_n\} + \frac{1}{2}(F_\lambda)^{np}\{d\alpha_n\}\{d\alpha_p\} \tag{28}$$

$$F_t = (F_t)^0 + (F_t)^n \{d\alpha_n\} + \frac{1}{2}(F_t)^{np}\{d\alpha_n\}\{d\alpha_p\}$$

The repetition of the index n two times implies a summation.

The averages are given by:

$$E[\lambda_i] = (\lambda_i)^0 + \frac{1}{2}(\lambda_i)^{(2)} \qquad E[F_\lambda] = (F_\lambda)^0 + \frac{1}{2}(F_\lambda)^{(2)} \qquad E[F_t] = (F_t)^0 + \frac{1}{2}(F_t)^{(2)} \tag{29}$$

With:

$$(\lambda_i)^{(2)} = (\lambda_i)^{np}\, cov(\alpha_n, \alpha_p) \qquad (F_\lambda)^{(2)} = (F_\lambda)^{np}\, cov(\alpha_n, \alpha_p) \qquad (F_t)^{(2)} = (F_t)^{np}\, cov(\alpha_n, \alpha_p)$$

$$\text{For} \qquad (n, p = 1, ...P)$$

The variances are given by:

$$var(\lambda_i) = (\lambda_i)^n (\lambda_i)^p\, cov(\alpha_n, \alpha_p) \qquad var(F_\lambda) = (F_\lambda)^n (F_\lambda)^p\, cov(\alpha_n, \alpha_p) \qquad var(F_t) = (F_t)^n (F_t)^p\, cov(\alpha_n, \alpha_p) \tag{30}$$

Muscolino perturbation method

The Muscolino perturbation method (Muscolino and al, 1999) could be used for a mechanical system whose the random parameters are independent. It is based on a development into a first-order Taylor series:

$$[K_T] = [K_T]^0 + [K_T]^n \{d\alpha_n\} \qquad [M_T] = [M_T]^0 + [M_T]^n \{d\alpha_n\} \tag{31}$$

$$\lambda_i = (\lambda_i)^0 + (\lambda_i)^n \{d\alpha_n\} \qquad F_\lambda = (F_\lambda)^0 + (F_\lambda)^n \{d\alpha_n\} \qquad F_t = (F_t)^0 + (F_t)^n \{d\alpha_n\}$$

The repetition of the index n two times implies a summation.

The averages are given by:

$$E[\lambda_i] = (\lambda_i)^0 \qquad E[F_\lambda] = (F_\lambda)^0 \qquad E[F_t] = (F_t)^0 \tag{32}$$

The standard deviations are given by:

$$Et(\lambda_i) = \left|(\lambda_i)^n Et(\alpha_n)\right| \qquad Et(F_\lambda) = \left|(F_\lambda)^n Et(\alpha_n)\right| \qquad Et(F_t) = \left|(F_t)^n Et(\alpha_n)\right| \tag{33}$$

The Muscolino method has the benefits of requiring less calculation than the one based on the development into a second-order Taylor series.

Perturbation methods with modal synthesis

For a coupled system whose dofs are reduced by the modal synthesis method, it is assumed that the modal bases are deterministic for the both perturbation methods. This assumption is justified, since the perturbation method is only applied to systems, whose parameters vary weakly. Therefore:

$$[M_y]^0 = [S]^t[\varphi_g]^t[M_g]^0[\varphi_g][S] \quad [M_y]^n = [S]^t[\varphi_g]^t[M_g]^n[\varphi_g][S]$$

And:

$$[K_y]^0 = [S]^t[\varphi_g]^t[K_g]^0[\varphi_g][S] \quad [K_y]^n = [S]^t[\varphi_g]^t[K_g]^n[\varphi_g][S] \tag{34}$$

NUMERICAL VALIDATION

In order to validate the proposed methods, Matlab was used in order to elaborate calculation codes. We will study the simple example of a beam coupled with an incompressible fluid. Below are diagram as well as the problem data:

