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Damage Detection Using Controllability Grammian Matrices

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Abstract: Some industries have great interest in damage identification using no destructive tests. A complete procedure should be able to detect the damage, to foresee the probable time of occurrence, and to diagnosis the type of fault in order to plan the maintenance operation in a convenient form and occasion. In this paper, it is proposed to locate the damage through controllability analyses. Since, this technique is based on controllability grammian matrix, it should be advantageous for practical structural health monitoring (SHM) in cases where the number of available sensors is small.

Keywords: SHM, Controllability, Grammian Matrix, input indices, few sensors

NOMENCLATURE

$\mathbf{A}$ = dynamic matrix
$\mathbf{B}$ = input matrix
$\mathbf{C}$ = output matrix
$\mathbf{D}$ = feed-through matrix
$\mathbf{C}_o$ = Controllability matrix
$\mathbf{W}_c$ = Cont. Grammian matrix
$\mathbf{x}(t)$ = state vector
$\mathbf{u}(t)$ = input vector
$\mathbf{y}(t)$ = output vector
n = number of modes
s = number of inputs
r = number of outputs
trace(W) = trace of grammian matrix
t = relative to time
exp( ) = exponential function

Greek Symbols

ω = natural frequency
ζ = modal damping
σ_s = grammian input indexes of structure

Subscripts

i relative to ith mode
m relative to modal representation
r relative to retained states
t relative to truncated states
sd structure with damage
sh intact structure
j relative to jth input position
0 relative to initial time
1 relative to time instant

INTRODUCTION

Structural Health Monitoring (SHM) denotes a system with the ability to detect and interpret adverse "changes" in a structure in order to improve reliability and reduce life-cycle costs. The greatest challenge in designing a SHM system is knowing what "changes" to look for and how to identify them. Different approaches for SHM have been proposed for damage location, each one with advantages and drawbacks. Most of the methods take advantage of the dynamical behaviour of the structure (Friswell and Penny, 1997). Up to now, there is no acknowledged best technique for general applications. Methods that have been developed tend to be case dependent.

Aerospace structures have one of the highest payoffs for SHM applications since damage can lead to catastrophic and expensive failures, and the vehicles involved undergo regular costly inspections. Currently 27% of an average aircraft's life cycle cost, both for commercial and military vehicles, is spent on inspection and repair; and it excludes the opportunity cost associated with the time the aircraft is grounded for scheduled maintenance (Hall and Conquest, 1999). SHM systems are getting strong attention for controlling or reducing risks associated with civil and mechanical structures due to natural hazards such as large earthquakes as reviewed by Mita (1999). This trend has been accelerated after the 1994 Northridge Earthquake and the 1995 Hyogo-Ken Nanbu Earthquake.

In general, a considerable number of sensors are necessary for damage location with some precision (Lopes Jr. *et al.*, 2001; Marqui et al., 2006). However, in practical situations, the number of sensors is limited. Sensors are used to record variables such as strain, acceleration, sound waves, electrical or magnetic impedance, pressure or temperature. In the literature it has been estimated that a SHM system for an aerospace vehicle would require between 100 and 1000 sensors, depending on its size and desired coverage area (Marantidis *et al.*, 1994). In this context, this paper proposes a novel approach using controllability grammian matrices. The paper concludes with numerical and experimental tests in a beam like structure. The damaged locations were detected using only one accelerometer varying the input excitation position on the structure.

DAMAGE DETECTION USING CONTROLLABILITY GRAMMIANS

A linear differential inclusion (LDI) system, in modal state-space form, considering the matrices with appropriate dimensions and assumed to be known is given by:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\ \mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\ \mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned} \tag{1}$$

The state vector $\mathbf{x}(t)$ of the modal coordinates system consists of n independent components, $\mathbf{x}_i(t)$, that represent a state of each mode. The $\mathbf{x}_i(t)$ (*ith* state component), related to Eq. (1), is defined (Gawronski, 1998):

$$\mathbf{x}_i(t) = \begin{Bmatrix} \mathbf{q}_{mi}(t) \\ \mathbf{q}_{moi}(t) \end{Bmatrix}, \quad \text{where } \mathbf{q}_{moi}(t) = \zeta_i \mathbf{q}_{mi}(t) + \dot{\mathbf{q}}_{mi}(t) \big/ \omega_i \tag{2}$$