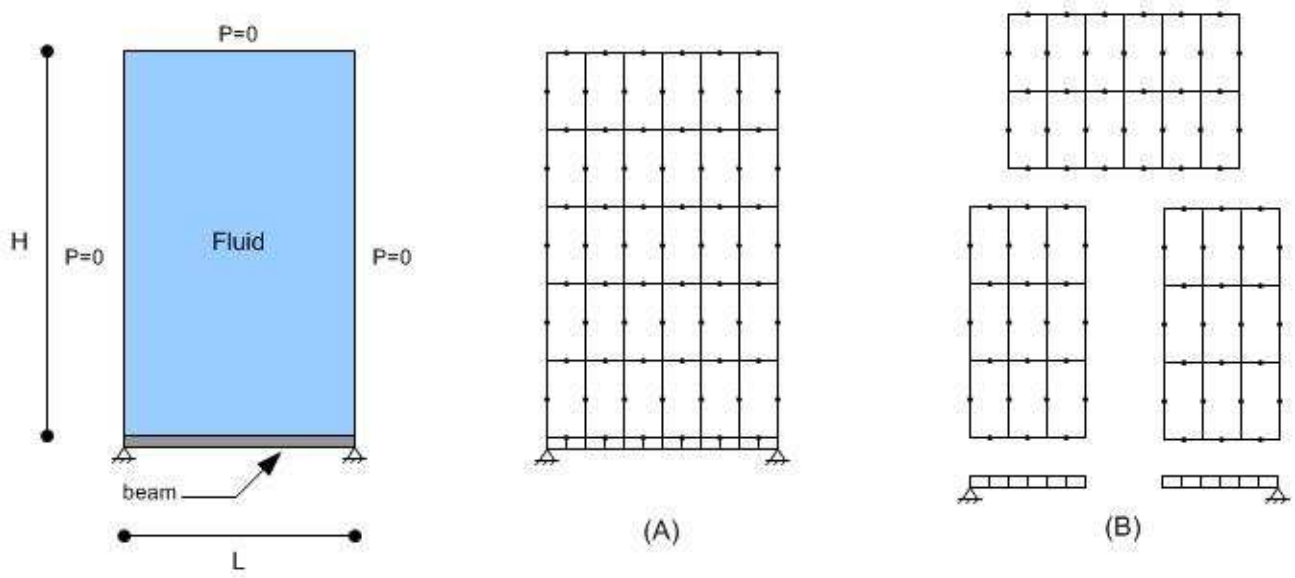

Figure1 – beam coupled to a fluid.

Where:

For the beam: L = 3 m I_z = O.333 e^{-4} m^4 S = 0.01 m^2 E = 2.1 e^{11} Pa ρ_s = 7800 Kg/m^3
b = 0.05 m (the beam's width)

For the fluid: H = 6 m ρ_f = 1000 Kg/m^3 c_f = 1500 m/s (the celerity)

For the finite elements calculation, the fluid was meshed with 8 nodes rectangular elements and the beam with 2 nodes linear elements. The configuration (B) of figure1 (sub-structuring: 3 fluid sub-domains and 2 sub-structures) was chosen for the dofs reduction by the modal synthesis method.

Deterministic calculation

The table below shows the results (**eigen frequencies**) of the analytical calculation (published in (Pavanello, 1991)) used as a reference, as well as the numerical results obtained by direct calculations and by modal synthesis method.

Table 1 – Analytical and numerical calculation

	Analytical Calculation		Numerical Calculation		
				coupling fluid-structure	
Mode	dry beam	coupling fluid-structure	dry beam	direct calculation	Modal synthesis
1	52.3	41.2	52.3 (0%)	41.2 (0%)	41.2 (0%)
2	209.1	182.9	209.2 (0%)	183 (0%)	183 (0%)
3	470.6	428.7	470.7 (0%)	429.5 (0.1%)	429.9 (0%)
4	836.6	778.7	837.2 (0%)	782.2 (0.4%)	782.8 (0%)
5	1307.1	1233.2	1309.7 (0.1%)	1247 (1.1%)	1254 (0.5%)
6	1882.3	1792.2	1889.7 (0.3%)	1850 (3.4%)	1869 (1.0%)

error in relation to the analytical calculation — error in relation to the direct calculation