The modal state-space realization is characterized by the block-diagonal dynamic matrix and the related input and output matrices, (Gawronski, 1998)

$$\mathbf{A} = \mathrm{diag}(\mathbf{A}_{mi}), \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_{m1} \\ \mathbf{B}_{m2} \\ \vdots \\ \mathbf{B}_{mn} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \mathbf{C}_{m1} & \mathbf{C}_{m2} & \cdots & \mathbf{C}_{mn} \end{bmatrix} \tag{3}$$

where i =1,2,…,n, $\mathbf{A}_{mi}$, $\mathbf{B}_{mi}$ and $\mathbf{C}_{mi}$ are 2x2, 2xs and rx2 blocks, respectively. These blocks can be obtained in several different forms and also it is possible to convert in another realization through a linear transformation. One possible form to block $\mathbf{A}_{mi}$ can be written by:

$$\mathbf{A}_{mi} = \begin{bmatrix} -\zeta_i\omega_i & \omega_i \\ -\omega_i(\zeta_i^2 - 1) & -\zeta_i\omega_i \end{bmatrix} \tag{4}$$

Usually, models obtained by FEM demands high number of degrees of freedom (dof). So, the order of the representation is generally very large, causing numeric difficulties. Besides, only a frequency range, or specified modes, is needed to characterize damages. Therefore, the synthesis of a low order plant is fundamental for practical applications.

A reduced-order model is obtained by truncating the states. Let $\mathbf{x}(t)$ and the state ($\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$) be partitioned considering the canonical modal decomposition. From the Jordan canonical form can be obtained:

$$\begin{aligned}\begin{Bmatrix} \dot{\mathbf{x}}_r(t) \\ \dot{\mathbf{x}}_t(t) \end{Bmatrix} &= \begin{bmatrix} \mathbf{A}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_t \end{bmatrix} \begin{Bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_t(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_r \\ \mathbf{B}_t \end{bmatrix} \mathbf{u}(t) \\ \mathbf{y}(t) &= \begin{bmatrix} \mathbf{C}_r & \mathbf{C}_t \end{bmatrix} \begin{Bmatrix} \mathbf{x}_r(t) \\ \mathbf{x}_t(t) \end{Bmatrix}\end{aligned} \tag{6}$$

where $\mathbf{A}_r$ is represented in a block-diagonal form as in Eq. (4). If the analytical model is not available, the dynamic properties can be experimentally determined. There are different identification methods proposed in the literature, as for instance, the parameter identification method by polynomial functions (Daniel *et al.*, 2005); complex exponential (Maia *et al.*, 1996); and eigensystem realization algorithms (ERA) (Juang, 1994; Juang and Minh, 2001). In this paper, these matrices were determined through ERA algorithm.

Damage Detection Approach Using Controllability Grammian Matrices

Controllability and observability are structural properties that carry useful information for structural testing and control, yet are often overlooked by structural engineers (Gawronski, 1998). A structure is controllable if the installed actuators excite all the structural modes of interest. It is observable if the installed sensors detect the motion of all the modes of interest. This information, although essential in many applications, is too limited; it answers the question of excitation or detection in terms of yes or no. The quantitative answer is supplied by the controllability and observability of each mode. This paper explores these properties for proposing a novel approach for structural health monitoring.

Controllability and observability properties of a linear time-invariant system can be heuristically described as follows. The system dynamics described by the state variable $\mathbf{x}(t)$ is excited by the input $\mathbf{u}(t)$ and measured by the output $\mathbf{y}(t)$. However, the input may not be able to excite all states (or, equivalently, move it in an arbitrary direction), and not all states are represented at the output (or, equivalently, the system state cannot be recovered from a record of the output measurements). If the input excites all states, the system is controllable. If all the states are represented in the output, the system is observable.

Controllability, as a coupling between the input and the states, involves the system matrix $\mathbf{A}$ and the input matrix $\mathbf{B}$. A linear system, or the pair ($\mathbf{A}$, $\mathbf{B}$), is controllable at t_0 if it is possible to find a piecewise continuous input $\mathbf{u}(t)$, t Є [t_0,

t_1], that will transfer the system from initial state, $\mathbf{x}(t_0)$, to the origin $\mathbf{x}(t_1) = 0$, at finite $t_1 > t_0$. If this is true for all initial $\mathbf{x}(t_0)$ the system is completely controllable. Otherwise, the system, or the pair (**A**, **B**) is uncontrollable.