Stochastic calculation

The tables below show the numerical averages and the standard deviations of the eigen frequencies. The results are obtained assuming that the Young modulus and the fluid density are random variables:

$$E = 2.1e^{11}(1+0.02\delta)\ (Pa) \quad \text{and} \quad \rho_f = 1000(1+0.05\delta)\ (Kg/m^3)$$

δ is a normal random variable such as : $E(\delta)=0$ and $Et(\delta)=1$

The calculation was carried out using the Muscolino perturbation method, with and without dofs reduction. *The Monte Carlo simulation will be used as a reference*.

"Ref." means Reference and "s.d." means standard deviation.

Table 2 – the averages of the eigen frequencies.

	Monte Carlo simulation	perturbation with direct calculation		perturbation with Modal synthesis	
mode	average (Ref.)	average	error in relation to the Ref.	average	error in relation to the Ref.
1	41.183	41.179	0.01 %	41.198	0.04 %
2	183.17	183.05	0.06 %	183.05	0.06 %
3	429.91	429.48	0.10 %	429.89	0.00 %
4	783.68	782.75	0.12 %	782.77	0.12 %
5	1248.9	1247.2	0.14 %	1254.2	0.42 %
6	1853.6	1850.6	0.16 %	1869.4	0.85 %

Table 3 – the standard deviation of the eigen frequencies.

	Monte Carlo simulation	perturbation with direct calculation		perturbation with Modal synthesis	
mode	standard deviation (Ref.)	s.d.	error in relation to the Ref.	s.d.	error in relation to the Ref.
1	1.318	1.312	0.45 %	1.330	0.91 %
2	16.76	16.67	0.54 %	16.67	0.54 %
3	46.56	46.31	0.54 %	46.51	0.11 %
4	92.14	91.62	0.56 %	91.63	0.55 %
5	155.4	154.4	0.64 %	157.8	1.54 %
6	249.5	247.9	0.64 %	257.1	3.05 %

The calculation time needed by each method:

- Monte Carlo Simulation : CPU time = 2.54 s
- Muscolino Method without dofs reduction : CPU time = 0.07 s
- Muscolino Method with the dofs reduction : CPU time = 0.25 s

The Muscolino method with dofs reduction needed more calculation time that the one without dofs reduction, because we proceeded to a matrix inversion for each sub-structure and each fluid sub-domain. Modal synthesis is effective for bigger size systems.

CONCLUSION

A synthesis modal method is proposed for the resolution of the great size deterministic fluid-structure interaction problems. The developed method couple a dynamic sub-structuring method of Craig and Bampton and a method of fluid sub-domains based on a pressure formulation. This formulation enables a direct extension of the Craig and Bampton method to the fluid. However, the resolution of the eigen values problem by classical algorithms necessitates a symmetrization of the global matrices.

Next we proposed perturbation methods for the resolution of great size stochastic fluid-structure interaction problems. Despite convergence problems, the perturbation methods have the benefits of necessitating less calculation time than the Monte Carlo method.

The obtained results in the case of a beam coupled to a fluid show the validity and the potentialities of the proposed methods. In the deterministic calculation the numerical results agree with the analytical ones and the results of the calculation with dofs reduction agree with the one without dofs reduction. Next we calculated the averages and the standard deviations of the eigen frequencies of the coupled system when the Young modulus and the fluid density are assumed random variables. The calculation was done with Monte Carlo method and the perturbation method with and without dofs reduction. The results obtained using the three methods do agree. The Muscolino method with dofs reduction needed more calculation time than the one without dofs reduction. This is because it was applied on a small size system (we proceeded to a matrix inversion for each sub-structure and each fluid sub-domain). However, for bigger size systems the opposite will be obtained.

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