There are many criteria to determine system controllability and observability (Kailath, 1980; Zhou, 1995). A linear time-invariant system (**A**, **B**, **C, D**), with s inputs is completely controllable if and only if the N x sN matrix of

$$\mathbf{C_o} = \begin{bmatrix}\mathbf{B} & \mathbf{AB} & \mathbf{A}^2\mathbf{B} & \dots & \mathbf{A}^{N-1}\mathbf{B}\end{bmatrix} \tag{7}$$

has rank N = size(**A**).

An alternative approach uses grammians to determine the system properties. Grammians express the controllability properties qualitatively, and avoid numerical difficulties. The controllability grammian is defined as (Kailath, 1980)

$$\mathbf{W}_c(t) = \int_0^t \exp(\mathbf{A}t)\mathbf{B}\mathbf{B}^T \exp(\mathbf{A}^T t)dt \tag{8}$$

Alternatively, it can be determined from the following system of differential equations

$$\dot{\mathbf{W}}_c(t) = \mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^T + \mathbf{B}\mathbf{B}^T \tag{9}$$

For a stable system, the stationary solutions of the above equations are obtained by assuming $\dot{\mathbf{W}}_c = 0$. In this case, the grammian matrix is determined from the following Lyapunov equation

$$\mathbf{A}\mathbf{W}_c + \mathbf{W}_c\mathbf{A}^T + \mathbf{B}\mathbf{B}^T = \mathbf{0} \tag{10}$$

For a stable **A**, the grammian $\mathbf{W_c}$ is positive definite. Denoting the jth controllability input index of the healthy structure by $\|W_{shj}\| = \text{trace}(\mathbf{W_c})$, and the jth controllability input index of the damaged structure by $\|W_{sdj}\|$. The jth input index to characterize structural damage is defined as the weighted difference between the jth input of the healthy and damaged structure. The input index reflects the impact of that specific structural damage on the jth input position

$$\sigma_{sj} = \frac{\left|\|W_{shj}\|^2 - \|W_{sdj}\|^2\right|}{\|W_{shj}\|^2} \tag{11}$$

NUMERICAL AND EXPERIMENTAL TESTS

The proposed methodology was applied numerically in a beam-like structure, as shown in Fig. 1. The beam was modelled through finite element method with 10 elements and 11 structural nodes. A cantilever beam was considered, resulting 20 structural dofs (2 dofs per node). The physical and geometric properties of the beam are given in Table. 1.

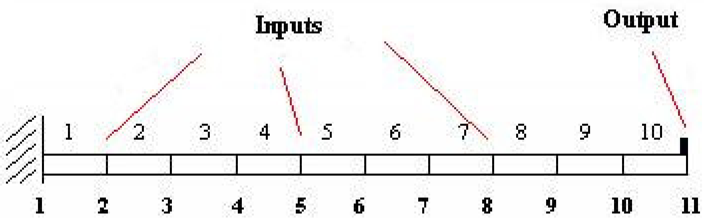

Figure 1 – Finite element model for a cantilever beam.

Table 1 – Dimensions and material properties of the cantilever beam.

Dimensions (m)	Length	Width	Thickness
	0.25	0.02	0.005
Density (kg.m^{-3})	2710		
Young's Modulus (GPa)	70		

The damaged situation was considered as a reduction of 20% in the stiffness of the element 2. One sensor was placed in node 11 and the vertical impulse forces were applied in nodes 2, 5 and 8 (figure 1). Figure 2 shows the input indexes. In this figure, one can see that the index is more sensitive when the force was applied next of the damaged location.

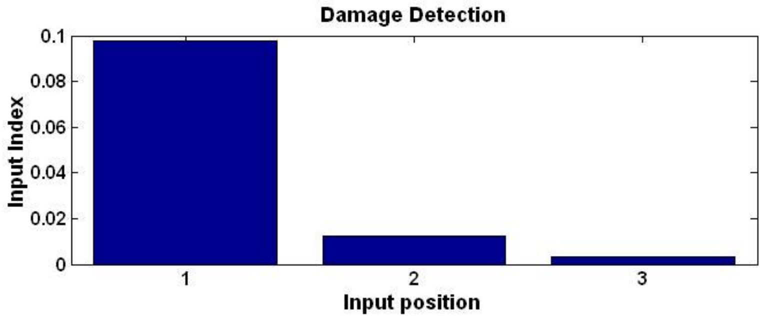

Figure 2 – Input indexes – damage is located.

The same methodology was applied experimentally in a beam-like structure, as shown in Fig. 3. The properties of the beam are given in table 2. Tests were performed by exciting the structure with an impact hammer, Fig. 4b. The output signals were measured with an accelerometer, model 352C22 PCB Piezotronics®. The measurements were obtained five times for each case of damage to verify the repeatability of the results. In these experiments the software SignalCalc ACE® II was used to realize the data acquisition. The parameters of the system were identified by using the Eigensystem Realization Algorithm (ERA) for all three different input excitation positions. The damage was applied by adding additional mass on the beam.

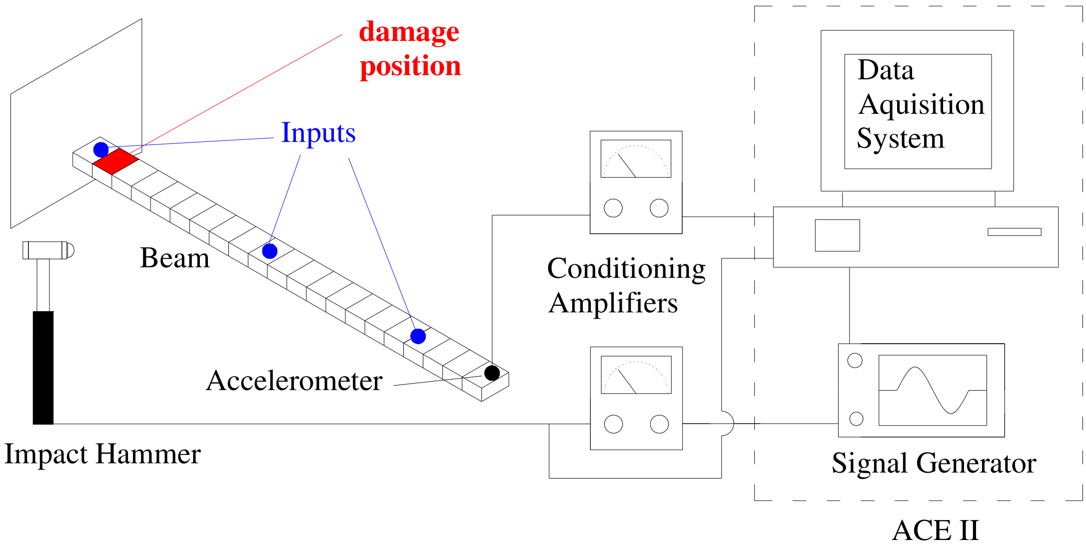

Figure 3 – Disposition of experimental setup.

Table 2 – Beam properties and dimensions – experimental application.

Young Modulus (GPa)	210
Area of transversal session (mm^2)	185
Length L (mm)	420
Density (Kg/m^3)	7800

Figure 4a shows the beam structure and the equipment used in the experimental setup. In figure 4b, one can see the impact hammer and the accelerometer used for signal acquisition.

(a) Beam structure and experimental setup; the additional mass = 16g is shown in the circle.

(b) beam structure, impact hammer in first position and accelerometer.

Figure 4 – Details of the experimental setup.

Figures 5 and 6 show the signals in time domain for the input excitation and the output for the intact and damaged structure, respectively. These signals were obtained with excitation in the first input position; the others are similar. Using these signals the system matrices were identified by ERA for each input position. Figures 7 to 9 show the frequency response functions before and after the damage was introduced.

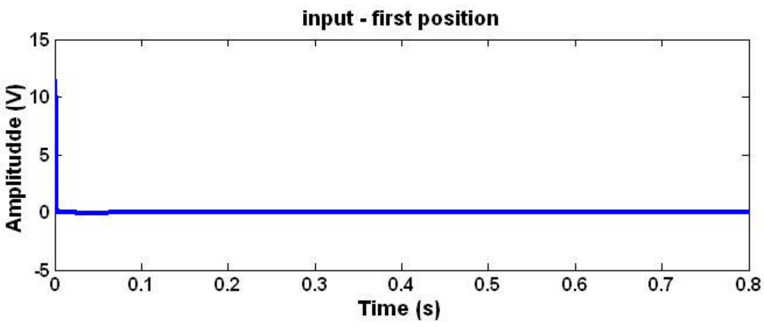

Figure 5 – Excitation in the structure through the impact hammer.

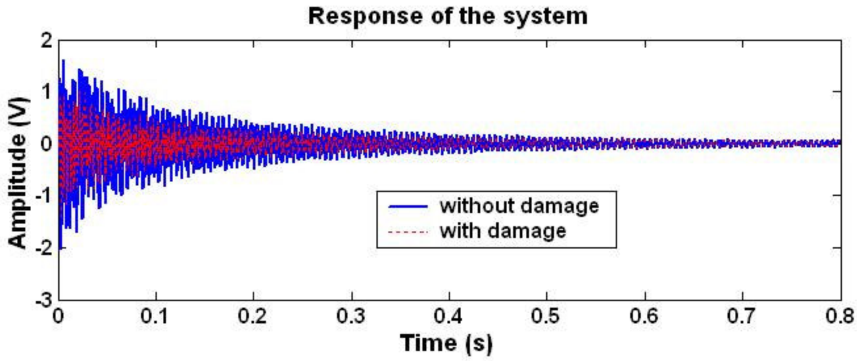

Figure 6 – Response of the system with excitation in the first input position – intact and damaged structure.

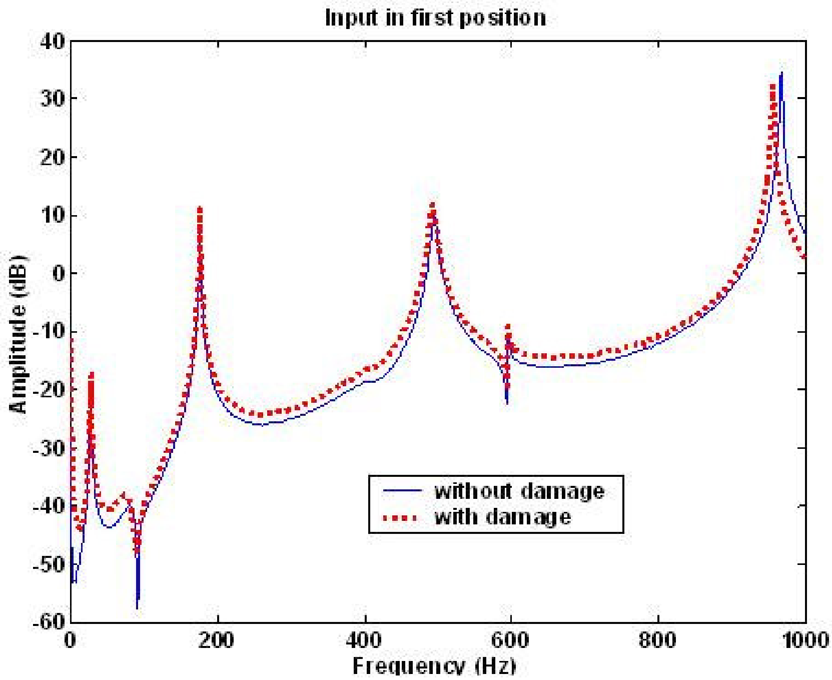

Figure 7 – Experimental Frequency Response Function of the structure – excitation in the first position.

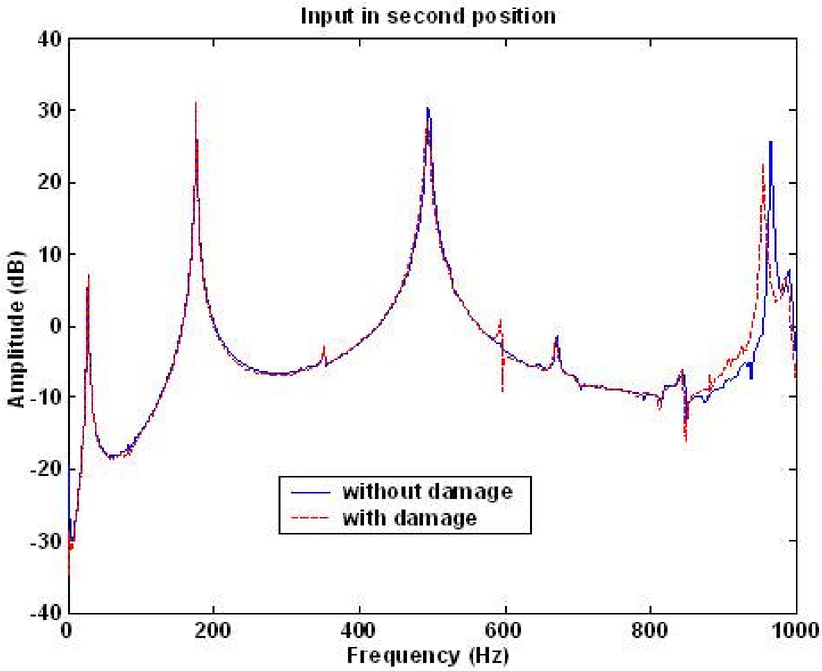

Figure 8 – Experimental Frequency Response Function of the structure – excitation in the second position.

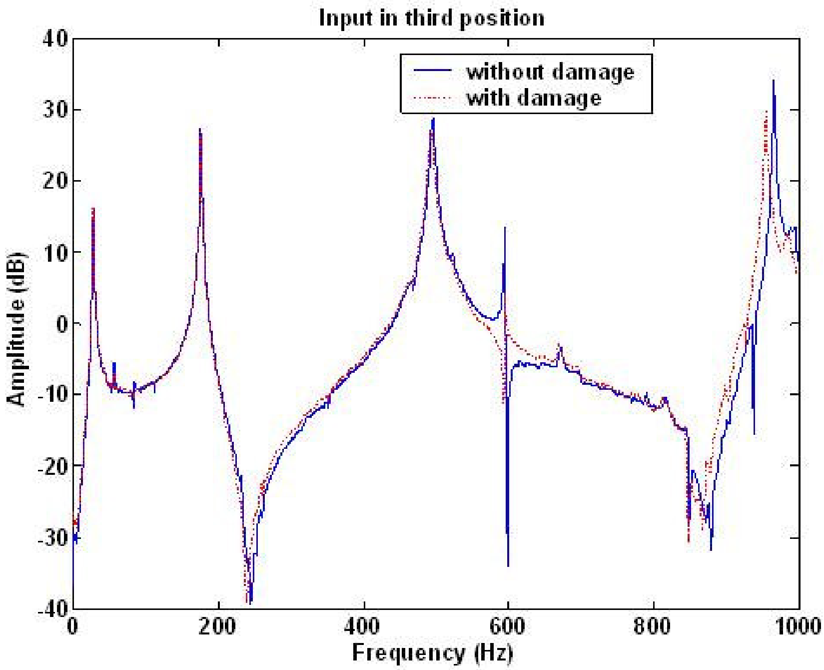

Figure 9 – Experimental Frequency Response Function of the structure – excitation in the third position.

Using the grammian input indexes, equation (11), the damage was experimentally located. Figure 10 shows the input indexes calculated for all three positions of inputs for the intact and the damaged structure. One can see that the damage location was correctly identified.

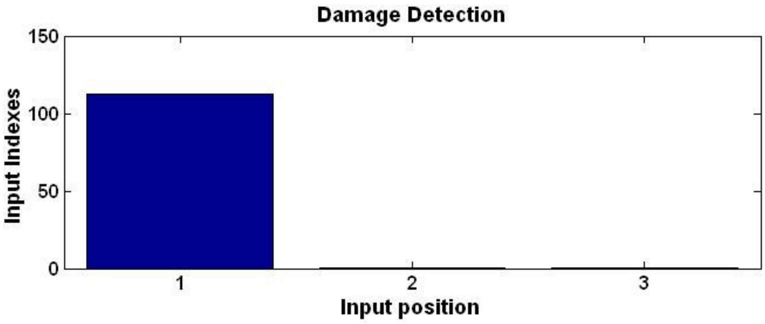

Figure 10 – Input grammian indexes, additional mass = 16g.

FINAL REMARKS

In literature, the main applications of controllability grammian matrices have been for control design and optimal placement of actuators in smart structures. This paper presents a novel approach for structural health monitoring involving controllability concepts. SHM is needed for an intelligent maintenance procedure, where the structure is repaired only if it is really necessary.

In practical situations, usually, the number of sensors is limited, so, this approach permits to locate the damage using a small number of sensors. It is also worth to define a threshold value to specify initial damages. This value should be taken from experience or using simulated data, in order to avoid that noises or variation in operational conditions should be misinterpreted as damage occurrence.

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